# Physics 5714 – Problem set 6

1. For the matrix $A$ below, either diagonalize or put in Jordan normal form. In other words, compute a matrix $P$ and a matrix $\Lambda$ that is either diagonal or in Jordan normal form, such that $A = P\Lambda P^{-1}$.

$$A = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 3/2 \end{bmatrix}$$

2. For the matrix $A$ below, either diagonalize or put in Jordan normal form. In other words, compute a matrix $P$ and a matrix $\Lambda$ that is either diagonal or in Jordan normal form, such that $A = P\Lambda P^{-1}$.

$$A = \begin{bmatrix} 1 & 1 \\ -4 & 5 \end{bmatrix}$$

3. For the matrix $A$ below, either diagonalize or put in Jordan normal form. In other words, compute a matrix $P$ and a matrix $\Lambda$ that is either diagonal or in Jordan normal form, such that $A = P\Lambda P^{-1}$.

$$A = \begin{bmatrix} 5/2 & -1/2 & 0 \\ 1/2 & 3/2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

4. Compute

$$\exp \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 3/2 \end{bmatrix}$$

5. Compute

$$\exp \begin{bmatrix} 1 & 1 \\ -4 & 5 \end{bmatrix}$$

6. (AWH 2.2.34) If $A$, $B$ are Hermitian matrices, show that $(AB + BA)$, $i(AB - BA)$ are also Hermitian.

7. (AWH 2.2.37) Two matrices $A$, $B$ are each Hermitian. Find a necessary and sufficient condition for their product $AB$ to be Hermitian.

8. Two matrices $U$, $H$ are related by $U = \exp(iaH)$ with $a$ real. If $H$ is Hermitian, show that $U$ is unitary.