Target Space Duality from Gauge-Gravity Pair Creation

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Based on work with
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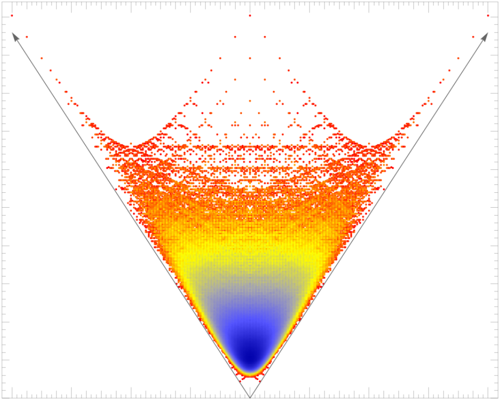
Motivation #1

Topological transitions

- Need to compactify string theory
  - And we'll here consider Calabi-Yau compactifications
- And different topologies cause drastic differences
  - Physics in external 4d critically depends on topology
- But mathematically, there are topological transitions
  - E.g. conifold transitions, in which we shrink a subspace to create a singularity, and then smooth this out

Topological transitions



(Figure: density map of Kreuzer-Skarke CY_3s, reproduced from arXiv:1808.09993)

- Appears possible to transition between all known CY_3s
  - Specifically, possible with conifold transitions
  - Maybe even true for all CY_3s ('Reid's conjecture')

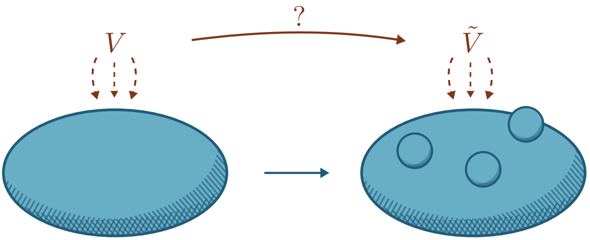
Physical transitions

- Natural to ask: can this also happen physically?
  - During topological transition, geometry becomes singular
  - And 4d effective theory appears to become singular too ...
- Yes - string theory can smooth out the singularity
  - New states appear from shrinking cycles
  - Cures the singular behaviour of the effective theory
  - So smooth physical transition
- Connects compactifications into larger moduli space
  - Theory continuously connected to distinct compactification

The heterotic case

- The above is correct in Type II string theory
    - Smoothness of transition has been shown in Type IIB
    - New states appear from wrapped D-branes
    - Argument for smoothness from mirror symmetry
- But ...
- In heterotic compactifications there is extra ingredient
    - There is also a gauge field background
    - Captured mathematically by vector bundle
- So: additional question of how gauge field goes through
- And at present this is still not understood (!)

The heterotic case



- Until this is understood, we will not have a good picture of the moduli space of heterotic compactifications
  - Not clear which theories are connected to each other
  - Might be masking very important structure
- So this is the task of this work
  - Here I will present a solution to this problem

The heterotic case

- Note previous efforts have focused on ‘spectators’
  - Bundles which essentially don’t notice the transition
  - But these have failed at anomaly condition
- So it is clear that on the contrary, the gauge sector must interact intimately with the geometry
  - This will be at the core of the solution I will present

Motivation #2

(0,2) Gauged Linear Sigma Models (GLSMs)

- From worldsheet perspective instead, compactification described by (0,2) non-linear sigma model
  - Theory living on 2d worldsheet
  - Fields on worldsheet describe embedding coordinates
  - Fermionic couplings describe gauge field background
- But also exist structurally similar non-geometric theories
  - Theories with mathematically similar structure
  - But no geometric interpretation as worldsheet embedding
  - (Landau-Ginzburg models, as well as hybrids)
- Witten: Just different phases of overarching theory (!) [Witten '93]
  - GLSMs - very useful and rich structure
  - But won't need details here, just one key idea:
  - Allows to continuously interpolate between geometric and non-geometric theories

Target space duality

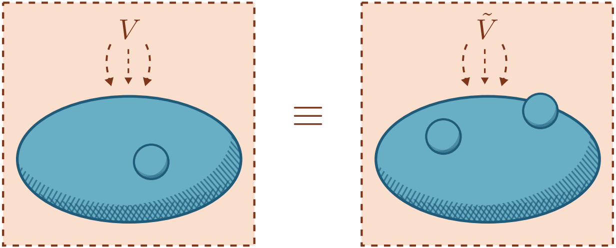
- In non-geometric phases, one finds curious fact . . .
  - Possible to exchange roles of fields
  - In way that leaves theory invariant

[Distler, Kachru '95]
[Chiang, Distler, Greene '97]
[Blumenhagen, Rahn '11]

- But going back to geometric phase gives different theory (!)
  - Gives theory on totally different geometry
  - Different topology, different gauge field background
  - So produce very different compactification data, by going via non-geometric phase of the GLSM
- So produces pairs of distinct compactifications . . .
  - Why? Do they have a special relationship?

Target space duality

- Pairs appear to give same external physics (!) [Distler, Kachru '95]
  - First evidence was that spectra match [Chiang, Distler, Greene '97] [Blumenhagen '98], [Blumenhagen '98]
  - And recently also evidence that potentials match [Anderson, Feng '16]
  - So by now strong evidence theories are in fact dual
  - (Phenomenon called 'target space duality')



- Intriguing observation, but remains to be explained ...

Geometric explanation?

- But one finds another intriguing property about target space dual pairs ...
- Geometries related by target space duality seem to be related by topological transitions [Blumenhagen '98], [Blumenhagen '98] [Blumenhagen, Rahn '11]
- And actually, there aren't only pairs ...
  - Target space duality procedure can produce whole chains
  - (Unlike mirror symmetry, where duality is for pair only)
- And find that whole chain are also connected by topological transitions ...

Geometric explanation?

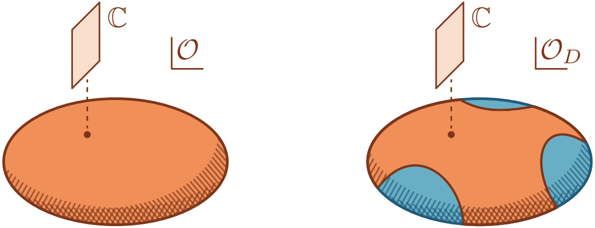
- But this suggests possible explanation . . .
- Could target space dual theories be related by process of traversing topological transition?
  - Would explain geometry relationships, plus chains
- Could be evidence for a positive answer to whether heterotic theories can traverse topological transitions
- And conversely, such a description would provide a geometric explanation for heterotic target space duality

Talk Outline

- Background
- Rethinking topological transitions
- Gauge-gravity pair creation
- The gauge sector transition
- Conclusions

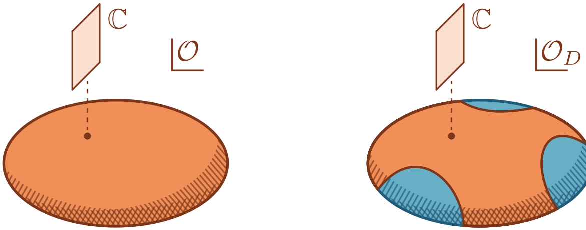
Background

Skyscraper sheaves and 5-branes



- What is a skyscraper sheaf?
  - Essentially restriction of bundle to subspace of base
  - E.g. begin with trivial line bundle $\mathcal{O}$, for which fibres are $\mathbb{C}$
  - Restriction to subspace $D \subset \mathrm{CY}_3$ has fibers only over D
  - Notation: denoted by subscript, so here $\mathcal{O}_D$ (will use a lot)

Skyscraper sheaves and 5-branes



- Skyscraper sheaves appropriately describe 5-branes
  - If wrapped on $C \subset$ CY$_3$ then describe by $\mathcal{O}_C$
  - Particularly useful description for certain aspects
  - Including: emission from / absorption into gauge field

Small instanton transitions

- Exists process of small instanton transition
  - See 5-brane as small instanton in gauge field background
  - Then can absorb it into background and smooth back out
  - Called ‘small instanton transition’
- Mathematically described by Hecke transform [Ovrut, Pantev, Park ’00]
  - Consider absorption of 5-brane on C into bundle V
  - Gives new configuration $\tilde{V}$ given by short exact sequence,

$$0 \to \tilde{V} \to V \to \mathcal{O}_C \to 0$$

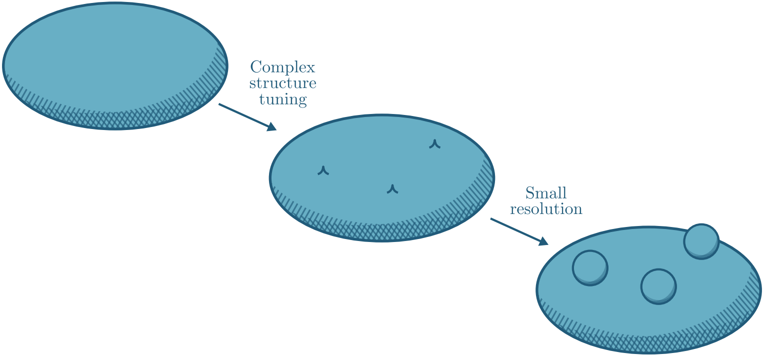
  - And finally $\tilde{V}$ is smoothed out to give vector bundle
  - Also exist rank-changing transitions, given by e.g.

$$0 \to \tilde{V} \to V \oplus \mathcal{O} \to \mathcal{O}_C \to 0$$

- This process will enter crucially in the following story

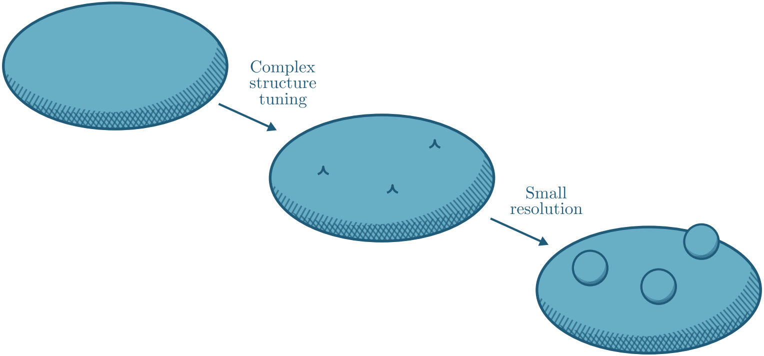
A Gravitational Small Instanton Transition

Conifold transitions



- Perhaps the simplest topological transition of CY_3s
  - Complex structure tuned, giving singular points
  - New Kähler modulus resolves points into $\mathbb{P}^1$s (spheres)
  - (Or conversely: small contraction, then deformation)
- But powerful: expect to possibly connect all CY_3s

Conifold transitions

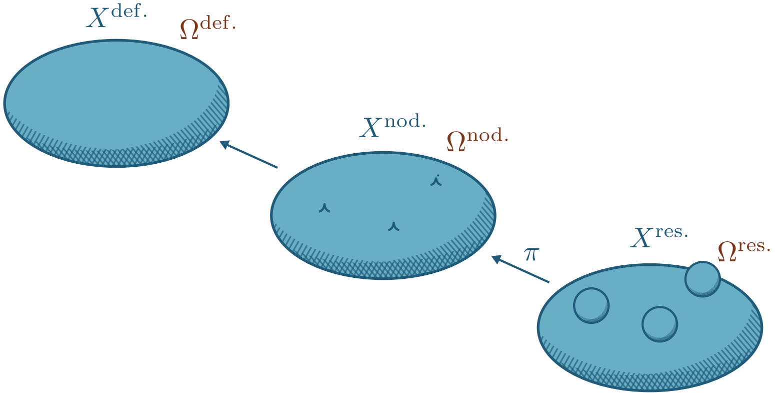


- Note: creates drastic changes
  - Chern character: $\mathrm{ch}_2 \to \mathrm{ch}_2 + [\mathbb{P}^1\mathrm{s}]$
  - Hodge numbers: $h^{1,1} \to h^{1,1} + 1$ and $h^{2,1} \to h^{2,1} - \Delta$
- I.e. the gravitational sector is altered significantly

Conifold transitions

- Goal: describe how gauge sector traverses conifold
  - And it is known that a pure ‘spectator’ is inconsistent
  - So must be interaction of gauge and gravitational sectors
- $\Rightarrow$ First want precise description of tangent bundle $\mathcal{T}$
  - This is the description of the gravitational sector
  - Clearly prerequisite for understanding the interaction
- Will turn out to be more useful to use cotangent bundle Ω
  - Note Ω and $\mathcal{T}$ carry same information, and $\mathrm{c}_2(\Omega) = \mathrm{c}_2(\mathcal{T})$
  - But description of cotangent bundle Ω nicer than tangent $\mathcal{T}$

Notation



$X^{\mathrm{def.}}$
$\Omega^{\mathrm{def.}}$
$X^{\mathrm{nod.}}$
$\Omega^{\mathrm{nod.}}$
π
$X^{\mathrm{res.}}$
$\Omega^{\mathrm{res.}}$

Relative cotangent sequence

- Relationship of $X^{\text{res.}}$ and $X^{\text{nod.}}$ is resolution, so ...
  - Perhaps simple relationship between $\Omega^{\text{res.}}$ and $\Omega^{\text{nod.}}$?
- Yes (!) one can show description is short exact sequence

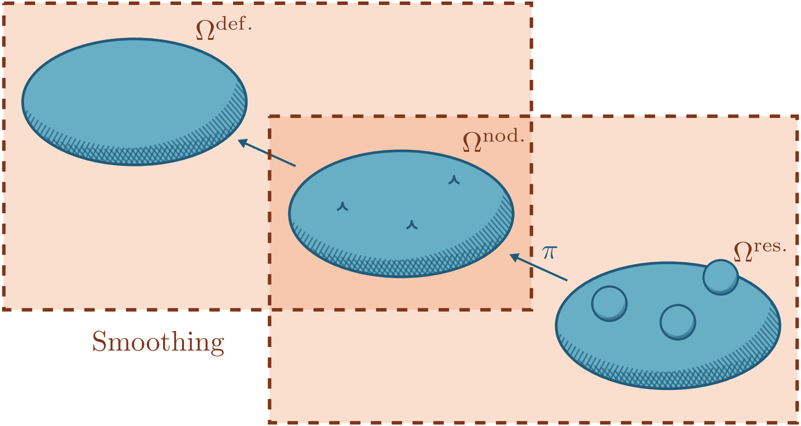
Relationship is 'relative cotangent sequence'

$$0 \to \pi^*(\Omega^{\text{nod.}}) \to \Omega^{\text{res.}} \to \mathcal{O}_{\mathbb{P}^1\text{s}}(-2) \to 0$$

where $\pi\colon X^{\text{res.}} \to X^{\text{nod.}}$ is small contraction

  - (Last term has a 'twist', because it's $i_*(\Omega^{\mathbb{P}^1\text{s}})$)
  - (π^* acts to represent $\Omega^{\text{nod.}}$ on resolution geometry $X^{\text{res.}}$)
- Then remaining part simple: $\Omega^{\text{nod.}}$ smoothed to give $\Omega^{\text{def.}}$

Relative cotangent sequence



Relative cotangent sequence

$$0 \to \pi^*(\Omega^{\mathrm{nod.}}) \to \Omega^{\mathrm{res.}} \to \mathcal{O}_{\mathbb{P}^1\mathrm{s}}(-2) \to 0$$

The gravitational small instanton transition

So change in cotangent bundle $\Omega^{\text{res.}}$ to $\Omega^{\text{nod.}}$ described by

$$0 \to \pi^*(\Omega^{\text{nod.}}) \to \Omega^{\text{res.}} \to \mathcal{O}_{\mathbb{P}^1\text{s}}(-2) \to 0$$

and then a smoothing process from $\Omega^{\text{nod.}}$ to $\Omega^{\text{def.}}$

- But this is just like small instanton transition (!)
  - Precisely the same structure as the Hecke transform
  - Small instanton piece wraps the exceptional $\mathbb{P}^1$s
  - Absorption of small instanton $\sim$ shrinking $X^{\text{res.}} \to X^{\text{nod.}}$
  - And smoothing of instanton $\sim$ deformation $X^{\text{nod.}} \to X^{\text{def.}}$

So the conifold transition can be viewed as the gravitational sector undergoing a small instanton transition

Gauge-Gravity Pair Creation

Heterotic anomaly cancellation

- In heterotic string compactification, there is an anomaly cancellation condition to be satisfied by the background
- Specifically, this is captured by the topological relation

$$\mathrm{c}_2(\Omega) = \mathrm{c}_2(V) + [C]$$

where V is the gauge bundle and $[C]$ are the gauge 5-branes

- So gravitational and gauge backgrounds must balance
    - Gravity and gauge backgrounds enter on opposite sides
    - Hence, changes in gravity sector must be compensated

Heterotic conifold as gauge-gravity pair creation

- But conifold transition requires altering gravitational piece
  - Before transition have cotangent bundle $\Omega^{\mathrm{res.}}$
  - But recall relationship of $\Omega^{\mathrm{res.}}$ and $\Omega^{\mathrm{nod.}}$,

$$0 \to \pi^*(\Omega^{\mathrm{nod.}}) \to \Omega^{\mathrm{res.}} \to \mathcal{O}_{\mathbb{P}^1\mathrm{s}}(-2) \to 0$$

  - Small instanton piece $\mathcal{O}_{\mathbb{P}^1\mathrm{s}}(-2)$ didn't exist before ...
  - It is added in the process of the conifold transition
- This change must be balanced by gauge sector

$$\begin{array}{llcl} \mathrm{Before:} & \mathrm{c}_2(\Omega^{\mathrm{res.}}) & = & \mathrm{c}_2(V^{\mathrm{res.}}) \\ \mathrm{After:} & \mathrm{c}_2(\Omega^{\mathrm{res.}}) + [\mathbb{P}^1\mathrm{s}] & = & \mathrm{c}_2(V^{\mathrm{res.}}) + \,? \end{array}$$

  - (Note smoothing $\Omega^{\mathrm{nod.}} \to \Omega^{\mathrm{def.}}$ doesn't further alter $\mathrm{c}_2(\Omega)$)
- So need to add gauge sector object with same class ...

Heterotic conifold as gauge-gravity pair creation

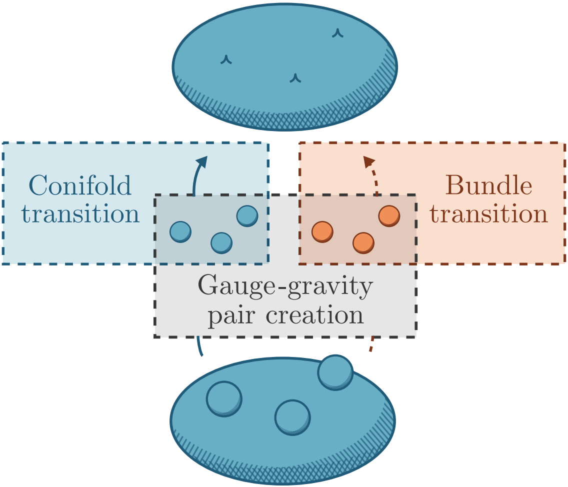
- But then the candidate is clear
  - Namely: introduce gauge small instanton on $\mathbb{P}^1$s
  - This has right c_2 to preserve anomaly cancellation
  - And is introduced at same place as gravitational effect
  - But which twist? Clearly very natural to match $\mathcal{O}_{\mathbb{P}^1\text{s}}(-2)$
  - Because in particular, then very natural characterisation (!)

Conjecture

The heterotic conifold transition is a process of gauge-gravity pair creation of small instantons $\mathcal{O}_{\mathbb{P}^1\text{s}}(-2)$

- This pair creation process is novel and so requires some comments, which we will make below
- Note: this proposal is in stark contrast with the spectator proposals that have been made in the past

Heterotic conifold as gauge-gravity pair creation



Comments

- Process resembles brane-anti-brane pair-creation
  - Objects are produced on same locus
  - Objects have opposite ‘charges’
- But moreover supersymmetry remains unbroken
  - Brane-anti-brane pair-creation breaks supersymmetry
  - But not the case for gauge-gravity pair-creation
- So appears to be at least natural and consistent ...
- But what is the evidence?
  - Consistency is not enough - can this really occur?

Evidence

Evidence

- Difficult to obtain direct evidence
  - At singular geometry, so less control, harder to prove
  - (Will be the subject of future work)
- But the focus of the rest of talk will be to provide some remarkable indirect evidence, namely:
- This process connects target space dual theories (!)
  - These are what one expects may connect through conifold
  - And gauge small instanton $\mathcal{O}_{\mathbb{P}^1\mathrm{s}}(-2)$ precisely does this (!)
- Before we turn to illustrating this, let's first be clear of the logic of this evidence

Evidence

- Correctly transitions *every* known target space dual pair
  - (Whenever geometries are related by conifold)
  - Have conjectured heterotic conifold is described by pair creation of gravitational and gauge small instantons
  - So form of gauge small instanton is forced on us: same form that describes the transition of the cotangent bundle
  - And yet find gauge sectors always related by absorbing this small instanton in the process we will describe below (!)
  - Would be amazing if this was all pure coincidence

Evidence

- Conversely, whenever bundle can undergo process below, target space dual exists and matches result of this process
  - Have proven this for examples of conifold transitions
  - Take example where all stable monads are known
  - Can collect all cases where bundle can undergo absorption of $\mathcal{O}_{\mathbb{P}^1\text{s}}(-2)$ in process we will describe below
  - Then find that: in every such case, can perform target space duality procedure, and correctly reproduces $V^{\text{res.}}$
  - So further evidence process we describe below indeed describes theory passing through transition to dual theory

Absorption of the Gauge Small Instanton

Naive attempt at absorption

- Conjectured description of the conifold implies the heterotic bundle is taken through by the small instanton $\mathcal{O}_{\mathbb{P}^1\mathrm{s}}(-2)$
- So we now want to check this
- Let's try absorbing this small instanton into a candidate
  - Take an example of a bundle we have reason to believe could be taken through the conifold
  - And see if what is produced by absorption of $\mathcal{O}_{\mathbb{P}^1\mathrm{s}}(-2)$ is consistent, sensible, etc
- Theories with target space dual are obvious candidates
  - So let's start here

Naive attempt at absorption

- Try to absorb small instanton by Hecke transform ...
  - Take $V^{\text{res.}}$ from theory on $X^{\text{res.}}$ with TSD on $X^{\text{def.}}$
  - Absorbing $\mathcal{O}_{\mathbb{P}^1\text{s}}(-2)$ will give $V^{\text{def.}}$ described by
    $$0 \to V^{\text{def.}} \to V^{\text{res.}} \to \mathcal{O}_{\mathbb{P}^1\text{s}}(-2) \to 0$$
  - But what one finds is $H^0\big(V^{\text{res.}\vee}, \mathcal{O}_{\mathbb{P}^1\text{s}}(-2)\big) = 0$
  - I.e. map from $V^{\text{res.}}$ to $\mathcal{O}_{\mathbb{P}^1\text{s}}(-2)$ doesn't exist (!)
  - This is true for every example with a target space dual (!)
- Actually, quite reasonable that map never exists
  - For bundle to limit under $X^{\text{res.}} \to X^{\text{nod.}}$ to something not too nasty, should be trivial on the $\mathbb{P}^1$s (and in fact, one finds that this is indeed true whenever TSD exists!)
  - But then on $\mathbb{P}^1$s, map from $V^{\text{res.}}$ to $\mathcal{O}_{\mathbb{P}^1\text{s}}(-2)$ cannot exist
- So appears that gauge small instanton $\mathcal{O}_{\mathbb{P}^1\text{s}}(-2)$ cannot be absorbed into any sensible candidates ...

Structure of target space duality

- Is there another absorption process?
  - Can't directly absorb the gauge small instanton by Hecke transform - is there another way for it to be absorbed?
- In fact the answer is 'yes'
  - And in particular this is actually due to a very special property of target space dual theories . . .
  - This is what we will describe now

Structure of target space duality

- In target space duality, one finds curious fact ...

Observation

For any pair of target space dual theories which live on geometries related by a conifold transition:

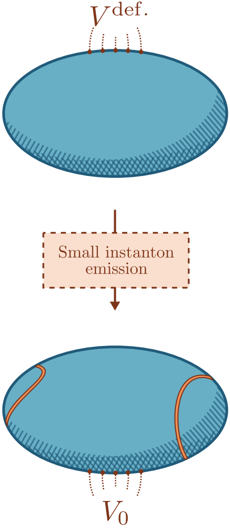
The gauge bundles are both able to emit a small instanton to leave behind the same spectator bundle

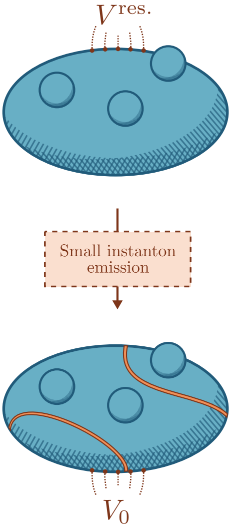
I.e. one can show that there are Hecke transforms,

$$0 \to \quad \tilde{V}^{\text{res.}} \to V^0 \to \mathcal{O}_{C^{\text{res.}}} \to 0$$
$$0 \to \quad \tilde{V}^{\text{def.}} \to V^0 \to \mathcal{O}_{C^{\text{def.}}} \to 0$$

where $\tilde{V}^{\text{res.}}$ and $\tilde{V}^{\text{def.}}$ are deformations of $V^{\text{res.}}$ and $V^{\text{def.}}$

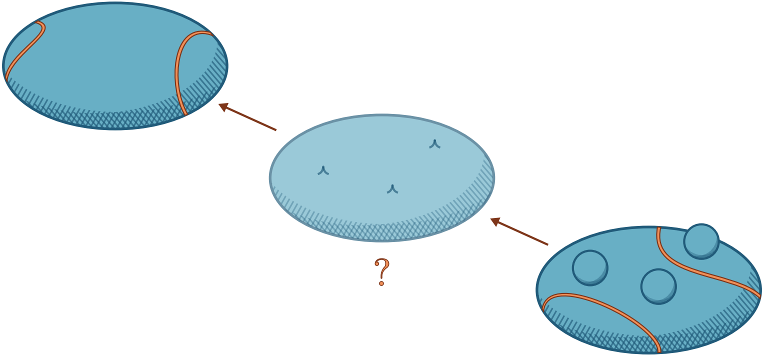
Structure of target space duality





Structure of target space duality

- One can always tune $V^{\text{res.}}$ and $V^{\text{def.}}$ then emit small instantons $\mathcal{O}_{C^{\text{res.}}}$ and $\mathcal{O}_{C^{\text{def.}}}$ leaving same spectator
- But spectator bundle has trivial behaviour through conifold
- So the story of how the gauge background traverses the conifold is purely a story about these small instantons



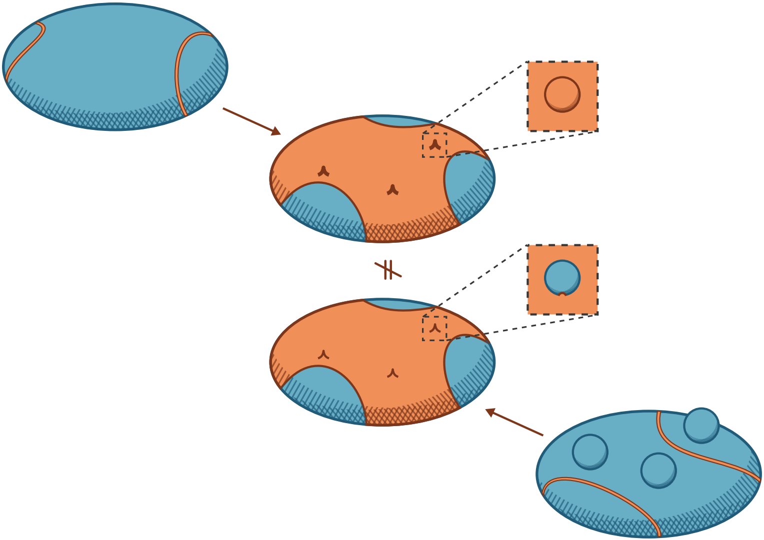
The meeting of a target space dual pair

- TSD theories seem to pick out these curves - but why?
  - Do they have special relationship with conifold geometry?
- Yes! In particular, special behaviour in the nodal limit ...
- They become divisors on the nodal geometry (!)
  - Algebraic description causes jump in dimension
  - (Will return to physical interpretation of dimension jump)
- (Actually there is a tuning story here)
  - They are the curves which can become divisors upon tuning
  - But we will see this tuning crucially allows the traversal
- In particular, they become *almost* the same divisor ...

The meeting of a target space dual pair

- In fact as loci they limit to exactly the same thing ...
  - Namely, the locus of a Weil non-Cartier divisor
  - Associated to the nature of the singularities of $X^{\mathrm{nod.}}$
- But: same locus, different objects
  - Curve $C^{\mathrm{def.}}$ limits to object $D^{\mathrm{def.}}$ 'fat' at nodal points
  - Curve $C^{\mathrm{res.}}$ limits to object $D^{\mathrm{res.}}$ which is not 'fat'
- Said differently ...
  - On infinitesimal resolution, one contains $\mathbb{P}^1$s ...
  - I.e. objects they limit to differ by the $\mathbb{P}^1$s (!)

The meeting of a target space dual pair



The Transition

Brane recombination

- Seen that target space dual theories almost meet
  - Can view as spectator plus small instantons
  - At nodal geometry, small instantons almost the same ...
- In particular, meet up to small instanton on the $\mathbb{P}^1$s ...
  - (I.e. on infinitesimal resolution, support loci differ by the $\mathbb{P}^1$s)
- But we have precisely such a small instanton (!)
  - Conjectured heterotic conifold is gauge-gravity pair creation
  - Which produces gauge small instanton $\mathcal{O}_{\mathbb{P}^1\mathrm{s}}(-2)$
  - Has right locus to be difference of the small instantons (!)

Brane recombination

- But can the recombination process actually occur?
  - Want to combine '$\mathcal{O}_{D^{\text{res.}}} + \mathcal{O}_{\mathbb{P}^1{}_{\text{S}}}(-2) = \mathcal{O}_{D^{\text{def.}}}$'
  - Support of sheaves correct, but this is not enough ...
  - Can recombination actually happen?
- Yes it can (!)
  - Easiest to discuss on infinitesimal resolution
  - Find that there is exact sequence

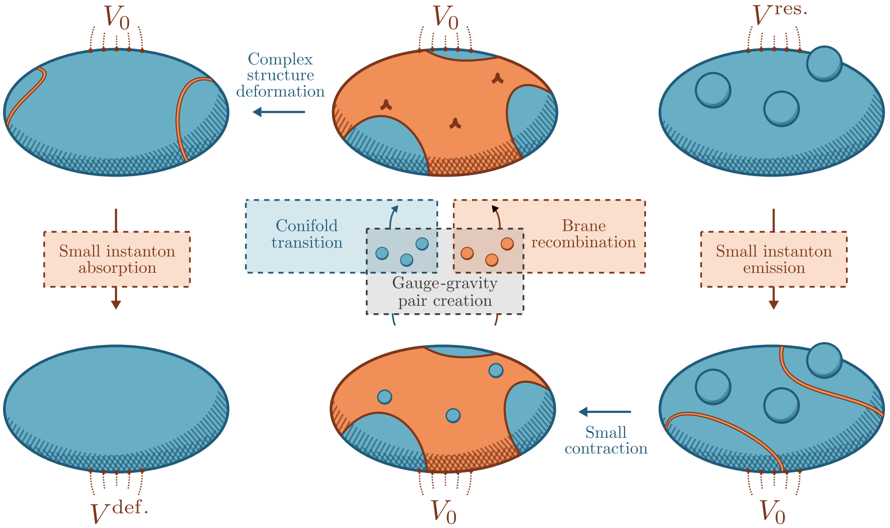
$$0 \to \mathcal{O}_{\mathbb{P}^1{}_{\text{S}}}(-2) \to \mathcal{O}_{D^{\text{def.}}}(-1) \to \mathcal{O}_{D^{\text{res.}}} \to 0\,.$$

    and moreover that $\text{Ext}^1\big(\mathcal{O}_{D^{\text{res.}}}\,,\,\mathcal{O}_{\mathbb{P}^1{}_{\text{S}}}(-2)\big) \neq 0$
  - (Twist on middle term is subtlety from doing on infinitesimal resolution, won't discuss here but explained in paper)
- So small instantons $\mathcal{O}_{C^{\text{res.}}}$ and $\mathcal{O}_{C^{\text{def.}}}$ related by recombination process across conifold transition (!)

The complete transition

- Conclusion: we now have a description of how the gauge small instanton $\mathcal{O}_{\mathbb{P}^1{}_{\rm s}}(-2)$ precisely performs the change taking a gauge bundle into its target space dual
- So we can now give a complete description of how the heterotic theory crosses the conifold transition ...

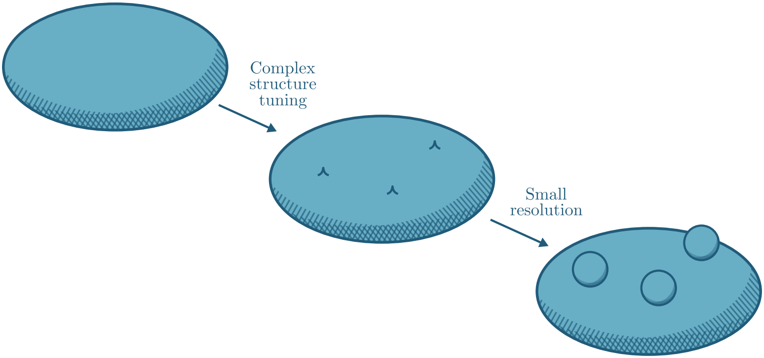
The complete transition



Significance of divisors?

- So far haven't discussed the interpretation of the jumping in dimension of the gauge sector objects at the nodal limit
- Short answer: physical interpretation is unclear
    - Clear the supports of the sheaves jump to divisors
    - But not clear if interpretation exists as extended object
    - May only be effect arising within small instanton limit
- But no clear problem for supersymmetry
    - Correct conditions to impose in singular limit are not known
- So currently: mathematical aspect of description whose physical significance remains to be understood

Example



$$X^{\text{def.}} \in \left[\; \mathbb{P}^4[y] \;\middle|\; 5 \;\right] \qquad X^{\text{nod.}} \in \left[\; \mathbb{P}^4[y] \;\middle|\; 5 \;\right] \qquad X^{\text{res.}} \in \left[\begin{array}{c|cc} \mathbb{P}^1[x] & 1 & 1 \\ \mathbb{P}^4[y] & 1 & 4 \end{array}\right]$$

$$\left\{\; Q(y) = 0 \;\right\} \qquad \left\{\; l_0(y)q_1(y) - l_1(y)q_0(y) = 0 \;\right\} \qquad \left\{\begin{array}{c} l_0(y)x_0 + l_1(y)x_1 = 0 \\ q_0(y)x_0 + q_1(y)x_1 = 0 \end{array}\right\}$$

16 nodal points at
$l_0 = l_1 = q_0 = q_1 = 0$

16 exceptional $\mathbb{P}^1$s at
$l_0 = l_1 = q_0 = q_1 = 0$

Example

$$X^{\text{def.}} \in \left[\begin{array}{c|c} \mathbb{P}^4 & 5 \end{array} \right]$$

$$0 \to V^{\text{def.}} \to \begin{array}{c} \mathcal{O}(1)^{\oplus 4} \\ \oplus \\ \mathcal{O}(1) \\ \oplus \\ \mathcal{O}(4) \end{array} \to \begin{array}{c} \mathcal{O}(4) \\ \oplus \\ \mathcal{O}(5) \end{array} \to 0$$

$$0 \to \tilde{V}^{\text{def.}} \to V_0 \oplus \mathcal{O} \to \mathcal{O}_{C^{\text{def.}}} \to 0$$

where $0 \to V_0 \to \mathcal{O}(1)^{\oplus 4} \to \mathcal{O}(4) \to 0$

and $0 \to \mathcal{O}(-5) \to \begin{array}{c} \mathcal{O}(-1) \\ \oplus \\ \mathcal{O}(-4) \end{array} \to \mathcal{I}_{C^{\text{def.}}} \to 0$

i.e. $C^{\text{def.}} \in \left[\begin{array}{c|cc} \mathbb{P}^4 & 1 & 4 \end{array} \right] \cap X^{\text{def.}}$

$$X^{\text{res.}} \in \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array} \right]$$

$$0 \to V^{\text{res.}} \to \begin{array}{c} \mathcal{O}(0,1)^{\oplus 4} \\ \oplus \\ \mathcal{O}(1,0) \\ \oplus \\ \mathcal{O}(0,5) \end{array} \to \begin{array}{c} \mathcal{O}(0,4) \\ \oplus \\ \mathcal{O}(1,5) \end{array} \to 0$$

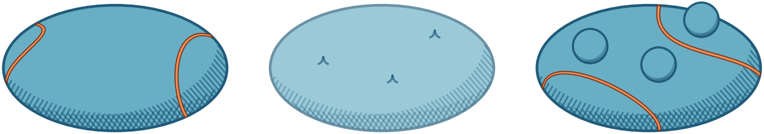
$$0 \to \tilde{V}^{\text{res.}} \to V_0 \oplus \mathcal{O} \to \mathcal{O}_{C^{\text{res.}}} \to 0$$

where $0 \to V_0 \to \mathcal{O}(0,1)^{\oplus 4} \to \mathcal{O}(0,4) \to 0$

and $0 \to \mathcal{O}(-1,-5) \to \begin{array}{c} \mathcal{O}(-1,0) \\ \oplus \\ \mathcal{O}(0,-5) \end{array} \to \mathcal{I}_{C^{\text{res.}}} \to 0$

i.e. $C^{\text{res.}} \in \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 0 \\ \mathbb{P}^4 & 0 & 5 \end{array} \right] \cap X^{\text{res.}}$

Example



$X^{\text{def.}} \in \left[\ \mathbb{P}^4[y] \ \middle|\ 5\ \right]$ $\quad$ $X^{\text{nod.}} \in \left[\ \mathbb{P}^4[y] \ \middle|\ 5\ \right]$ $\quad$ $X^{\text{res.}} \in \left[\begin{array}{c|cc} \mathbb{P}^1[x] & 1 & 1 \\ \mathbb{P}^4[y] & 1 & 4 \end{array}\right]$

$\{\ Q(y) = 0\ \}$ $\quad$ $\{\ l_0(y)q_1(y) - l_1(y)q_0(y) = 0\ \}$ $\quad$ $\left\{\begin{array}{c} l_0(y)x_0 + l_1(y)x_1 = 0 \\ q_0(y)x_0 + q_1(y)x_1 = 0 \end{array}\right\}$

$C^{\text{def.}} \in \left[\ \mathbb{P}^4 \ \middle|\ 1 \quad 4\ \right] \cap X^{\text{def.}}$ $\quad$ $C^{\text{res.}} \in \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 0 \\ \mathbb{P}^4 & 0 & 5 \end{array}\right] \cap X^{\text{res.}}$

$\downarrow$ Tune $\downarrow$ $\quad$ $\downarrow$ Tune $\downarrow$

$D^{\text{def.}}:\ \{l_0 = q_0 = 0\}$ $\quad$ $D^{\text{res.}}:\ \{x_1 = l_0q_1 - l_1q_0 = 0\}$

Can check explicitly that

$$0 \to \mathcal{O}_{\mathbb{P}^1\text{s}}(-2) \to \mathcal{O}_{D^{\text{def.}}}(-1, 0) \to \mathcal{O}_{D^{\text{res.}}} \to 0$$

is exact and that $\text{Ext}^1\big(\mathcal{O}_{D^{\text{res.}}}, \mathcal{O}_{\mathbb{P}^1\text{s}}(-2)\big) = \mathbb{C}$

Final Comments

Novel 5-brane theory duality

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- This story also gives novel duality of 5-brane theories
  - Process above involves bundle emitting / absorbing gauge small instantons $C^{\mathrm{def.}}$ and $C^{\mathrm{res.}}$
  - Can instead just begin with these, as background 5-branes
  - Can also replace spectator bundle with spectator 5-brane
  - Then: have pure 5-brane theories on both geometries
- Indeed, able to prove moduli matching quite generally
  - For large classes we have been able to prove moduli matching of these 5-brane theories
  - This is new duality - not readily accessible from GLSM
  - (So again would be incredible if not due to real structure)

Moduli matching

- And can then lift to moduli matching for full bundle theories
  - Reabsorb 5-branes into bundles via Hecke transform
  - And can understand decomposition of moduli
  - Key part turns out to be 5-brane computation, simply lifts
  - Allows us to prove moduli matching for bundle theories

Conclusions

- Understood which heterotic theories can pass through topological transitions, plus what they become afterwards
  - Bundle must admit appropriate small instanton emission
  - Described traversal explicitly (for conifold transition)
  - Concluded result is a target space dual theory
- Provided evidence for novel gauge-gravity pair creation
  - Conjectured as the description of the heterotic conifold
  - Resulted in specific gauge small instanton
  - Found this precisely connects target space dual theories
- As bonus, produced novel duality of 5-brane theories
  - Quite general proofs of duality
  - Novel, since not readily accessible from GLSM picture

Future directions

- Further study gauge-gravity pair creation process
  - Consider in simpler setting
  - Develop field theory description
- Look at other types of topological transitions
  - E.g. flops
  - Seem to have some qualitative differences
- Determine the F-theory dual of these processes
- Understand implications for moduli space of heterotic compactifications
  - Evidence that theories apparently essentially the same
  - Portions of moduli space seem to carry up to higher $h^{1,1}$