New Evidence for $(0,2)$ Target Space Duality

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Introduction

- Goal: further explore target space duality
- Generate examples with non-trivial D/F term potential $\Rightarrow$
  - Count matter spectrum as previous work did
  - Compare effective potential and explore vacuum spaces
  - Study structure group and enhanced symmetries
- More work
  - Provide complete list of target space dual chains
  - Develop new tools (repeated entry, etc.)

Review of Target Space Duality

- Abelian, massive 2D theory $\rightarrow$ $(0,2)$ GLSM
- Multiple $U(1)$ gauge fields $A^{(\alpha)}$ with $\alpha = 1, ..., r$
- Chiral superfields: $\{X_i | i = 1, ..., d\}$ with $U(1)$ charges $Q_i^{(\alpha)}$, and $\{P_l | l = 1, ..., \gamma\}$ with $U(1)$ charges $-M_l^{(\alpha)}$.
- Fermi superfields: $\{\Lambda^a | a = 1, ..., \delta\}$ with charges $N_a^{(\alpha)}$, and $\{\Gamma^j | j = 1, ..., c\}$ with charges $-S_j^{(\alpha)}$.
- Gauge and gravitational anomaly cancellation

$$\sum_{a=1}^{\delta} N_a^{(\alpha)} = \sum_{l=1}^{\gamma} M_l^{(\alpha)} \qquad \sum_{i=1}^{d} Q_i^{(\alpha)} = \sum_{j=1}^{c} S_j^{(\alpha)}$$

$$\sum_{l=1}^{\gamma} M_l^{(\alpha)} M_l^{(\beta)} - \sum_{a=1}^{\delta} N_a^{(\alpha)} N_a^{(\beta)} = \sum_{j=1}^{c} S_j^{(\alpha)} S_j^{(\beta)} - \sum_{i=1}^{d} Q_i^{(\alpha)} Q_i^{(\beta)} \tag{1}$$

for all $\alpha, \beta = 1, ..., r$.

Put the above data in a table

$$
\begin{array}{|cccc||cccc|}
\hline
\multicolumn{4}{|c||}{x_i} & \multicolumn{4}{c|}{\Gamma^j} \\
\hline
Q_1^{(1)} & Q_2^{(1)} & \dots & Q_d^{(1)} & -S_1^{(1)} & -S_2^{(1)} & \dots & S_c^{(1)} \\
Q_1^{(2)} & Q_2^{(2)} & \dots & Q_d^{(2)} & -S_1^{(2)} & -S_2^{(2)} & \dots & S_c^{(2)} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
Q_1^{(r)} & Q_2^{(r)} & \dots & Q_d^{(r)} & -S_1^{(r)} & -S_2^{(r)} & \dots & S_c^{(r)} \\
\hline
\end{array}
$$

$$
\begin{array}{|cccc||cccc|}
\hline
\multicolumn{4}{|c||}{\Lambda^a} & \multicolumn{4}{c|}{p_l} \\
\hline
N_1^{(1)} & N_2^{(1)} & \dots & N_\delta^{(1)} & -M_1^{(1)} & -M_2^{(1)} & \dots & -M_\gamma^{(1)} \\
N_1^{(2)} & N_2^{(2)} & \dots & N_\delta^{(2)} & -M_1^{(2)} & -M_2^{(2)} & \dots & -M_\gamma^{(2)} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
N_1^{(r)} & N_2^{(r)} & \dots & N_\delta^{(r)} & -M_1^{(r)} & -M_2^{(r)} & \dots & -M_\gamma^{(r)} \\
\hline
\end{array}
\quad (2)
$$

GLSM is defined via a superpotential:

$$S = \int d^2 z d\theta \Big[\sum_j \Gamma^j G_j(x_i) + \sum_{l,a} P_l \Lambda^a F_a^l(x_i) \Big] \tag{3}$$

G_j and F_a^l are quasi-homogeneous polynomials with multi-degrees:

G^j			
S_1	S_2	$\dots$	S_c

$F_a{}^l$			
$M_1 - N_1$	$M_1 - N_2$	$\dots$	$M_1 - N_\delta$
$M_2 - N_1$	$M_2 - N_2$	$\dots$	$M_2 - N_\delta$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$M_\gamma - N_1$	$M_\gamma - N_2$	$\dots$	$M_\gamma - N_\delta$

(4)

F_a^l satisfies transversality condition: all $F_a^l(x) = 0$ only when all $x_i = 0$
F-term potential:

$$V_F = \sum_j \left| G_j(x_i) \right|^2 + \sum_a \Big| \sum_l p_l F_a^l(x_i) \Big|^2 \tag{5}$$

D-term potential:

$$V_D = \sum_{\alpha=1}^{r} \left(\sum_{i=1}^{d} Q_i^{(\alpha)} |x_i|^2 - \sum_{l=1}^{\gamma} M_l^{(\alpha)} |p_l|^2 - \xi^{(\alpha)} \right)^2 \tag{6}$$

Fayet-Iliopoulos (FI) parameter controls the phase, consider a single U(1):

For $\xi > 0$, not all x_i are zero thus not all F_a are zero, $G_j(x_i) = 0$ and $< p >= 0 \Rightarrow$ "geometric" phase
(X, V) where X is a CY and V is a bundle, $V = \frac{ker(F_a^l)}{im(E_i^a)}$ in the monad:

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r\nu} \xrightarrow{\otimes E_i^a} \bigoplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a^l} \bigoplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \to 0 \quad (7)$$

For $\xi < 0$, $< p > \neq 0$ thus all $< x_i >= 0 \Rightarrow$ "nongeometric" phase
Landau-Ginzburg orbifold with a superpotential:

$$\mathcal{W}(x_i, \Lambda^a, \Gamma^i) = \sum_j \Gamma^j G_j(x_i) + \sum_a \Lambda^a F_a(x_i) \quad (8)$$

For multiple U(1)'s, hybrid phase

Target space duality

For Landau-Ginzburg orbifold with a superpotential:

$$\mathcal{W}(x_i, \Lambda^a, \Gamma^i) = \sum_j \Gamma^j G_j(x_i) + \sum_a \Lambda^a F_a(x_i) \tag{9}$$

Observation (Distler, Kachru): An exchange/relabeling of the functions G_j and F_a will not affect the Landau-Ginzburg model, as long as anomaly cancellation conditions are satisfied.

Procedure:
Geometric to nongeometric phase: find phase with one $\langle p_l \rangle \neq 0$ for some l, say $l = 1$.
Rescale: $\tilde{\Lambda}^{a_i} := \frac{\Gamma^{j_i}}{\langle p_1 \rangle}$, $\tilde{\Gamma}^{j_i} := \langle p_1 \rangle \Lambda^{a_i}$ s.t. $\sum_i ||G_{j_i}|| = \sum_i ||{F_{a_i}}^1||$.
Move to a region where Λ^{a_i} appear only with P_1, i.e. choose $F^l_{a_i} = 0 \; \forall l \neq 1$, $i = 1, \dots k$.
Leave non-geometric phase: $||\tilde{\Lambda}^{a_i}|| = ||\Gamma^{j_i}|| - ||P_1||$ and $||\tilde{\Gamma}^{j_i}|| = ||\Lambda^{a_i}|| + ||P_1||$, return to a generic pt. and get new $(\tilde{X}, \tilde{V})$.

Example

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−2	−2	1	0	0	2	−3
1	1	1	2	2	2	0	−4	−5	0	1	1	6	−8

(10)

- Here $||G_1|| = (2,4)$, $||G_2|| = (2,5)$, $||F_1^1|| = (2,8)$, $||F_2^1|| = (3,7)$, $||F_3^1|| = (3,7)$, $||F_4^1|| = (1,2)$.
- Sum of third and fourth F equals sum of two G's.
- Redefine: $\tilde{\Gamma}^1 = \langle p_1 \rangle \Lambda^3$, $\tilde{\Gamma}^2 = \langle p_1 \rangle \Lambda^4$, $\tilde{\Lambda}^3 = \frac{\Gamma^1}{\langle p_1 \rangle}$, $\tilde{\Lambda}^4 = \frac{\Gamma^2}{\langle p_1 \rangle}$, $\tilde{G} = F_3^1$, $\tilde{G}_2 = F_4^1$, $\tilde{F}_3^1 = G_1$, $\tilde{F}_4^1 = G_2$
- then the new geometry is given by: $||\tilde{G}_1|| = (3,7)$, $||\tilde{G}_2|| = (1,2)$, $||\tilde{F}_3^1|| = (2,4)$, $||F_4^1|| = (2,5)$

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−3	−1	1	0	1	1	−3
1	1	1	2	2	2	0	−7	−2	0	1	4	3	−8

(11)

Compare degree of freedom:

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−2	−2	1	0	0	2	−3
1	1	1	2	2	2	0	−4	−5	0	1	1	6	−8

(10)

$dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 2 + 68 + 322 = 392,$
$h^*(V) = (0, 120, 0, 0)$

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−3	−1	1	0	1	1	−3
1	1	1	2	2	2	0	−7	−2	0	1	4	3	−8

(11)

$dim(\widetilde{\mathcal{M}}_0) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 2 + 95 + 295 = 392,$
$h^*(\widetilde{V}) = (0, 120, 0, 0)$

Landscape scan by Blumenhagen + Rahn, agreement in nearly all ∼80,000 examples.

TS duality with extra U(1)

Add a new coord y_1 with multi-degree B and a hypersurface of degree B.
Perform previous procedure (e.g. $||B|| = ||F_1^1|| + ||F_2^1|| - S_1$)
Resolve singularities (Distler, Greene, Morrison) by formally adding a $\mathbb{P}^1$ (another coord y_2)
Set constraint $G^B = y_1 = 0$ to eliminate y_1. Use additional $U(1)$ and D-term to fix y_2 to a real constant.$\leftrightarrow X\times$ a single pt.

x_1	$\dots$	x_d	y_1	y_2	Γ^1	$\dots$	Γ^c	Γ^B
0	$\dots$	0	1	1	0	$\dots$	0	-1
Q_1	$\dots$	Q_d	B	0	$-S_1$	$\dots$	$-S_c$	$-B$

Λ^1	Λ^2	$\dots$	Λ^δ	p_1	p_2	$\dots$	p_γ
0	0	$\dots$	0	-1	0	$\dots$	0
N_1	N_2	$\dots$	N_δ	$-M_1$	$-M_2$	$\dots$	$-M_\gamma$

End up with new geometry:

x_1	$\dots$	x_d	y_1	y_2	$\tilde{\Gamma}^1$	$\dots$	Γ^c	$\tilde{\Gamma}^B$
0	$\dots$	0	1	1	-1	$\dots$	0	-1
Q_1	$\dots$	Q_d	B	0	$-(M_1 - N_1)$	$\dots$	$-S_c$	$-(M_1 - N_2)$

$\tilde{\Lambda}^1$	$\tilde{\Lambda}^2$	$\dots$	Λ^δ	p_1	p_2	$\dots$	p_γ
1	0	$\dots$	0	-1	0	$\dots$	0
0	$M_2 - B$	$\dots$	N_δ	$-M_1$	$-M_2$	$\dots$	$-M_\gamma$

More TS Duality with Redundant Entry

We consider $V = ker(F_a^l)$ defined by a short exact sequence

$$0 \to V \to \bigoplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{F_a^l} \bigoplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \to 0 \tag{12}$$

Adding a redundant entry can lead to non trivial results after TS duality

$$\begin{aligned} 0 \to V \to B \xrightarrow{F} C \to 0 \\ 0 \to V' \to B \oplus L \xrightarrow{F'} C \oplus L \to 0 \end{aligned} \tag{13}$$

where the new defining map F' is given by

$$F' = \begin{pmatrix} F & \alpha \\ \beta & \mathbb{C} \end{pmatrix} \tag{14}$$

This repeated L is bounded, because of well-defined map F_a^l;
Too many L's won't enroll in the transformation so keep redundant.

Bundle stability/holomorphy and D/F-term

$N = 1$ Supersymmetry in $4D \Rightarrow$ Hermitian-Yang Mills Eqns

$$F_{ab} = F_{\overline{a}\overline{b}} = g^{a\overline{b}} F_{\overline{b}a} = 0 \tag{15}$$

$g^{a\overline{b}} F_{\overline{b}a} = 0 \Leftrightarrow$ Donaldson-Uhlenbeck-Yau Thm: V is stable (poly-stable).
$F_{ab} = F_{\overline{a}\overline{b}} = 0 \Leftrightarrow V$ is holomorphic.

Stability $\Leftrightarrow$ $4D$ D-terms

Holomorphy $\Leftrightarrow$ $4D$ F-terms

Our work:
Test TS duality with bundles not stable/holomorphic everywhere
See if the stability/holomorphy properties (etc.) carry through

D-term and stability

Thanks to recent progress (Sharpe, Anderson, Gray, Lukas, Ovrut)
The slope, $\mu(V)$, of a vector bundle is

$$\mu(V) \equiv \frac{1}{\mathrm{rk}(V)} \int_X c_1(V) \wedge \omega \wedge \omega \tag{16}$$

where $\omega = t^k \omega_k$ is the Kahler form on X (ω_k a basis for $H^{1,1}(X)$).

V is Stable if for every sub-sheaf $\mathcal{F} \subset V$ s.t. $\mu(\mathcal{F}) < \mu(V)$
V is Poly-stable if $V = \bigoplus_i V_i$, where V_i stable s.t. $\mu(V) = \mu(V_i)\ \forall i$. Problem: hard to find all sub-sheaves.
V is stable if $\forall$ sub-line bundles $\mathcal{L}$, $\mu(\mathcal{L}) < \mu(\wedge^k V) = 0$, where $0 < k < n$.

If there is a sub-bundle $\mathcal{L} = \mathcal{O}(a, b)$, where $ab < 0$, then V is stable in the region

$$\mu(\mathcal{L}) = \frac{1}{rk(\mathcal{L})} d_{ijk} c_1^i(\mathcal{L}) t^j t^k = \frac{1}{rk(\mathcal{L})} s_i c_1^i(\mathcal{F}) = s_1 a + s_2 b < 0 \tag{17}$$

Consider the following rank 5 bundle V on a CICY: $\begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^3 \end{matrix} \left[\begin{matrix} 2 \\ 4 \end{matrix} \right]$, anomaly cancellation condition: $c_1(TX) = c_1(V) = 0$, $c_2(TX) = c_2(V)$

$$\begin{array}{|cccccc|c||ccccccc|cc|} \hline & & x_i & & & & \Gamma^j & & & & \Lambda^a & & & & p_l & \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 & 1 & 1 & -4 & 1 & 1 & 1 & 2 & -1 & 1 & 2 & -4 & -3 \\ \hline \end{array} \tag{18}$$

The bundle V is given by SES:

$$\begin{aligned} 0 \to V \to \mathcal{O}(0,1)^{\oplus 3} \oplus \mathcal{O}(0,2) \oplus \mathcal{O}(1,-1) \oplus \mathcal{O}(1,1) \oplus \mathcal{O}(1,2) \oplus \mathcal{O}(3,2) \\ \to \mathcal{O}(3,2) \oplus \mathcal{O}(1,4) \oplus \mathcal{O}(2,3) \to 0 \end{aligned} \tag{19}$$

The "maximally destabilizing" sub-bundle is a rank 4 bundle Q_4 with $c_1(Q_4) = -J_1 + J_2$, so that

$$0 \to Q_4 \to V \to \mathcal{L} \to 0 \tag{20}$$

where

$$\mathcal{L} = \mathcal{O}(1,-1) \tag{21}$$

V is stable in region $s_2 < s_1$.

On the stability wall ($s_2 = s_1$), V is poly-stable and can break into a sum of two pieces: $V = Q_4 \oplus \mathcal{L}$. The structure group of an $SU(5)$ will become $S[U(4) \times U(1)] \simeq SU(4) \times SU(1) \times U(1)$.

To explore 4D vacuum space through D-term potential (Sharpe, Lukas, Stelle, Blumenhagen, Weigand, Honecker, ...):

$$D^{U(1)} \sim \frac{\mu(\mathcal{F})}{Vol(X)} - \frac{1}{2} \sum_i Q_i G_{L\bar{M}} C_i^L \bar{C}_i^{\bar{M}} \tag{22}$$

In this case, the D-term looks like:

$$D^{U(1)} \sim \frac{\mu(Q_4)}{Vol(X)} - \frac{1}{2} q_1 G_{L\bar{M}} C_1^L C_1^{\bar{M}} + \frac{1}{2} q_2 G_{L\bar{M}} C_2^L C_2^{\bar{M}} \tag{23}$$

with

$$C_1 \in H^1(X, \mathcal{L} \otimes Q_4^*) \quad C_2 \in H^1(X, Q_4 \otimes \mathcal{L}^*) \tag{24}$$

In region V stable, $< C_1 >= 0, < C_2 > \neq 0$.

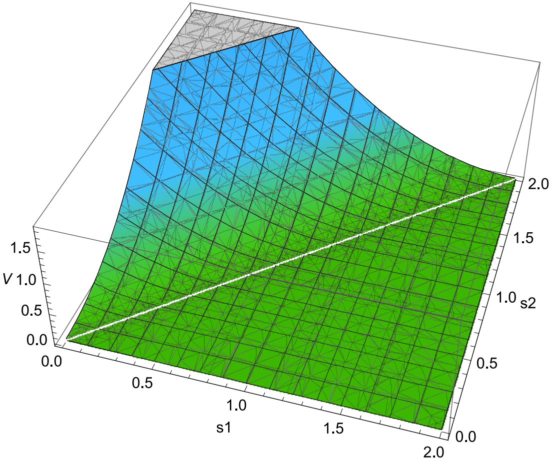

Figure: *D-term potential for bundle V, stable in region* $s_2 < s_1$

Question

Start from the wall, take infinitesimal fluctuation to leave the wall, and take TS duals, is this fluctuation preserved?

$$
\begin{array}{ccccc}
 & V_1 & \xrightarrow{dual} & \widetilde{V}_1 & \\
\langle C \rangle & \downarrow & & \downarrow & \langle \tilde{C} \rangle ?? \\
 & V_2 & \xrightarrow{dual} & \widetilde{V}_2 &
\end{array}
$$

Deform V_1 to get V_2, and take duals, is $\tilde{V}_2$ the same deformation of $\tilde{V}_1$?

How to build the geometry?

Example

Start from an example which is only stable on a line, where $c_2(V) = c_2(TX) = \{24, 44\}$

x_i						Γ^j	Λ^a								p_l		
1	1	0	0	0	0	−2	1	−1	0	0	2	1	1	2	−3	−1	−2
0	0	1	1	1	1	−4	−1	1	1	1	1	2	2	2	−2	−4	−3

(25)

$$\begin{aligned} dim(\mathcal{M}_0) &= h^{1,1}(X) + h^{2,1}(X) + h^1(X, End_0(V)) = 2 + 86 + 340 = 428 \\ dim(\mathcal{M}_1) &= dim(\mathcal{M}_0) - 1 = 427 \quad \text{(restricted on the wall)} \end{aligned} \tag{26}$$

the stability condition writes:

$$\begin{aligned} 0 \to Q_4 \to V \to \mathcal{O}(1, -1) \to 0 \\ 0 \to \tilde{Q}_4 \to V \to \mathcal{O}(-1, 1) \to 0 \end{aligned} \tag{27}$$

On the stability wall, V breaks into three parts:

$$V \to U_3 \oplus L \oplus L^\vee \quad \text{where } L = \mathcal{O}(1, -1) \tag{28}$$

Structure group: seems like $SU(5)$ bundle $\Rightarrow SU(5)$ 4d effective theory
Non-Abelian Enhancement: $S[U(1) \times U(1)] \times SU(3) \subset E_8 \Rightarrow SU(6) \times U(1)$, with $U(1)$ symmetry visible in 4d theory.

Field	Cohom.	Multiplicity	Field	Cohom.	Multiplicity
$\mathbf{1}_{+2}$	$H^1(L \otimes L)$	0	$\mathbf{1}_{-2}$	$H^1(L^\vee \otimes L^\vee)$	10
$\mathbf{15}_0$	$H^1({U_3}^\vee)$	0	$\overline{\mathbf{15}}_0$	$H^1(U_3)$	80
$\mathbf{20}_{+1}$	$H^1(L)$	0	$\mathbf{20}_{-1}$	$H^1(L^\vee)$	0
$\mathbf{6}_{+1}$	$H^1(L \otimes U_3)$	72	$\mathbf{6}_{-1}$	$H^1(L^\vee \otimes U_3)$	90
$\overline{\mathbf{6}}_{+1}$	$H^1(L \otimes {U_3}^\vee)$	0	$\overline{\mathbf{6}}_{-1}$	$H^1(L^\vee \otimes {U_3}^\vee)$	2
$\mathbf{1}_0$	$H^1(U_3 \otimes {U_3}^\vee)$	166			

Table: *Particle content of the $SU(6) \times U(1)$ theory associated to the bundle along its reducible and poly-stable locus $V = \mathcal{O}(-1,1) \oplus \mathcal{O}(1,-1) \oplus U_3$ (i.e. on the stability wall).*

Target Space Dual

A target space dual with $c_2(\tilde{V}) = c_2(T\tilde{X}) = \{24, 24, 44\}$

x_i	Γ^j		Λ^a								p_l		
$\mathbb{P}^1$	-1	-1	0	0	1	0	0	0	0	0	0	0	-1
$\mathbb{P}^1$	-2	0	1	-1	0	0	2	1	1	2	-3	-1	-2
$\mathbb{P}^3$	-2	-2	-1	1	-1	1	3	2	2	2	-2	-4	-3

(29)

$$
\begin{aligned}
dim(\tilde{\mathcal{M}}_0) &= h^{1,1}(\tilde{X}) + h^{2,1}(\tilde{X}) + h^1(X, End_0(\tilde{V})) = 3 + 55 + 370 = 428 \\
dim(\tilde{\mathcal{M}}_1) &= dim(\tilde{\mathcal{M}}_0) - 1 = 427 \quad \text{(restricted on the wall)}
\end{aligned} \tag{30}
$$

the stability condition writes:

$$
\begin{aligned}
&0 \to \tilde{\mathcal{F}}_1 \to \tilde{V} \to \mathcal{O}(0,1,-1) \to 0 && c_1(\tilde{\mathcal{F}}_1) = (0,-1,1) \\
&0 \to \tilde{\mathcal{F}}_2 \to \tilde{V} \to \mathcal{O}(0,-1,1) \to 0 && c_1(\tilde{\mathcal{F}}_2) = (0,1,-1) \\
&0 \to \tilde{\mathcal{F}}_3 \to \tilde{V} \to \mathcal{O}(1,0,-1) \to 0 && c_1(\tilde{\mathcal{F}}_3) = (-1,0,1)
\end{aligned} \tag{31}
$$

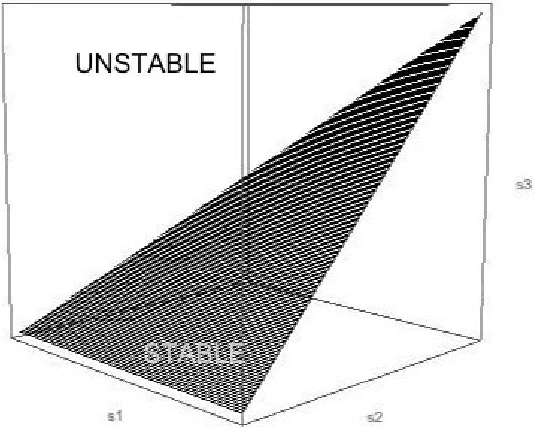

Figure: *Stable region for $\tilde{V}$ $(s_3 < s_1$ and $s_2 = s_3)$*

V breaks the same way: $\tilde{V} \to \tilde{L} \oplus \tilde{L}^\vee \oplus \tilde{U}_3$
Identical Non-Abelian Symmetry Enhancement: $S[U(1) \times U(1)] \times SU(3) \subset E_8$
$\Rightarrow SU(6) \times U(1)$

Field	Cohom.	Multiplicity	Field	Cohom.	Multiplicity
$\mathbf{1}_{+2}$	$H^1(\tilde{L} \otimes \tilde{L})$	0	$\mathbf{1}_{-2}$	$H^1(\tilde{L}^\vee \otimes \tilde{L}^\vee)$	10
$\mathbf{15}_0$	$H^1(\tilde{U}_3^\vee)$	0	$\overline{\mathbf{15}}_0$	$H^1(\tilde{U}_3)$	80
$\mathbf{20}_{+1}$	$H^1(\tilde{L})$	0	$\mathbf{20}_{-1}$	$H^1(\tilde{L}^\vee)$	0
$\mathbf{6}_{+1}$	$H^1(\tilde{L} \otimes \tilde{U}_3)$	72	$\mathbf{6}_{-1}$	$H^1(\tilde{L}^\vee \otimes \tilde{U}_3)$	90
$\overline{\mathbf{6}}_{+1}$	$H^1(\tilde{L} \otimes \tilde{U}_3^\vee)$	0	$\overline{\mathbf{6}}_{-1}$	$H^1(\tilde{L}^\vee \otimes \tilde{U}_3^\vee)$	2
$\mathbf{1}_0$	$H^1(\tilde{U}_3 \otimes \tilde{U}_3^\vee)$	196			

Table: *Particle content of the $SU(6) \times U(1)$ theory* $\leftrightarrow \tilde{V} = \tilde{L} \oplus \tilde{L}^\vee \oplus \tilde{U}_3$.

Branch Structure

Search for breaking: $SU(6) \to SU(5)$ stable off the wall, i.e. glue the components together. $(L + L^{\vee} + U_3 \to V_5)$
But how? Consider D-term potential:

$$D_{GS}^{U(1)} \sim \frac{3}{16} \frac{\epsilon_S {\epsilon_R}^2 \mu(L^{\vee})}{{\kappa_4}^2 \mathcal{V}} - \frac{1}{2} \left((-2)|C_{-2,0}|^2 + (+1)|C_{+1,-5}|^2 + (-1)|C_{-1,-5}|^2 + (-1)|C_{-1,+5}|^2 \right) \tag{32}$$

$$D_{SU(6)}^{U(1)} \sim \frac{1}{2} \left((-5)|C_{+1,-5}|^2 + (-5)|C_{-1,-5}|^2 + (+5)|C_{-1,+5}|^2 \right) \tag{33}$$

Previous case corresponds to $< C >= 0$ so $\mu(\mathcal{F}) = 0$.

To find new branch, choose $< C > \neq 0$, take the second D-term potential to 0 and substitute into the first D-term potential, to make it to 0 requires: $\mu(L^{\vee}) < 0$, Observation: $L^\vee = \mathcal{O}(-1,1)$ itself can be written as a monad:

$$0 \to L_{new} \to \mathcal{O}(0,1)^{\oplus 2} \xrightarrow{g} \mathcal{O}(1,1) \to 0 \tag{34}$$

because line bundles on CY 3-folds are classified by their first Chern class (here $c_1(L_{new}) = -J_1 + J_2$).

Replace $L^\vee$ with new expression and mix them up:

x_i	Γ^j	Λ^a									p_l			
$\mathbb{P}^1$	-2	1	0	0	0	0	2	1	1	2	-1	-3	-1	-2
$\mathbb{P}^3$	-4	-1	1	1	1	1	1	2	2	2	-1	-2	-4	-3

(35)

Degree of freedom count gives:

$$dim(\mathcal{M}_0) = dim(\mathcal{M}_1) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 2 + 86 + 338 = 426 \tag{36}$$

compared to 427 of the on-wall branch.

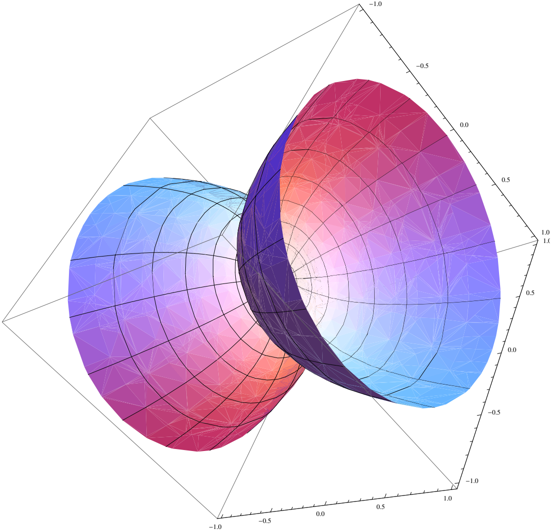

Figure: *Two bundle moduli spaces touch*

Similarly replace $\tilde{L} = \mathcal{O}(0,-1,1)$ with new expression

$$0 \to \tilde{L}_{new} \to \mathcal{O}(0,0,1)^{\oplus 2} \xrightarrow{\tilde{g}} \mathcal{O}(0,1,1) \to 0 \tag{37}$$

This leads at last to the bundle

x_i	Γ^j		Λ^a									p_l			
$\mathbb{P}^1$	−1	−1	0	0	0	1	0	0	0	0	0	0	0	0	−1
$\mathbb{P}^1$	−2	0	1	0	0	0	0	2	1	1	2	−1	−3	−1	−2
$\mathbb{P}^3$	−2	−2	−1	1	1	−1	1	3	2	2	2	−1	−2	−4	−3

(38)

Again degree of freedom count gives:

$$dim(\mathcal{M}_0) = dim(\mathcal{M}_1) = 426 \tag{39}$$

Interestingly, the off-wall branch of the TS dual is also a TS dual of the off-wall branch, which gives the commutative diagram:

$$\begin{array}{ccccc} & V_1 & \xrightarrow{dual} & \widetilde{V}_1 & \\ \langle C \rangle & \downarrow & & \downarrow & \langle \tilde{C} \rangle \\ & V_2 & \xrightarrow{dual} & \widetilde{V}_2 & \end{array}$$

Isomorphic geometry in TS duality

Compare numbers of TS duals of two manifolds of the same homotopy type: Among all TS duals of the original bundle on the wall, 3 and 5 results on the following two manifolds, respectively: $\begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^1 \\ \mathbb{P}^3 \end{matrix} \left[\begin{matrix} 1 & 1 \\ 0 & 2 \\ 2 & 2 \end{matrix} \right]$ and $\begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^1 \\ \mathbb{P}^3 \end{matrix} \left[\begin{matrix} 1 & 1 \\ 1 & 1 \\ 2 & 2 \end{matrix} \right]$.

TS duality can result in the same manifold: consider the following dual to our original bundle:

x_i	Γ^j		Λ^a									p_l			
$\mathbb{P}^1$	-1	-1	1	0	0	0	0	0	0	0	0	0	0	-1	0
$\mathbb{P}^1$	-1	-1	0	-1	0	0	2	1	1	2	2	-3	-1	-2	-1
$\mathbb{P}^3$	-4	0	-1	1	1	1	1	2	2	2	3	-2	-4	-3	-3

(40)

base manifold is the same as the $\{2, 4\}$ on $\mathbb{P}^1 \times \mathbb{P}^3$, but bundle is not trivially related to the original V (need some lemma to prove this).

Two seemingly different bundle can be related by an isomorphism, e.g.

x_i						Γ^j	Λ^a							p_l	
1	1	0	0	0	0	−2	0	0	0	0	1	1	1	−1	−2
0	0	1	1	1	1	−4	1	1	1	2	−1	1	2	−4	−3

(41)

This bundle shares *identical topology* with the bundle of the off-wall branch,

x_i	Γ^j	Λ^a									p_l			
$\mathbb{P}^1$	−2	1	0	0	0	0	2	1	1	2	−1	−3	−1	−2
$\mathbb{P}^3$	−4	−1	1	1	1	1	1	2	2	2	−1	−2	−4	−3

(35)

because these two bundle share a stability wall and stable in the same region:

$$
\begin{aligned}
0 \to Q_4 \to V_5 \to \mathcal{O}(1,-1) \to 0 \\
0 \to U_4 \to V_5' \to \mathcal{O}(1,-1) \to 0
\end{aligned} \tag{42}
$$

a calculation gives:

$$dim(Hom(\mathcal{Q}_4, U_4)) = 1 \tag{43}$$

Corollary: (Morphism Lemma) if $\phi : V_1 \to V_2$ homomorphism, $rk(V_1) = rk(V_2)$, $c_1(V_1) = c_1(V_2)$, V_1 or V_2 stable, then ϕ is an isomorphism.

F-term and holomorphy

Next consider 4D F-terms in a supersymmetric Minkowski vacuum

$$F_{C_i} = \frac{\partial W}{\partial C_i} \sim \int_X \frac{\partial \omega^{3YM}}{\partial C_i} \tag{44}$$

where the Gukov-Vafa-Witten superpotential is given by

$$W = \int_X \Omega \wedge H \tag{45}$$

Geometrically this is associated with complex structure.

However consider a holomorphic bundle and vary the complex structure $\Rightarrow$ bundle may not stay holomorphic.

Precisely, complex moduli $\neq$ bundle moduli + complex structure moduli, but rather the mix of the two.

Question: Can we see this property in TS duals? How to engineer non-trivial F-term geometry?

- $Def(X)$: complex structure deformations of X, parameterized by $H^1(TX) = H^{2,1}(X)$.
- $Def(V)$: bundle moduli of V, deformation of V for fixed C.S. moduli, measured by $H^1(End(V)) = H^1(V \otimes V^\vee)$.
- $Def(V, X)$: Simultaneous holomorphic deformations of V and X. The tangent space is $H^1(X, \mathcal{Q})$ where $\mathcal{Q}$ is defined by Atiyah Sequence:

$$0 \to V \otimes V^\vee \to \mathcal{Q} \xrightarrow{\pi} TX \to 0 \tag{46}$$

- $H^1(X, \mathcal{Q})$ are the actual complex moduli of a heterotic theory
- Long exact sequence in cohomology

$$0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q}) \xrightarrow{d\pi} H^1(TX) \xrightarrow{\alpha} H^2(V \otimes V^\vee) \to \dots \tag{47}$$

$$H^1(\mathcal{Q}) \xrightarrow{?} H^1(V \otimes V^\vee) \oplus H^1(TX) \quad \text{decided by Atiyah Class } \alpha \tag{48}$$

An explicit way of calculating Atiyah class is to use "jumping" phenomena. Consider line bundle $\mathcal{O}(-2,4)$ on the $\{2,4\}$ hypersurface in $\mathbb{P}^1 \times \mathbb{P}^3$:

$$h^0(X, \mathcal{O}(-2,4)) = 0 \ \text{ for generic values of complex structure} \tag{49}$$

As computed in Anderson, Gray, Lukas, Ovrut: arXiv:1107.5076, on a 53-dim sub-locus of the 86-dim CS moduli space, this cohomology can "jump" to

$$\text{On } \mathcal{C}S_{jump}, \quad h^0(X, \mathcal{O}(-2,4)) = 1 \tag{50}$$

Now consider a bundle V

$$0 \to V \to \mathcal{O}(\mathbf{b}_1) \oplus \ldots \mathcal{O}(\mathbf{b}_{n+1}) \xrightarrow{F} \mathcal{O}(\mathbf{c}) \to 0 \tag{51}$$

s.t. a given map element, say $h^0(X, \mathcal{O}(\mathbf{c} - \mathbf{b_1})) = h^0(X, \mathcal{O}(-2,4))$ then V is reducible in the 33 dimensions: $V \to \mathcal{O}(\mathbf{b_1}) \oplus V'$.

Example

Consider the following bundle:

x_i						Γ^j	Λ^a					p_l	
1	1	0	0	0	0	-2	2	-1	-1	1	0	0	-1
0	0	1	1	1	1	-4	0	2	2	0	2	-4	-2

(52)

the map F takes the form:

$$F_a^l = \begin{pmatrix} f_{(-2,4)} & f_{(1,2)} & f'_{(1,2)} & f_{(-1,4)} & f_{(0,2)}) \\ 0 & f_{(1,0)} & f'_{(1,0)} & f_{(0,2)} & f_{(1,0)} \end{pmatrix} \tag{53}$$

where $h^0(X, \mathcal{O}(-2,4)) = 1$ fixes 33 CS moduli:

$$\begin{aligned} dim(\mathcal{M}_0) &= h^{1,1} + h^{2,1} + h^1(X, End_0(V)) = 2 + 86 + 92 = 180 \\ dim(\mathcal{M}_1) &= dim(\mathcal{M}_0) - 33 = 147 \end{aligned} \tag{54}$$

to complete degree of freedom count

$$\begin{aligned} h^1(X, V) &= 41 \quad (\text{no. of } \mathbf{27}) \\ h^1(X, V^\vee) &= 1 \quad (\text{no. of } \overline{\mathbf{27}}) \end{aligned} \tag{55}$$

Construct the TS dual for the bundle above:

x_i								Γ^j		Λ^a					p_l	
0	0	0	0	0	0	1	1	−1	−1	0	1	0	0	0	−1	0
1	1	0	0	0	0	0	0	−1	−1	2	−2	0	1	0	0	−1
0	0	1	1	1	1	0	0	−2	−2	0	0	4	0	2	−4	−2

(56)

where

$$dim(\mathcal{M}_0) = 3 + 55 + 122 = 180 \tag{57}$$

In this case there are two jumping map components: $h^0(\widetilde{X}, \mathcal{O}(0, -2, 4)) = 1$ fixes 15 CS moduli, $h^0(\widetilde{X}, \mathcal{O}(1, -2, 4)) = 1$ fixes 18 CS moduli

$$dim(\mathcal{M}_1) = dim(\mathcal{M}_0) - 33 = 147 \tag{58}$$

degree of freedom count

$$\begin{aligned} h^1(X, V) &= 41 \quad (\text{no. of } \mathbf{27}) \\ h^1(X, V^\vee) &= 1 \quad (\text{no. of } \overline{\mathbf{27}}) \end{aligned} \tag{59}$$

(2,2) Locus Preserved

To study the (2,2) locus of a (0,2) theory, consider tangent bundle:

x_i						Γ^j	Λ^a						p_l
1	1	0	0	0	0	−2	1	1	0	0	0	0	−2
0	0	1	1	1	1	−4	0	0	1	1	1	1	−4

(60)

TS duality gives the following:

x_i								Γ^j		Λ^a							p_l	
0	0	0	0	0	0	1	1	−1	−1	1	0	0	0	0	0	0	−1	0
1	1	0	0	0	0	0	0	−1	−1	0	1	0	0	0	0	2	−2	−1
0	0	1	1	1	1	0	0	−4	0	0	0	1	1	1	1	4	−4	−4

(61)

this manifold is unchanged. Known that $\mathcal{O}(a,b,c)$ on the second manifold the same as $\mathcal{O}(a+b,c)$ on the first manifold, rewrite the dual theory:

x_i						Γ^j	Λ^a							p_l	
1	1	0	0	0	0	−2	1	1	0	0	0	0	2	−3	−1
0	0	1	1	1	1	−4	0	0	1	1	1	1	4	−4	−4

(62)

thus can prove the two configuration are the same:

$$dim(Hom(V,\tilde{V})) = h^0(X, V \otimes \tilde{V}^\vee) = 1 \quad (63)$$

Conclusion and Future Work

- In our non-trivial D/F term examples, TS duality preserves not only the matter spectrum, but also the effective potentials and vacuum spaces.
- Beginning at given points in moduli space infinitesimal fluctuations are preserved, which gives the commutative diagram.
- Loci of enhanced symmetry - stability walls, and (2,2) loci are preserved.
- TS duality may indicate a true (0, 2) string duality
- Future work: Study the behavior in non-geometric phases
- Understand TS duality in Het/F-theory duality (Blumenhagen)

$$\begin{array}{ccc} & Y_4 & \\ {\scriptstyle \mathbb{E}}\swarrow{\scriptstyle \pi_1} & & {\scriptstyle \pi_2}\searrow{\scriptstyle \mathbb{E}} \\ \mathcal{B}_3 & & \widetilde{\mathcal{B}}_3 \end{array} \tag{64}$$