Physics 5455 – Problem set 3

1. **Independent subsystems and probability interpretations.** Consider the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} \;=\; H\psi$$

with a Hamiltonian that describes two non-interacting independent subsystems: $H = H_1 + H_2$, $[H_1, H_2] = 0$.

   (a) Show that this equation is solved by the product wavefunction $\psi = \psi_1\psi_2$, where $\psi_{1,2}$ solve the time-dependent Schrödinger equation for the subsystem Hamiltonian $H_{1,2}$.

   (b) Why is this result consistent with the probability amplitude interpretation of the wavefunction?

   (c) Suppose we modified the Schrödinger equation to instead have the form

$$\kappa\frac{\partial^2\psi}{\partial t^2} \;=\; H\psi$$

   for some constant κ. For non-interacting subsystems, would the same factorization give a solution to this equation?

2. Schwabl 2.6.

3. Schwabl 2.7.

4. Write the inner product

$$(\psi_1, \psi_2) \;=\; \int d^3x\,\psi_1(x,t)^*\psi_2(x,t)$$

on momentum space.

5. **Schrödinger equation Green function / propagator**

   (a) Show that a general solution of the time-dependent Schrödinger equation can be written in terms of the initial wavefunction at $t = 0$ in the form

$$\psi(x,t) \;=\; \int d^3x'\,K(\vec{x},\vec{x'};t)\psi(\vec{x'},0)$$

   where $K(\vec{x},\vec{x'};t)$ satisfies the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}K(\vec{x},\vec{x'};t) \;=\; H(\vec{x})K(\vec{x},\vec{x'};t)$$

   with initial condition $K(\vec{x},\vec{x'};0) = \delta^3(\vec{x}-\vec{x'})$. K is known as the *Green's function* or *propagator*.

(b) Demonstrate that

$$K(\vec{x},\vec{x'};t) \;=\; \int d^3y\, K(\vec{x},\vec{y};t-t')K(\vec{y},\vec{x'};t')$$

(c) Let $\{\psi_n(\vec{x})\}$ be a complete orthonormal set of energy eigenfunctions:

$$H\psi_n(\vec{x}) \;=\; E_n\psi_n(\vec{x})$$

Show that

$$K(\vec{x},\vec{x'};t) \;=\; \sum_n \psi_n^*(\vec{x'})\psi_n(\vec{x})\exp(-iE_nt/\hbar)$$