Physics 5456 – Problem set 6

1. Consider scattering in the potential

\[ V(r) = \begin{cases} 
  0 & r > R, \\
  V_0 & r < R, 
\end{cases} \]

for some constant \( V_0 \). Use the Born approximation to compute the scattering amplitude \( f_k(\theta, \phi) \).

2. **Hard sphere scattering** Consider scattering off a rigid sphere defined by the potential

\[ V(r) = \begin{cases} 
  0 & r > R, \\
  \infty & r < R. 
\end{cases} \]

(a) By considering the boundary condition on the wavefunction, derive an exact expression for \( \tan \delta_\ell \).

(b) Derive a simple expression for the phase shift \( \delta_\ell \) for \( \ell = 0 \). (HInt: refer to Schwabl section 17.2 for information on spherical Bessel functions.) (For credit, do not merely refer to your answer to the previous part and say, set \( \ell = 0 \).)

(c) Estimate \( \delta_\ell \) for small \( kR \), and show that in this regime, the largest phase is \( \delta_0 \).

(d) Compute the s-wave contribution to the total scattering cross section, and compare to the geometric cross-section of the rigid sphere in the limit of small \( kR \).