THE BRANES BEHIND GENERALIZED SYMMETRY OPERATORS

2209.03343, 2212.09743, 2203.10102, 2304.xxxxx (tomorrow) & WIP

w/ ~~Cvetic~~, Acharya, Cvetic, Del Zotto, Heckman, ~~Hübner~~, Torres, Yu, Zhang

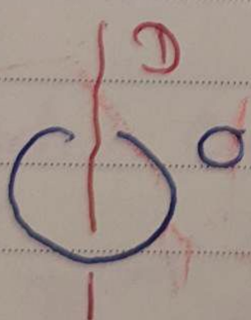
Now Program: Lagrangian

$\boxed{\text{3-phases}}$

Theme: Generalized Global Symmetries + Geometric Engineering

$\rightsquigarrow$ Talks by Bah, Choi

QFT: Defect ~~and~~ Ops and Symmetry Ops



Eg. Wilson Lines
Flux Operators

[GKSW, 14] [S, 15]

Generalized Global Sym. $\Longleftrightarrow$ Topological Sym. Operators

No.
Date / /

QFT $\Leftrightarrow$ String / M-theory

Q: How do $\mathcal{O}$, $\mathcal{D}$ lift?

A: Branes wrapped over non-compact cycles

[GE, 22] [ABBS, 22] [HHTZ, 22]

Outline: ① "Branes at infinity"

② Examples

a) 6d $\mathcal{N} = (2,0)$ Theory

b) 4d $\mathcal{N} = 4$ SYM

c) 7d $\mathcal{N} = 1$ SUGRA

d) 5d $\mathcal{N} = 1$ SCFT Junctions

①

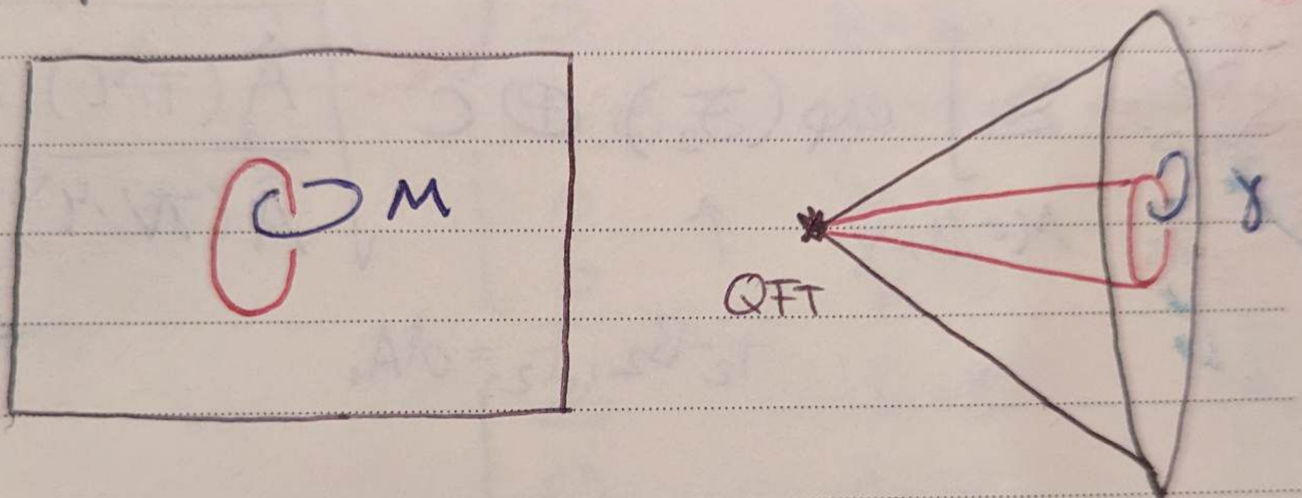
Geometric Engineering:

non-compact

Geometry $X \xrightarrow[\text{IIA / IIB / M / F}]{}$ "QFT $\mathcal{T}_X$"

(+ ~~other duality frames~~)

Spacetime $\mathbb{R}^{\#} \times X$ ∂X



p-Brane wrappings:

Defect Ops: $\mathbb{D} = \oplus_m \mathbb{D}^{(m)}$ [DZ, H, P, R, 15]

$$\mathbb{D}^{(m)} = \bigoplus_{p-k=m-1} H_k(X,\partial X) / H_k(X)$$

No.
Date / /

Mo Tu We Th Fr Sa Su

Symmetry Ops.: $\mathbb{D} = \bigoplus_n \mathbb{D}^{(n)}$

$$\mathbb{D}^{(n)} = \bigoplus_{p-k=n-1} H_k(\partial X)$$

$$\sigma(M) = \int DA_1 \exp\left(2\pi i\, S^{D_p}_{top}\right)$$

$$S^{D_p}_{top} = \cancel{Z}\int_{M = M\times\gamma} \exp(\mathcal{F}_2) \oplus C \sqrt{\frac{\hat{A}(TM)}{\hat{A}(NM)}}$$

$\mathcal{F}_2 - B_2,\ F_2 = dA_1$

⇒ Symmetry Operators support TFT

⇒ Non-invertible Fusion Rules

2. Examples

a) IIB on $X = \mathbb{C}^2/\Gamma$, $\Gamma \subseteq SU(2)$ /
6d $\mathcal{N} = (2,0)$ Theory

Cycles for Dp-branes:

$$\frac{H_k(X, \partial X)}{H_k(X)} = \begin{cases} \mathbb{Z} & k=4 \\ 0 & k=3 \\ Ab(\Gamma) & k=2 \quad \leftarrow \\ 0 & k=1 \\ 0 & k=0 \end{cases}$$

$$H_k(\partial X) = \begin{cases} \mathbb{Z} & k=3 \\ 0 & k=2 \\ Ab(\Gamma) & k=1 \quad \leftarrow \\ \mathbb{Z} & k=0 \end{cases}$$

$$H^2(\partial X) \cong Ab(\Gamma) \cong \langle u_2 \rangle$$

Set $\Gamma = \mathbb{Z}_N$ for exposition.

Date / /

KK Reduction:

$$S^{D3}_{top} = \int_{M_3 \times Y} \breve{G}_5 + \breve{G}_3 \wedge (\breve{F}_2) + \ldots$$

$C_4 \quad C_2 \quad F_2 - B_2$

$$\breve{F}_2 = \breve{C}_0 * \breve{u}_2 + \breve{C}_2 * \breve{1}, \quad \text{etc.}$$

$$\breve{G}_5 = g_3 * \breve{u}_2 + \ldots$$

$$\sigma(M_3) = \# \int Dc_0' \, Dc_1' \, e^{\frac{2\pi i}{N} \int_{M_3} g_3 + c_0' \cup b_3' + c_2' \cup b_1'}$$

$$= U_0(M_3) \, U_1(M_3) \, U_3(M_3)$$

Flux operator

Condensation Ops

$$U_1 = \frac{1}{|H_1(M_3, \mathbb{Z}_N)|} \sum_{\ell \in H_1(M_3, \mathbb{Z}_N)} \exp\left(\frac{2\pi i}{N} \int_\ell b_1'\right)$$

b) N D3-branes probing $X = \mathbb{C}^3 /$
4D $\mathcal{N}=4$ SYM $SU(N)$

Punchline: 7-branes on ∂X realize 4D duality interfaces/defects

Interlude: Symmetry TFTs

[A, B, GE, H, S, 2021], [F, M, T, 2022]

$$B_{phys.} \;\Big|\; TFT_{d+1} \;\Big|\; B_{top} \;=\; QFT_d$$

$\Rightarrow$ Isolation of top syms. of QFT_d

String theory naturally expands QFT_d in this sandwich.

Collapse X onto ~~its~~ radial direction / integrate out radial shells tracking topological data:

$$\mathcal{L}_{TFT_{d+1}} = \int_{\partial X} \mathcal{L}^{SUGRA}_{Top}$$

$$\propto F_5 \wedge B_2 \wedge dC_2$$

4D $\mathcal{N}=4$ SYM Sandwich:

4D 5D 4D

$$S_0 = \frac{N}{2\pi} \int_{\mathbb{R}^{1,3} \times [0,\infty)_r} B_2 \wedge dC_2$$

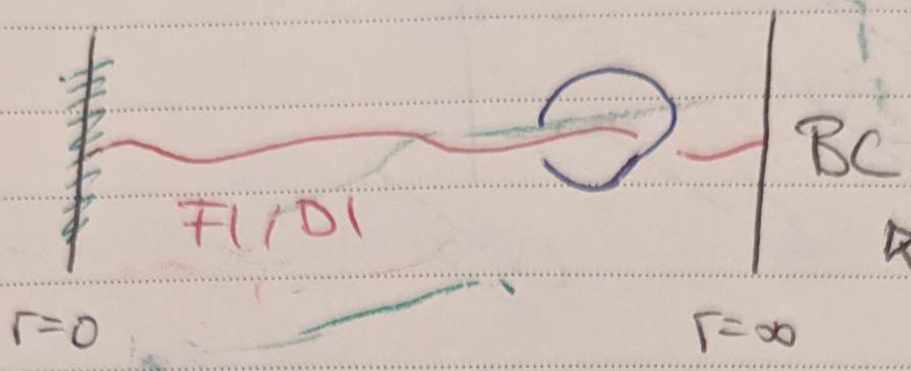
D3s, $\mathfrak{g} = SU(N)$

Neumann / Dirichlet

[W, 98]

Determine global form of gauge group

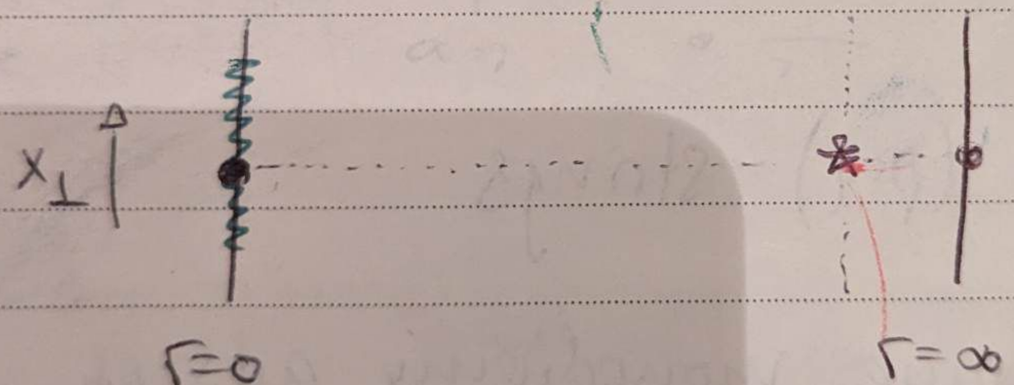
Defect & Sym Ops.:



determines defects

Wilson / 't Hooft Lines
=

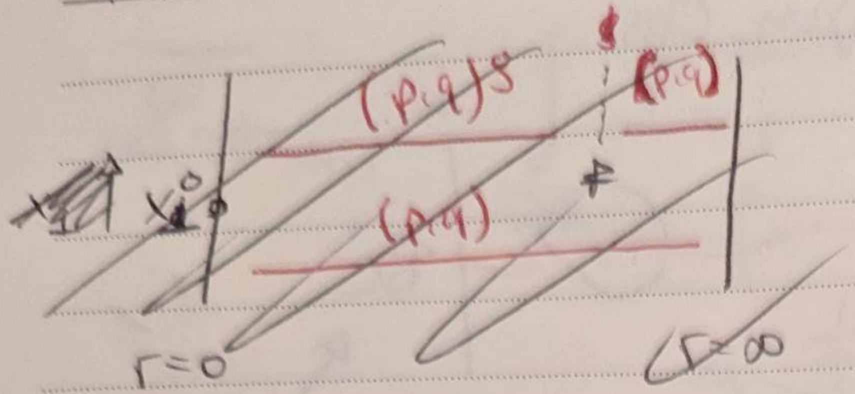
7-brane on S^5 + branchcut choice



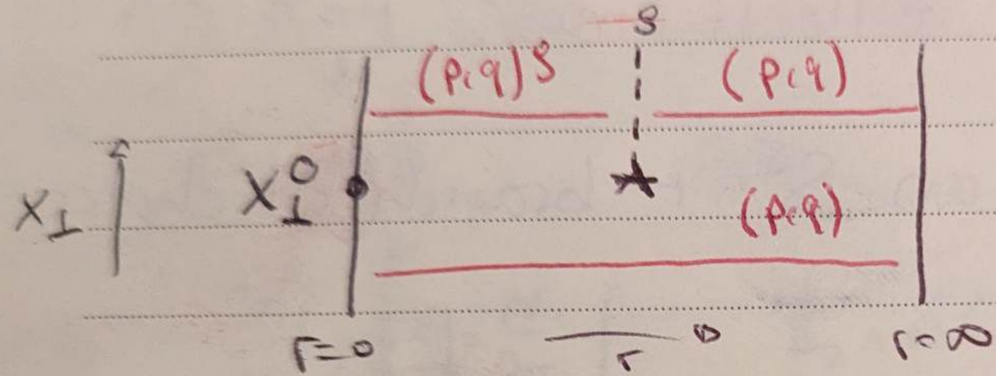
$$S_{TFT} = S_0 + S_1 + S_2 \quad (\text{index} = \text{codim})$$

Branchcut 7-brane / S^5

Date



Branchcut w/ const. r^0



Defects: (p,q)-strings

⇒ 7-brane monodromy acts on Defects

⇒ Half spaces w/ $x_\perp > x_\perp^0$; $x_\perp < x_\perp^0$ different global forms

⇒ Half-space gauging construction for duality Interface / defect

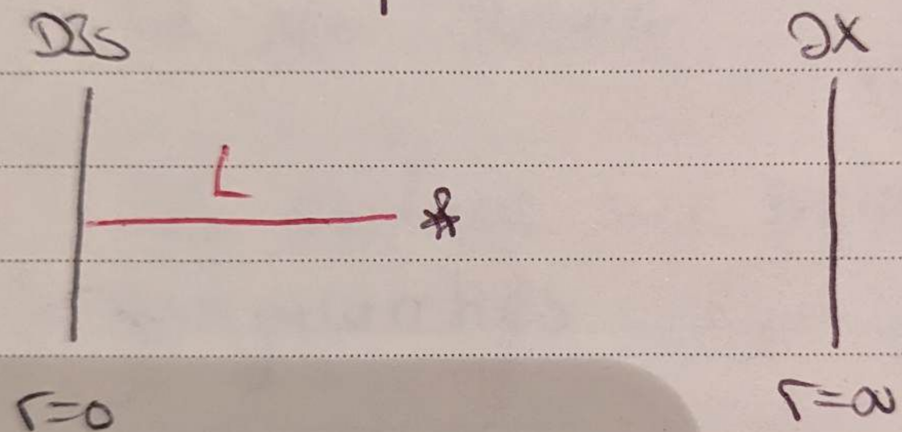
Branch cut action:

$$S_1 = \frac{2\pi i}{N} \int_{\text{cut}} \frac{\mathcal{P}(B_2^g)}{2}$$

w/ monodromy eigenvector B_2^g

7-brane ~~action~~ / S^5 : 3d TFT

$\rightsquigarrow$ study 3d lines L



$$S_2 = \mathcal{A}^{k,p}(B_2^g) \otimes \mathbb{I}$$

minimal abelian 3d TFT

completely decoupled