1. **Independent subsystems and probability interpretations.** Consider the time-dependent Schrödinger equation

\[ i\hbar \frac{\partial \psi}{\partial t} = H\psi \]

with a Hamiltonian that describes two non-interacting independent subsystems: \( H = H_1 + H_2, \ [H_1, H_2] = 0. \)

(a) Show that this equation is solved by the product wavefunction \( \psi = \psi_1 \psi_2, \) where \( \psi_{1,2} \) solve the time-dependent Schrödinger equation for the subsystem Hamiltonian \( H_{1,2}. \)

(b) Why is this result consistent with the probability amplitude interpretation of the wavefunction?

(c) Suppose we modified the Schrödinger equation to instead have the form

\[ \kappa \frac{\partial^2 \psi}{\partial t^2} = H\psi \]

for some constant \( \kappa. \) For non-interacting subsystems, would the same factorization give a solution to this equation?

2. Schwabl 2.6.


4. Write the inner product

\[ (\psi_1, \psi_2) = \int d^3 x \psi_1(x, t)^* \psi_2(x, t) \]

on momentum space.

5. **Schrödinger equation Green function / propagator**

(a) Show that a general solution of the time-dependent Schrödinger equation can be written in terms of the initial wavefunction at \( t = 0 \) in the form

\[ \psi(x, t) = \int d^3 x' K(\bar{x}, \bar{x}' ; t) \psi(\bar{x}', 0) \]

where \( K(\bar{x}, \bar{x}' ; t) \) satisfies the time-dependent Schrödinger equation

\[ i\hbar \frac{\partial}{\partial t} K(\bar{x}, \bar{x}' ; t) = H(\bar{x})K(\bar{x}, \bar{x}' ; t) \]

with initial condition \( K(\bar{x}, \bar{x}' ; 0) = \delta^3(\bar{x} - \bar{x}'). \) \( K \) is known as the Green’s function or propagator.
(b) Demonstrate that

\[ K(\vec{x}, \vec{x}'; t) = \int d^3y K(\vec{x}, \vec{y}; t - t')K(\vec{y}, \vec{x}'; t') \]

(c) Let \( \{\psi_n(\vec{x})\} \) be a complete orthonormal set of energy eigenfunctions:

\[ H\psi_n(\vec{x}) = E_n\psi_n(\vec{x}) \]

Show that

\[ K(\vec{x}, \vec{x}'; t) = \sum_n \psi_n^*(\vec{x}')\psi_n(\vec{x}) \exp(-iE_n t/\hbar) \]