Phys. 2306 Exam 2 F09 Form A

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In Thomson’s atomic model of Helium, electrons are considered point charges and are like chocolate chips in cookie dough. In this case the dough is the alpha particle, treated as a sphere of radius R with total charge 2e and uniform volume charge density. The electrons can move freely inside the alpha. Place the origin at the center of the alpha and the two electrons, each of charge -e inside the alpha at \((x,y,z)= (±d,0,0)\), where \(d < R\).

1) Consider the electron at \(x= d\), what magnitude of force does it feel due to the alpha?

a) \(\frac{e^2R}{2\pi\varepsilon_0d^2}\)  b) \(\frac{e^2d}{2\pi\varepsilon_0R^2}\)  c) \(\frac{e^2}{2\pi\varepsilon_0d^2}\)  d) \(\frac{e^2(R-d)}{2\pi\varepsilon_0d^2}\)  e) none of these

The electric field at radius \(r\) inside a sphere of radius \(R\) with uniform charge density \(\rho = \frac{2e}{4\pi R^3} \text{ C/m}^3\) depends only on the radial distance \(r\) and is directed radially outwards. So draw a Gauss sphere of radius \(r=d\) and get,

\[
E(d) = \frac{2ed}{4\pi\varepsilon_0R^3} = \frac{ed}{2\pi\varepsilon_0R^3}, \quad \text{(b)}.
\]

2) What magnitude of force does it feel from the other electron?

a) \(\frac{e^2R}{4\pi\varepsilon_0d^2}\)  b) \(\frac{e^2}{16\pi\varepsilon_0d^2}\)  c) \(\frac{e^2}{4\pi\varepsilon_0d^2}\)  d) \(\frac{e^2R}{16\pi\varepsilon_0d^2}\)  e) none of these

This force is between two point charges so \(F_e = \frac{e^2}{4\pi\varepsilon_0(2d)^2} = \frac{e^2}{16\pi\varepsilon_0d^2}, \quad \text{(c)}\).

3) If the electrons are at rest in equilibrium, what is \(d\)?

a) \(R/\sqrt{2}\)  b) \(R/\sqrt{3}\)  c) \(R/2\)  d) \(R/(2)^{1/3}\)  e) none of these
\( F_{\alpha,e} \) is attractive, repulsive so they can add to zero for equilibrium.

\[
0 = \frac{e^2 d}{2\pi\varepsilon_0 R^3} - \frac{e^2}{16\pi\varepsilon_0 d^2}
\]

\[
0 = \frac{d}{R^3} - \frac{1}{8d^2}
\]

\[
d^3 = \frac{R^3}{8} \quad \text{so} \quad d = R/2, \ (c).
\]

4) Suppose there were no electrons, this is a doubly ionized atom, what is the potential at the surface of the alpha? The potential far away is zero.

a) \( \frac{e}{4\pi \varepsilon_0 R^2} \)  

b) \( \frac{e}{2\pi \varepsilon_0 R^2} \)  

c) \( \frac{e^2}{4\pi \varepsilon_0 R} \)  

d) \( \frac{e}{4\pi \varepsilon_0 R} \)  

e) none of these

Outside the sphere of radius \( R \), \( E(r) = \frac{2e}{4\pi \varepsilon_0 \cdot r^3}. \)

\[
V(R) - V(\infty) = V(R) = -\int_{\infty}^{R} E \cdot dr
\]

\[
= -\frac{e}{2\pi \varepsilon_0} \int_{\infty}^{R} dr/r^2 = \frac{e}{2\pi \varepsilon_0} (1/R - 1/\infty) = \frac{e}{2\pi \varepsilon_0 R}, \ (e).
\]

5) Like the last problem, but now we want the ratio of the potential inside the sphere to the potential at \( R \). This ratio \( r = V(< R)/V(R) \) is

a) \(< 1 \)  

b) \( = 1 \)  

c) \( 1 < r \leq 1.5 \)  

d) \( 1.5 < r < 2 \)  

e) none of these

We now integrate from \( \infty \) to \( D \), where \( D < R \).

\[
V(D) = -\int_{\infty}^{D} E \cdot dr = V(R) - \int_{R}^{D} E_{in} \cdot dr
\]

\[
= V(R) - \frac{e}{2\pi \varepsilon_0 R^3} \int_{R}^{D} rdr = V(R) + \frac{e}{4\pi \varepsilon_0 R^3} (R^2 - D^2)
\]

\[
= \frac{e}{2\pi \varepsilon_0 R} (1 + (1 - (D/R)^2)/2)
\]

\[
= 1.5 \frac{e}{2\pi \varepsilon_0 R} \quad \text{when} \quad D = 0.
\]

Thus \( 1 < V(< R)/V(R) < 1.5, \ (c). \)
The potential difference, $V_a - V_b \equiv V_{ab} = 1.0 \times 10^2$ V. Capacitors C are identical and their capacitance $= 1.00 \times 10^{-8}$ F. The capacitors were uncharged before the connection to the power source.

6) When switch S is open, what is the potential difference $V_a - V_e \equiv V_{de}$ in V?

a) 0 b) 25 c) 50 d) 75 e) none of these

Here all 4 capacitors are in series so the equivalent capacitance is C/4 and the charge on the positive plate of each is $V_{ab}C/4$. So $V_{de} = 2(V_{ab}C/4)/C = V_{ab}/2 = 50$ V, (c).

7) When switch S is closed, what is the potential difference $V_{de}$ in V?

a) 0 b) 25 c) 50 d) 75 e) none of these

Here the two capacitors to the right of the switch are in series so the equivalent capacitance is C/2. This equivalent capacitance is in parallel with the capacitor in the switch line, so the equivalent capacitance is C’=3C/2. The latter is in series with the two capacitances closest to $V_{ab}$ so the total equivalent capacitance is $1/C'' = 1/C + 1/C + 2/(3C)$ or $C''=3C/8$. The charge on each C and C’ is $3CV_{ab}/8$ and so the voltage across C’ is $(3CV_{ab}/8)/(3C/2)=V_{ab}/4=25$ V, (b).

8) Consider the energy U stored in all capacitors. What is the difference in the stored energy $U(S$ closed$)-U(S$ open$)$ in $10^{-5}$ J?

a) -5.0 b) 5.0 c) -0.63 d) 0.63 e) none of these

$U(open)=V_{ab}^2(C/4)/2=CV_{ab}^2/8$. $U(closed)=V_{ab}^2(3C/8)/2=3CV_{ab}^2/16$. The difference, $U(S$ closed$)-U(S$ open$)=(CV_{ab}^2/8)(3/2-1)=CV_{ab}^2/16=(1.00\times10^{-8})(10^4)/16$ J $= 0.63\times10^{-5}$ J, (d).

9) The power source is now disconnected and an identical uncharged capacitor is connected between points a and b. What charge does this added capacitor acquire in $10^{-8}$ C?
a) 27 b) 13 c) 6.8 d) 0 e) none of these

When the power source is disconnected equivalent capacitance $C''$ keeps its charge $3CV_{ab}/8$ (where could it go?). The new capacitance $C$ and $C''$ are in parallel and so the have the same potential difference. This means charge must rearrange so that some of it is now on the added capacitance. Thus,

\[
Q + Q'' = 3CV_{ab}/8 \\
Q/C = Q''/C'' \quad \text{so} \quad Q'' = QC''/C \\
Q(1 + C''/C) = Q(1.375) = 3CV_{ab}/8 \\
Q = 0.375CV_{ab}/1.375 = 0.27 \times 10^{-8}\text{100} \quad C = 27. \times 10^{-8} \quad C, (a).
\]

10) Due to this added capacitor, the energy stored in all the capacitors is, in $10^{-5}$ J?

a) 0.36 b) 0.81 c) 1.8 d) 2.6 e) none of these

\[
U = 0.5(Q^2/C + Q''^2/C'') = 0.5(Q^2/C + (QC''/C)^2/C'') \\
= (0.5Q^2/C)(1 + C''/C) = 0.5(27^2)10^{-8}(1.375) = 0.50 \times 10^{-5} \quad J, (e).
\]

The energy stored went down because some was converted to heat and radiation due to the charge flow.

11) Suppose the capacitors were concentric cylinders with air between the cylinders. If the added capacitor of problem 9) were to have an insulator of dielectric constant $K=2$ between the cylinders, what would happen to the results of problems 9 and 10?

a)Q in- U de-creases Q b) Q and U decrease c)Q and U increase d) Q de- U in-creases e) insufficient information to tell.

The capacitance of the added capacitor is $2C$, instead of $C$ so following the solutions of problems 9 and 10 as below you note Q increases and U decreases, (a).
The vertical deflecting plates of a typical oscilloscope are a pair of parallel, square, metal plates carrying the signal you want to examine on one plate while grounding the other plate. Typical dimensions are 3.00 cm on a side with a spacing of 0.500 cm. The plates are close enough so that the fringing fields are negligible. The plates form part of a cathode ray tube (CRT). The plates are parallel to the x axis and the line perpendicular to the plates is parallel to the y axis. The screen of the CRT, parallel to the y axis is 12.0 cm from the nearest end of the plates. The CRT manufacturer projects a beam of electrons with a velocity $6.50 \times 10^6 \text{m/s}$ along the axis and midway between the plates ($y=0$).

12) How much time is an electron between the plates in ns?

a) 1.23 b) 2.76 c) 4.62 d) 5.97 e) none of these

$$T = \frac{L(\text{plate})}{v} = \frac{3.00 \times 10^{-2}}{6.50 \times 10^6} = 0.462 \times 10^{-8} = 4.62 \text{ ns, (c).}$$

13) If during this time the voltage on the ungrounded top plate is \(-1.50\) V, what is the y component of the electron’s velocity after passing through the plates in \(10^6\) m/s? The magnitudes of the electron’s charge and mass are $1.60 \times 10^{-19}, 9.11 \times 10^{-31}$ C,kg.

a) 0.351 b) 0.243 c) -0.351 d) -0.243 e) none of these

E points in +y direction as top plate is more negative than ground. Electron has negative charge so it moves in -y direction, opposite E.

$$v_y = -a_yT = \frac{FT}{m} = \frac{eET}{m} = \frac{eVT}{md}$$
\[ y_p = \frac{a_y T^2}{2} = \frac{v_y T}{2} \]
\[ = -0.243 \times 10^6 \left(\frac{4.62}{2}\right) \times 10^{-9} \times 10^3 \ mm = -0.561 \ mm, (c). \]

15) What is the y position in mm when the electrons hit the CRT screen?

a) 4.05 b) 2.16 c) -4.05 d) -2.16 e) none of these

The time from the exit of the plate to the screen is \( \frac{D}{v} = \frac{0.12}{6.50 \times 10^{-6}} = 18.46 \times 10^{-9} \) s. Outside the plates the electrons move with constant \( v_y = -0.243 \times 10^6 \) m/s. So \( y_o = -0.243(18.46) \ mm = 4.486 \ mm. \) The net \( y = y_p + y_o = -(4.486 + 0.561) \ mm = -5.05 \ mm, (e). \)

A voltmeter should be connected across the terminals of the device whose potential difference is desired. An ammeter should be connected in series with the device whose current is desired. In this case consider the voltmeter and ammeter as “ideal”. Now suppose a series circuit composed of a battery of \( E = 5.00 \ V \) with internal resistance \( r = 1.00 \ \Omega \) and an external resistance \( R=4.00 \ \Omega \). If the voltmeter is connected across the battery and the ammeter is connected in series with the battery and \( R \),

16) What does the ammeter read in A?

a) 5.00 b) 4.00 c) 1.00 d) 0 e) none of these

A ideal voltmeter has infinite resistance so it doesn’t draw current if connected across a device, while an ideal ammeter has zero resistance and so doesn’t change the current if connected in series. Therefore, in this case,
I = \frac{5.00}{4.00+1.00} \ A = 1.00 \ A, \ (c).

17) What does the voltmeter read in V?

a) 5.00  b) 4.00  c) 1.00  d) 0  e) none of these

V = E - Ir = 5.00 - 1.00 \times 1.00 \ V = 4.00 \ V, \ (b).

18) Now suppose a mistake is made. The voltmeter is connected in series with the battery and R and the ammeter is not used. What does the voltmeter read in V?

a) 5.00  b) 4.00  c) 1.00  d) 0  e) none of these

Since the voltmeter has infinite resistance, \( R_{eq} = \infty \), I = 0 and V = 5.00 V, \ (a).

19) Now suppose a different mistake is made. The ammeter is connected across the battery or equivalently R and the voltmeter is not used. What does the ammeter read in A?

a) 5.00  b) 4.00  c) 1.00  d) 0  e) none of these

Since the ammeter has zero resistance all the current flows through the ammeter and none flows through R. Then \( R_{eq} = r \) so I = \frac{5.00}{1.00} \ A = 5.00 \ A, \ (a).

20) What is the voltage across the battery in V, in 19)?

a) 5.00  b) 4.00  c) 1.00  d) 0  e) none of these

V = 5.00 - 5.00(1.00) = 0, \ (d).