Robustness of error-suppressing entangling gates in cavity-coupled transmon qubits

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(Received 30 March 2017; revised manuscript received 5 June 2017; published 27 July 2017)

Superconducting transmon qubits comprise one of the most promising platforms for quantum information processing due to their long coherence times and to their scalability into larger qubit networks. However, their weakly anharmonic spectrum leads to spectral crowding in multiqubit systems, making it challenging to implement fast, high-fidelity gates while avoiding leakage errors. To address this challenge, we use a protocol for speeding up waveforms by inducing phases to harmful transitions (SWIPHT) [S. E. Economou and E. Barnes, Phys. Rev. B 91, 161405(R) (2015)], which yields smooth, simple microwave pulses designed to suppress leakage without sacrificing gate speed through spectral selectivity. Here, we determine the parameter regimes in which SWIPHT is effective and demonstrate that in these regimes, it systematically produces two-qubit gate fidelities for cavity-coupled transmons in the range 99.6%–99.9% with gate times as fast as 23 ns. Our results are obtained from full numerical simulations that include current experimental levels of relaxation and dephasing. These high fidelities persist over a wide range of system parameters that encompass many current experimental setups and are insensitive to small parameter variations and pulse imperfections.

DOI: 10.1103/PhysRevB.96.035441

I. INTRODUCTION

Rapid progress in the coherence and control of superconducting qubits over the past decade has made them a frontrunner in the quest for viable quantum computing platforms [1–4]. High-fidelity single- and multiqubit operations [5–9], as well as initial demonstrations of algorithms and error-correcting codes [10–14], have been implemented in several multiqubit devices, and coherence times on the order of several tens of microseconds and above are now achieved regularly [15–20]. Perhaps the most promising of these is transmon qubits, in which insensitivity to charge noise is achieved by reducing the capacitive energy relative to the Josephson energy through the use of a large shunt capacitor, leading to a flattening of the charge dispersion of the energy levels [21–23].

There are two general approaches to implementing two-qubit gates in superconducting qubits. For tunable qubits such as two-dimensional (2D) transmons [21] or Xmons [23], dc magnetic fields are used to set qubit energies and other circuit parameters. In many systems, such fields are also used to implement gates by temporarily bringing the system to a special parameter regime (e.g., a two-qubit resonance), where it is held idle until different states accumulate the relative phases appropriate for a desired operation [24–26]. The main disadvantage of this approach is the reliance on flux-tunable qubits, which can have reduced coherence times due to flux noise [27].

The second general approach to gate implementation is to drive one or more qubits with modulated ac microwave pulses. This method leads to less noise since the qubit energies are held fixed, and it is the only option for systems with nontunable qubits [28–35]. The primary challenge with this approach stems from spectral crowding: a system of several coupled, weakly anharmonic qubits such as transmons possesses a dense energy spectrum with many closely spaced transitions. Faster gates are generally preferred since they allow for faster algorithms. However, faster pulses have broader bandwidth and can thus lead to the unintended excitation of transitions that are nearly degenerate with the target transition(s), causing phase and leakage errors. On the other hand, using spectrally narrower, slower pulses to avoid this problem increases exposure to relaxation and decoherence.

To date, there have been several works that address this problem in the context of single-qubit gates by devising pulses that avoid the harmful transitions, either by numerical pulse shaping [36] or by engineering the pulse spectrum to contain sharp holes at the frequencies of the unwanted transitions [37–41]. Recent experiments implementing microwave-driven two-qubit entangling gates in transmon devices have reported gate times and fidelities ranging from 300–500 ns and 87–97% [6,13,17]. While there has been recent progress in designing fast leakage-suppressing two-qubit gates using numerical pulse shaping [42], there remains a need for fast high-fidelity gates based on simple pulses.

Instead of attempting to avoid harmful unwanted transitions, two of us proposed a protocol for speeding up waveforms by inducing phases to harmful transitions (SWIPHT) [43] to achieve fast, high-fidelity two-qubit gates by purposely driving the nearest harmful transition such that the corresponding subspace undergoes trivial cyclic evolution. This minimizes leakage errors and significantly enhances gate fidelities without resorting to slow, spectrally selective pulses. While Ref. [43] demonstrated the efficacy of SWIPHT for a set of typical parameters, a full examination of its regime of validity and its robustness to parameter variations and decoherence has yet to be carried out.

In this paper, we fill this gap by providing a detailed investigation of the robustness of the SWIPHT protocol for two-qubit CNOT gates. We show that there exist wide fidelity plateaus in the qubit-frequency landscape where the fidelity remains above 99.9%. We also find that with our method, we are able to maintain the CNOT fidelity at 99.9% while decreasing the gate time to tens of nanoseconds by exploiting resonances between ground- and excited-state transitions. We further demonstrate the robustness of these results to decoherence and relaxation.

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Here, \( a^\dagger(a), b_{i,j}^\dagger(b_{i,j}) \) are creation (annihilation) operators for the cavity and transmons, respectively, \( \omega_{1,2} \) denote the energy splittings between the lowest two states of each transmon, \( \omega_{1,2} \) are the anharmonicities, and \( g_{1,2} \) are the coupling strengths between each transmon and the cavity. Working in the Fock basis \(|n,i,j\rangle\), where \( n \) is the number of cavity photons and \( i,j \) denote the energy levels of transmon 1 and 2, respectively, we diagonalize \( H_0 \) to obtain the dressed eigenstates. In the dispersive regime and with \( g_{1,2} \ll (\omega_c,\omega_p), \) each dressed state has a large overlap with one of the bare Fock states; hence, we use \( n,i,j \) to denote the dressed states, but with an additional tilde: \(|\tilde{n},\tilde{i},\tilde{j}\rangle\).

We define our computational two-qubit states to be the dressed states \(|000\rangle, |001\rangle, |010\rangle, |011\rangle\), which are very close to the bare states, \(|00\rangle, |01\rangle, |10\rangle, |11\rangle\), for typical system parameters. The splittings between the bare states \(|00\rangle, |01\rangle \) and between \(|10\rangle, |11\rangle \) are equal, as are those between \(|00\rangle, |10\rangle \) and between \(|01\rangle, |11\rangle \). These degeneracies are slightly broken in the dressed states due to the finite couplings \( g_{1,2} \), allowing one to perform two-qubit entangling gates by driving only one transition, e.g., driving the \(|000\rangle \leftrightarrow |010\rangle \) transition can implement a CNOT gate:

\[
\text{CNOT} = e^{i\phi_0} |000\rangle\langle 001| + e^{i\phi_1} |010\rangle\langle 000| + e^{i\phi_0} |001\rangle\langle 000| + e^{i\phi_1} |011\rangle\langle 011|.
\]

Here, we generalize the standard CNOT by including arbitrary phases \( \phi_0, \phi_1 \); this generalized CNOT is maximally entangling and is locally equivalent to the standard CNOT. In particular, the two are related by single-qubit Z gates, which have recently been experimentally demonstrated for fixed-frequency qubits [44,45].

The CNOT gate in Eq. (2) can be implemented by driving only the first transmon with a microwave \( \pi \) pulse that is resonant with the \(|000\rangle \leftrightarrow |010\rangle \) transition. The total Hamiltonian can be written in the bare eigenbasis as

\[
\mathcal{H}(t) = H_0 + b_1 \Omega(t)e^{i\omega_pt} + b_1^\dagger \Omega(t)e^{-i\omega_pt},
\]

where \( \Omega(t) \) and \( \omega_p \) are the pulse envelope and frequency, respectively. In the dispersive regime, the simplest way to ensure that this transition is the only one excited by the pulse is to use a very narrowband pulse—an approach which necessarily leads to long gate times. To avoid making this sacrifice in gate speed, we instead employ the SWIPHT method [43,46] to remove the effects of inadvertently driving unwanted transitions without resorting to spectrally narrow, slow pulses.

For typical experimental values of the qubit-cavity couplings \( g_{1,2} \), there is exactly one nearest “harmful” transition, namely the \(|001\rangle \leftrightarrow |011\rangle \) transition, which competes with the target transition, \(|000\rangle \leftrightarrow |010\rangle \). The SWIPHT protocol calls for purposely driving this transition in such a way that the net evolution operator in this subspace is proportional to an identity operation.

In the computational two-qubit subspace spanned by the states \(|000\rangle, |010\rangle, |001\rangle, |011\rangle \) (note the unconventional basis ordering), the Hamiltonian of the driven transmon-cavity system is approximately

\[
H_d \approx \begin{pmatrix}
-E/2 - \epsilon & \Omega(t)e^{i\omega_pt} & 0 & 0 \\
\Omega(t)e^{-i\omega_pt} & E/2 - \epsilon & 0 & 0 \\
0 & 0 & -\left(E - \delta\right)/2 & \Omega(t)e^{i\omega_pt} \\
0 & 0 & \Omega(t)e^{-i\omega_pt} & -\left(E - \delta\right)/2
\end{pmatrix},
\]

where \( E \) is the energy splitting between \(|000\rangle \) and \(|010\rangle \), and \( E - \delta \) is the splitting between \(|001\rangle \) and \(|011\rangle \). We have shifted the overall energy by \( -E/2 - \epsilon \), where \( \epsilon + \delta/2 \) is the energy of state \(|001\rangle \). We denote the pulse duration by \( \tau_p \). We have also neglected the subleading terms in the off-diagonal 2 \( \times \) 2 blocks (but not in the simulations). To implement a SWIPHT CNOT gate, we set \( \omega_p = E \) and engineer \( \Omega(t) \) such that the evolution operator generated by \( H_d \) coincides with the CNOT gate given in Eq. (2) with \( \phi_0 = 0 \). Matching the form of the upper-left 2 \( \times \) 2 subspace requires the area of the pulse to be given by \( \int_0^{\tau_p} dt \Omega(t) = \pi/2 \), as is consistent with a \( \pi \) pulse.

Engineering the evolution operator in the lower-right 2 \( \times \) 2 subspace to be an identity operation at time \( t = \tau_p \) is more challenging since it is not possible to analytically solve the Schrödinger equation for an off-resonant pulse with arbitrary envelope \( \Omega(t) \). We can overcome this difficulty by making use of a partial-reverse engineering formalism introduced in Refs. [47,48]. In Ref. [43], this formalism was used to obtain the pulse shown in Fig. 1, which implements a CNOT gate with fidelity >99% in 35.4 ns. A brief review of the construction of this pulse is given in Appendix A. The duration of the
pulse is given by $\tau_p = 5.87/|\Delta|$, where $\Delta = \omega_p - (E - \delta)$ is the detuning of the pulse relative to the harmful transition. For $\omega_p = E$, we have $\Delta = \delta$, and thus $\tau_p$ depends on the system parameters through the dependence on the transition frequency difference $\delta$, which is due to the cavity-mediated coupling. For the parameters considered in Ref. [43] (summarized in the caption of Fig. 1), $\delta = 26.4$ MHz.

### III. NUMERICAL RESULTS AND ROBUSTNESS

#### A. Dependence of gate fidelity on qubit frequencies

First, we study the dependence of the CNOT fidelity and gate speed on the transmon frequencies. For the moment, we neglect relaxation and dephasing, although these effects will be included below. In this case, we define the gate fidelity as in Ref. [49], which accounts for leakage outside the computational two-qubit subspace:

$$F_{\text{gate}} = \frac{1}{20} [\text{Tr}(MM^\dagger) + |\text{Tr}(M)|^2],$$

where $M = U_{\text{ideal}}U^\dagger$ and $U$ is the actual evolution operator, while $U_{\text{ideal}}$ is the target gate operation, here taken as the CNOT gate defined in Eq. (2). We solve the time-dependent Schrödinger equation for the evolution operator generated by our analytical SWIPHT pulse keeping three cavity and four transmon states, for a total of 48 states. The number of states was increased until convergence in the results was achieved. For each set of system parameters, we optimize over the phases $\phi_\mu$. Our numerical results for $F_{\text{gate}}$ and $\tau_p$ are shown in Fig. 2. The most important features of Fig. 2(a) are the large plateaus where $F_{\text{gate}}$ is well above 0.999 (dark red); these occur in regions where $\omega_{1,2}$ are detuned from the three sharp linear features evident in the figure. The central feature corresponds to the qubit-qubit resonance, $\omega_1 = \omega_2$, while the two “secondary” resonances occur where $\omega_1 = \omega_2 + \delta_1$ or $\omega_1 = \omega_2 - \delta_2$, corresponding to the $|0\rangle \leftrightarrow |1\rangle$ transition of one qubit becoming degenerate with the $|1\rangle \leftrightarrow |2\rangle$ transition of the other. Near these resonances, additional harmful transitions become important, causing a decrease in fidelity. Further details regarding these resonances can be found in Appendix B. This figure also exhibits an asymmetry between the dependencies on $\omega_{1,2}$ caused by the fact that only transmon 1 is driven. Since the high-fidelity plateau is broader for $\omega_1 < \omega_2$, we see that it is more advantageous to drive the transmon that is further detuned from the cavity.

Figure 2(b) reveals that there is significant overlap between the high-fidelity plateaus and the parameter regions where the gate times are below 150 ns (blue lines). The fastest pulses occur near the secondary resonances because these give rise to a larger splitting, $\delta$, between the target and harmful transitions, which in turn reduces the SWIPHT gate time since $\tau_p \sim 1/|\delta|$. Further details can be found in Appendix B. Figures 2(c) and 2(d) show the CNOT fidelity and gate time along two one-dimensional slices in qubit-frequency space. Importantly, we see that while the fidelity quickly increases up to above 0.999 as $\omega_1$ is tuned away from a secondary resonance, the gate time increases more slowly. Thus, the best combination of low gate time and high fidelity is achieved when the system lies close to a secondary resonance. From Fig. 2(d), which shows a slice closer to the cavity frequency, $\omega_c = 7.15$ GHz, we in fact see that as $\omega_1$ is reduced, the gate time saturates at around 150 ns while the fidelity continues to improve. Below, we show...
FIG. 3. (a) CNOT fidelity and (b) gate time (μs) vs qubit frequencies for $g_{1,2} = 250$ MHz.

that the gate time can be further reduced by more than a factor of 6 by adjusting system and pulse parameters appropriately.

Figure 3 shows zoomed-out versions of Figs. 2(a) and 2(b), where the full extent of the broad high-fidelity plateaus is more evident.

B. Performance under relaxation and dephasing

Next, we evaluate the impact of relaxation and decoherence on our gate by solving the Lindblad equation,

$$\dot{\rho} = i[\rho, \mathcal{H}(t)] + \sum_{\ell=1,2} \left( \frac{1}{T_1} \mathcal{D}[b_\ell] + \frac{1}{T_\phi} \mathcal{D}[b_\ell^\dagger b_\ell] \right).$$

with $\mathcal{D}[L] = L\rho L^\dagger - \frac{1}{2} [L^\dagger L, \rho]$. The first Lindblad term corresponds to qubit relaxation (time scale $T_1$), while the second corresponds to pure dephasing (time scale $T_\phi$) caused by charge fluctuations [21,22]. Here, $1/T_2 = 1/2T_1 + 1/T_\phi$.

We have neglected cavity decay in our simulation because its time scale is typically much larger than $T_1$ and $T_2$ and because our gate scheme causes minimal cavity excitation. With noise terms included, $\mathcal{F}^{\text{gate}}$ is no longer a suitable definition of fidelity and we instead perform quantum state tomography. We prepare 16 input states in total, chosen from the set $\{|0\rangle, |1\rangle, (|0\rangle + |1\rangle)/\sqrt{2}, (|0\rangle + i|1\rangle)/\sqrt{2}\}$ for each qubit. We calculate the average fidelity, defined as $\mathcal{F} = \frac{1}{16} \sum_{j=1}^{16} \text{Tr}(\rho^{\text{ideal}}_j \rho^{\text{sim}}_j)$, where $\rho^{\text{ideal}}_j$ is the ideal target state, while $\rho^{\text{sim}}_j$ is the final density matrix obtained by solving Eq. (5). We again use a 48-state Hilbert space to achieve convergence and optimize over $\phi_\mu$.

We first study the dependence of $\mathcal{F}$ on the pulse detuning $\Delta$. Figure 4(a) shows this dependence with and without noise, where it is clear that a slight detuning away from the idealized value based on $H_{\text{cs}}$ ($\Delta = \delta = 26.4$ MHz) down to $\Delta = 25.5$ MHz brings the fidelity up to 0.9999 without noise or up to 0.9963 with noise for typical experimental values of $T_1$, $T_2$. The figure also shows that this improvement comes with a slight increase in the gate time from 35.4 up to 36.6 ns.

In Fig. 4(b), we show the dependence of the fidelity on $T_\phi$ for $T_1 = T_2 = 20$ μs. The black crosses indicate $\Delta = \delta$. The dashed red line shows the gate time. (b) Average fidelity vs dephasing time $T_\phi$ for $\Delta = 25.5$ MHz and pulse duration is 36.6 ns, which are the optimal values found in (a).

C. Asymmetry of coupling strength and anharmonicity

In a real setup, the qubit-cavity coupling strengths $g_1, g_2$ may differ. Figure 5(a) shows that the fidelity remains > 0.995 even when the couplings differ by more than 20%. We also find that further optimization of the gate is possible if the coupling of the undriven transmon (here $g_2$) is tuned to be slightly larger than that of the driven qubit ($g_1$). The figure further shows that optimized local phases $\phi_\mu$ that enter into the generalized CNOT gate [Eq. (2)] are not sensitive to experimental uncertainties in parameter values.

FIG. 4. (a) Fidelity vs detuning $\Delta$ for system parameters in Fig. 1. The cyan line is the fidelity for the noiseless case, while the blue line is for $T_1 = T_2 = T_\phi/2 = 20$ μs. The black crosses indicate $\Delta = \delta$. The dashed red line shows the gate time. (b) Average fidelity vs dephasing time $T_\phi$ for $\Delta = 25.5$ MHz and pulse duration is 36.6 ns, which are the optimal values found in (a).
FIG. 5. (a) Fidelity and gate time vs coupling asymmetry and (b) fidelity vs anharmonicity asymmetry. The system parameters are as in Fig. 1, except as shown ($g_2$ and $\alpha_1$ are varied), and $T_1 = T_2 = 20 \mu s$.

the gate time is simultaneously reduced to as low as 23 ns, while the fidelity remains above 0.996 even in the presence of relaxation and dephasing ($T_1 = T_2 = 20 \mu s$).

SWIPHT is similarly robust against anharmonicity differences. So far, we assumed that both qubits share the same value of anharmonicity, $\alpha_1 = \alpha_2$, for simplicity. We have rerun the simulations shown in Fig. 3 for $\alpha_1 = 350$ MHz, $\alpha_2 = 300$ MHz. The results are essentially unchanged from those shown in Fig. 3 except for a shift in the location of one secondary resonance as follows trivially from the change in $\alpha_2$. In Fig. 5(b), we show the SWIPHT CNOT gate fidelity versus asymmetry in anharmonicity between the two transmons. It is clear from the figure that not only is the SWIPHT gate robust against anharmonicity differences, but that such differences can even lead to further improvement in the fidelity.

D. Pulse deformation

Next, we consider the robustness of the results to Gaussian-type pulse deformations of the form

$$\Omega(t) = (1 - p) \ast \Omega_{\text{SWIPHT}}(t) + p \ast \Omega_{\text{Gaussian}}(t),$$

where $\Omega_{\text{SWIPHT}}(t)$ is the pulse shown in Fig. 1. The Gaussian pulse, $\Omega_{\text{Gaussian}}$, is chosen to have the same area ($\pi/2$) and duration ($35.4$ ns) as the SWIPHT pulse. Explicitly, we use

$$\Omega_{\text{Gaussian}}(t) = A_G e^{-(t - t_G)/2\tau_G^2},$$

where $A_G = 2\pi \times 18.8$ MHz and $t_G = 0.15 \times 35.4$ ns. The resulting pulses for three different values of $p$ are shown in Fig. 6(a).

Fig. 6(a). Figure 6(b) shows the fidelity as a function of $p$ (with relaxation and dephasing included), where it is evident that the gate performance is essentially unchanged for deformations up to the 10% level, further highlighting the robustness of our gate. In Fig. 6(c), we show a comparison of the SWIPHT and pure Gaussian pulses; we see that the SWIPHT pulse performs dramatically better for gate times by the order of a few tens of nanoseconds.

Pulse deformations can also result from the finite-time resolution of a pulse generator. In Fig. 7, we show the SWIPHT fidelity versus time resolution. The plateau of fidelity that persists up to 4 ns shows that SWIPHT is very robust to these pulse deformations. These findings demonstrate

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that SWIPHT is effective even with modest pulse-shaping capabilities.

IV. CONCLUSION

In conclusion, we have shown that the SWIPHT method can produce CNOT gates in cavity-coupled transmon systems with fidelities well above 99.5% and gate times below 30 ns, even when realistic levels of decoherence, relaxation, and parameter uncertainties are taken into account. In general, we find that SWIPHT performs well when the degeneracy between target and harmful transitions is strongly broken, either through strong qubit-cavity couplings, reduced qubit-cavity detunings, or transition resonances. Our work is of immediate use to ongoing experimental efforts to optimize the performance of transmon systems operated with microwave control.

ACKNOWLEDGMENT

We would like to thank A. Lupascu for interesting discussions and helpful comments.

APPENDIX A: SWIPHT PULSE SHAPE

In this Appendix, we review how the analytical pulse used to implement the SWIPHT CNOT gate is derived. As described in the main text, in order to implement this gate, we must design a pulse that implements a π rotation about x on the target transition and an identity operation on the harmful transition. The former is achieved by making the pulse resonant with the target transition and choosing the pulse area and strength that achieves this evolution using the formula

$$\Omega(t) = \frac{\dot{x}}{2\sqrt{\Delta^2/4 - \dot{x}^2}} = \sqrt{\frac{\Delta^2}{4} - \dot{x}^2} \cot(2\chi).$$  \hspace{1cm} (A1)$$

where Δ is the detuning of the pulse relative to the harmful transition. Since the pulse is chosen to be resonant with the target transition, we have Δ = δ, where δ is the detuning between the target and harmful transitions. In Ref. [43], it was shown that achieving an identity operation on the harmful transition requires that the following conditions be satisfied:

$$\chi(0) = 0, \chi(t) = \pi/4, \dot{x}(t) = 0, \left|\chi(t)\right| \leq \frac{\pi}{2}, \text{ and } \psi_\pm(t) = \frac{\dot{x}^2}{4\Delta},$$

where $\psi_\pm(t) = \int_0^t dt' \sqrt{\Delta^2/4 - \dot{x}^2(t')} \csc(2\chi(t')) \pm \frac{x}{4} \arcsin(2\chi(t)/\Delta)$.

A choice of $\chi(t)$ satisfying these conditions was found to be

$$\chi(t) = A(t/t_p)^3(1 - t/t_p)^3 + \pi/4, \hspace{1cm} (A2)$$

FIG. 7. Fidelity vs time resolution of pulse envelope. Pulse and system parameters are as in Fig. 1; $T_1 = T_2 = 20 \mu s$.

FIG. 8. Diagram of bare (noninteracting) energy levels for the two secondary qubit resonances (SQRs) at which the $|0\rangle \leftrightarrow |1\rangle$ transition of one qubit is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of the other. These resonances give rise to the bright linear off-diagonal features evident in Fig. 2(a).

FIG. 9. Dressed state energy level diagram for the secondary qubit resonance at which the $|0\rangle \leftrightarrow |1\rangle$ transition of qubit 2 is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qubit 1, which is driven. The degeneracy between the bare states $|01\rangle$ and $|02\rangle$ at this resonance leads to a large mixing of these states when interactions are turned on, creating a large splitting between the dressed states $|01\rangle$ and $|02\rangle$. This in turn leads to a large detuning between the target and harmful transitions and hence a reduced gate time, which is inversely proportional to the target-harmful detuning under the SWIPHT protocol.
with $A = 138.9$, and where the pulse duration is $\tau_p = 5.87/|\delta|$. The pulse shape that results from plugging Eq. (A2) into Eq. (A1) is shown in Fig. 1.

**APPENDIX B: ANALYSIS OF SECONDARY RESONANCES**

In this Appendix, we provide a more detailed analysis of the secondary resonances, near which optimal gate performance can be achieved. We elucidate the origin of the gate speedup near the secondary qubit resonances that is evident in Fig. 2. At these resonances, the $|0\rangle \leftrightarrow |1\rangle$ transition of one qubit is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of the other (see Fig. 8).

For concreteness, we focus on the secondary qubit resonance at which the $|0\rangle \leftrightarrow |1\rangle$ transition of qubit 2 is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qubit 1, which is driven. The degeneracy between the bare states $|01\rangle$ and $|02\rangle$ at this resonance leads to a large mixing of these states when interactions are turned on. This gives rise to a large splitting between the dressed states $|01\rangle$ and $|02\rangle$, and in particular the state $|01\rangle$ gets pushed to an energy that is higher than what it would be further away from the resonance (see Fig. 9).

This means that the detuning between the target transition, $|00\rangle \leftrightarrow |01\rangle$, and the harmful transition, $|01\rangle \leftrightarrow |01\rangle$, becomes larger. Since in the SWIPHT protocol gate time is inversely proportional to this detuning, the gate time is reduced near this secondary resonance. This is evident in Figs. 2(b)–2(d).

**APPENDIX C: NUMERICAL SIMULATIONS USING PARAMETERS FROM EXPERIMENTAL CIRCUITS**

We examine the performance for several sets of parameters taken from experimental works and indicate ways to further improve results through minimal parameter adjustments. We have simulated the SWIPHT CNOT gate performance using parameters extracted from experimental works, including those of IBM [15–17], National Institute of Standards and Technology [50], Yale University [18], Delft University of Technology [19], ETH Zurich [20], and the Laboratory for Physical Sciences [50], as shown in the following tables.

We have optimized the fidelity over the pulse frequency for each row of data. Asterisks indicate parameters that have been adjusted relative to what was used in the corresponding paper in order to improve performance. We have increased the coupling in cases where the cavity was too far detuned from the qubits to yield feasible gate times within the SWIPHT scheme. In general, SWIPHT works when the degeneracy between the target and harmful transitions is strongly broken, which requires either strong qubit-cavity couplings, reduced qubit-cavity detunings, or tuning qubit parameters to lie near secondary resonances (see Appendix B). Fidelities outside the operational regime of SWIPHT are typically below 90%. Couplings up to 250 MHz are experimentally reasonable since there exist experimental filtering techniques that can enable one to increase the coupling strength without sacrificing $T_1$ times through Purcell effects [51–53]. In the column labeled Yale$^-$ in Table I, we show the performance without such filtering, where the relaxation time is reduced by a factor of 4 as a consequence of the factor-of-2 enhancement in qubit-cavity coupling.

We see that the performance is not significantly affected provided the original relaxation time is well above 10 $\mu$s. As described in the main text and in Appendix B, we have demonstrated a way to improve the gate quality {$F^+, T_1^+$, $\Delta^+$} by tuning one qubit transition so that the system lies near a secondary resonance. {$F_{\text{ideal}}^+, T_{1\text{ideal}}^+$, $\Delta_{\text{ideal}}^+$} are the results for the fidelity (obtained from quantum state tomography) without noise for the improved parameters. In the last row, $T_{1\text{ideal}}^{F=0.999}$ indicates a threshold of decoherence in order to reach a fidelity of 99.9% for a specific set of parameters with corresponding $\Delta^+$. Here we have assumed $T_{1\text{ideal}}^{F=0.999} = T_{1\text{ideal}}^{F=0.999}/2$. This threshold $T_1$ value provides an idea of the noise level needed for a specific

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<td>39</td>
<td>0.967</td>
<td>41</td>
</tr>
<tr>
<td>$F^+$</td>
<td>0.9936</td>
<td>0.9760</td>
<td>0.9951</td>
<td>0.9937</td>
<td>0.9942</td>
<td>0.8503</td>
<td>0.9900</td>
</tr>
<tr>
<td>$T_{1\text{ideal}}^+$ (ns)</td>
<td>57.0389</td>
<td>167.5833</td>
<td>41.8092</td>
<td>41.5766</td>
<td>35.9051</td>
<td>73.1032</td>
<td>73.6202</td>
</tr>
<tr>
<td>$\Delta^+ (MHz \times 2\pi)$</td>
<td>16.3790</td>
<td>5.5748</td>
<td>22.3453</td>
<td>22.4986</td>
<td>27.146</td>
<td>11.2462</td>
<td>12.6900</td>
</tr>
<tr>
<td>$T_{1\text{ideal}}^+$ (ns)</td>
<td>92.5</td>
<td>175.08</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>164.5</td>
<td>N/A</td>
</tr>
</tbody>
</table>
TABLE II. SWIPHT CNOT gate performance in the case where both qubits are driven (D2Q) at the same time. Numbers with asterisks indicate values that have been modified to yield improved results. Numbers in italics indicate values that were taken from other works since they were not provided in the paper.

<table>
<thead>
<tr>
<th>Reference</th>
<th>IBM</th>
<th>IBM(^{D2Q})</th>
<th>NIST</th>
<th>NIST(^{D2Q})</th>
<th>Yale</th>
<th>Yale(^{D2Q})</th>
<th>Delft</th>
<th>Delft(^{D2Q})</th>
<th>ETH</th>
<th>ETH(^{D2Q})</th>
<th>LPS</th>
<th>LPS(^{D2Q})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_c) (GHz (\times 2\pi))</td>
<td>6.494</td>
<td>6.494</td>
<td>6.3(5.6(^{*}))</td>
<td>7.5</td>
<td>7.5</td>
<td>6.8506</td>
<td>6.8506</td>
<td>7.347</td>
<td>7.347</td>
<td>6.6</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>(\omega_1) (GHz (\times 2\pi))</td>
<td>4.917(4.72(^{*}))</td>
<td>4.917(4.72(^{*}))</td>
<td>4.72</td>
<td>4.72</td>
<td>4.87(6.5(^{*}))</td>
<td>6.5(6.5(^{*}))</td>
<td>5.8899</td>
<td>5.8899</td>
<td>6.18</td>
<td>6.18</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>(\omega_2) (GHz (\times 2\pi))</td>
<td>5.415</td>
<td>5.415</td>
<td>5.1</td>
<td>5.1</td>
<td>6.18</td>
<td>6.18</td>
<td>6.477</td>
<td>6.477</td>
<td>7.0335</td>
<td>7.0335</td>
<td>6.3</td>
<td>6.3</td>
</tr>
<tr>
<td>(\alpha) (GHz (\times 2\pi))</td>
<td>3.30</td>
<td>3.30</td>
<td>2.84</td>
<td>2.84</td>
<td>2.12</td>
<td>2.12</td>
<td>3.50</td>
<td>3.50</td>
<td>90(300(^{*}))</td>
<td>90(300(^{*}))</td>
<td>211</td>
<td>211</td>
</tr>
<tr>
<td>(g, g_1) (MHz (\times 2\pi))</td>
<td>250(^{*})</td>
<td>250(^{*})</td>
<td>125</td>
<td>125</td>
<td>250(^{*})</td>
<td>250(^{*})</td>
<td>250(^{*})</td>
<td>250(^{*})</td>
<td>70(250(^{*}))</td>
<td>250(^{*})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T_1) ((\mu s))</td>
<td>30</td>
<td>30</td>
<td>20</td>
<td>20</td>
<td>60</td>
<td>60</td>
<td>25</td>
<td>25</td>
<td>1.33</td>
<td>1.33</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>(\tau_0) ((\mu s))</td>
<td>13.8</td>
<td>13.8</td>
<td>20</td>
<td>20</td>
<td>8.4</td>
<td>8.4</td>
<td>39</td>
<td>39</td>
<td>0.967</td>
<td>0.967</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>(T_2) ((\mu s))</td>
<td>0.9925</td>
<td>0.9910</td>
<td>0.9293</td>
<td>0.9334</td>
<td>0.8957</td>
<td>0.8870</td>
<td>0.9942</td>
<td>0.9981</td>
<td>0.6756</td>
<td>0.6440</td>
<td>0.6430</td>
<td></td>
</tr>
<tr>
<td>(r_p) (ns)</td>
<td>69.5868</td>
<td>61.7387</td>
<td>511.2219</td>
<td>473.2454</td>
<td>1512.3</td>
<td>1498.8</td>
<td>35.8913</td>
<td>35.8913</td>
<td>200.6993</td>
<td>200.6993</td>
<td>4328.7</td>
<td></td>
</tr>
<tr>
<td>(\Delta) (MHz (\times 2\pi))</td>
<td>13.4255</td>
<td>15.1322</td>
<td>1.8275</td>
<td>1.9741</td>
<td>0.6177</td>
<td>0.6233</td>
<td>25.7146</td>
<td>26.0297</td>
<td>4.6549</td>
<td>4.6549</td>
<td>0.2158</td>
<td></td>
</tr>
<tr>
<td>(\tau_1) ((\mu s))</td>
<td>0.9821</td>
<td>0.9828</td>
<td>0.9760</td>
<td>0.9741</td>
<td>0.9949</td>
<td>0.9555</td>
<td>0.9942</td>
<td>0.9981</td>
<td>0.8492</td>
<td>0.8586</td>
<td>0.9904</td>
<td></td>
</tr>
<tr>
<td>(r_{\tau}^*) (ns)</td>
<td>173.2139</td>
<td>168.9891</td>
<td>167.5833</td>
<td>186.1</td>
<td>41.1427</td>
<td>42.8220</td>
<td>35.8913</td>
<td>35.8913</td>
<td>72.7194</td>
<td>73.1099</td>
<td>73.6202</td>
<td>69.9620</td>
</tr>
<tr>
<td>(\tau_{\Delta^<em>}^</em>) ((\mu s))</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9978</td>
<td>0.9972</td>
<td>0.9983</td>
<td>0.993</td>
<td>0.9988</td>
<td>0.9977</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r_{\tau_{\Delta^<em>}^</em>}^*) (ns)</td>
<td>173.2139</td>
<td>172.5122</td>
<td>41.1427</td>
<td>40.8755</td>
<td>36.3311</td>
<td>36.3311</td>
<td>83.4134</td>
<td>83.4134</td>
<td>72.7289</td>
<td>69.9620</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta_{\text{ideal}}) (MHz (\times 2\pi))</td>
<td>5.3936</td>
<td>5.4155</td>
<td>22.7073</td>
<td>22.8557</td>
<td>25.7146</td>
<td>11.2001</td>
<td>12.4855</td>
<td>13.3535</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T_{1=0.999}^*) ((\mu s))</td>
<td>166.25</td>
<td>175.08</td>
<td>N/A</td>
<td>N/A</td>
<td>175</td>
<td>N/A</td>
<td>175</td>
<td>N/A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG. 10. (a) Local phases of the generalized CNOT as a function of qubit frequency. (b) Insensitivity of the SWIPHT gate fidelity with respect to local phases. The fidelity as a function of qubit frequency is shown for fixed values of local phases. The system parameters are as in Fig. 1 and \( T_1 = T_2 = 20 \mu s \).

A transmon system to achieve 0.999 fidelity for a CNOT gate based on our scheme. Table I shows that it is possible to obtain fidelities in excess of 0.99 while keeping pulse times below 100 ns in most cases even with realistic noise included. The ideal fidelity values further show that most of the residual gate error is caused by decoherence and relaxation. Table II gives similar results for additional parameter sets. The table further shows that the results are essentially the same when the driving is allowed to act on both qubits.

APPENDIX D: SENSITIVITY TO LOCAL PHASES OF THE GENERALIZED CNOT

We consider how the phases entering into the definition of our generalized CNOT gate, given by Eq. (2), depend on system parameters. These phases represent the trivial, local part of the entangling gate and can be corrected with local single-qubit gates. Due to the finite linewidths of the transmon excited states, there exists experimental uncertainty in the values of the transmon frequencies (of the order of 10 kHz), and this can in turn create uncertainty in the values of the local phases. In Fig. 10, we show that although the local phases are sensitive to qubit frequencies [Fig. 10(a)], the SWIPHT CNOT gate fidelity remains essentially constant as qubit frequencies are varied over a range of 20 kHz even when the local phases are held fixed, demonstrating that the gate performance is not sensitive to these phases or to typical levels of uncertainty in qubit frequencies [Fig. 10(b)].


[50] B. Palmer (private communication).

