# Physics 5455 – Problem set 1

1. A simple pendulum of length $L$ and bob mass $m$ is attached to a massless support moving horizontally with constant acceleration $a$. Find equations of motion of the bob.

2. A particle with mass $m$ moves in three dimensions in a spherically symmetric potential $V = V(r)$.

   (a) Show that the particle's kinetic energy can be written as

   $$T = \frac{p_r^2}{2m} + \frac{\vec{L}^2}{2mr^2}$$

   where $p_r$ is the radial momentum component and $\vec{L} \equiv \vec{r} \times \vec{p}$ the angular momentum.

   (b) Express the particle's Lagrangian and the angular momentum component $L_z$ in terms of spherical coordinates.

   (c) Determine the conjugate momenta for all coordinates and the Hamiltonian. Compare to (a). List all cyclic coordinates and the associated conserved quantities.

   (d) Show by direct differentiation that $d\vec{L}/dt = 0$.

   (e) Find Hamilton's equations of motion for the radial components $r$, $p_r$. What is $V_{\text{eff}}(r)$ here?

3. Poisson brackets. Recall that the Poisson bracket is defined by

   $$\{A, B\} = \sum_i \left( \frac{\partial A}{\partial q_i} \frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i} \frac{\partial B}{\partial q_i} \right)$$

   (a) Show that

   $$\{AB, C\} = A\{B, C\} + \{A, C\}B$$

   (b) Given the canonical Poisson brackets

   $$\{x_i, x_j\} = 0 = \{p_i, p_j\}, \quad \{x_i, p_j\} = \delta_{ij}$$

   for a particle in three dimensions, compute $\{L_i, x_j\}$, $\{L_i, p_j\}$, and $\{L_i, L_j\}$ where $\vec{L} = \vec{r} \times \vec{p}$ is angular momentum, and show that $\{L_i, A\} = 0$ for $A = \vec{r}^2, \vec{p}^2, \vec{L}^2$.

   (c) Suppose a point particle moves in a spherically symmetric potential $V(r)$. Use the Poisson brackets you have just computed to show that all components of angular momentum are conserved, *i.e.* $d\vec{L}/dt = 0$. For full credit you must use Poisson brackets.