# Physics 5456 – Problem set 9

1. **Klein-Gordon equation in spherical potential well** A scalar meson (mass $m$) inside an atomic nucleus can be described by the Klein-Gordon equation with a spherical potential well, which we can think of as an electromagnetic scalar potential

$$q\Phi(r) \;=\; V(r) \;=\; \begin{cases} -V_0 & r \leq a \\ 0 & r > a \end{cases}$$

for $V_0 > 0$. Consider bound states for this problem, for which $|E| < mc^2$.

   (a) What are the solutions of the time-independent Klein-Gordon equation for $r < a$ and $r > a$ ? Give an implicit condition that determines the allowed energy eigenvalues.

   (b) Specialize to s-waves ($\ell = 0$) and provide a condition on the potential strength $V_0$ for the existence of a bound state.

2. **Relativistic Landau levels** A charged relativistic spin 0 particle is moving in a uniform magnetic field: $\vec{E} = 0$, $\vec{B} = B\hat{e}_z$, $\vec{A} = Bx\hat{e}_y$.

   (a) Write down the Klein-Gordon equation in this background.

   (b) Solve for the energy levels of this system.

   (c) Compute the nonrelativistic limit of the positive branch of the energy levels above.

3. **Dirac equation in a homogeneous magnetic field** Determine the energy eigenvalues for a relativistic spin 1/2 particle (mass $m$, charge $q$) in a homogeneous magnetic field $\vec{B} = B\hat{e}_z$, by solving the time-independent Dirac equation. Compare the result with the corresponding energy levels for the Klein-Gordon equation.

   Hint: Write the four-component spinor

$$\psi \;=\; \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

   and eliminate the two-component spinor $\chi$. Then use the gauge $\vec{A} = Bx\hat{e}_y$, $\Phi = 0$.

4. **Quadratic form of the Dirac equation** Multiply the Dirac equation

$$\left[-\gamma^\mu \left(i\hbar\partial_\mu \;-\; \frac{q}{c}A_\mu\right) \;+\; mc\right]\psi \;=\; 0$$

   by

$$\gamma^\nu \left(i\hbar\partial_\nu \;-\; \frac{q}{c}A_\nu\right) \;+\; mc$$

to derive the quadratic Dirac equation

$$\left[\left(i\hbar\partial^{\mu} \;-\; \frac{q}{c}A^{\mu}\right)\left(i\hbar\partial_{\mu} \;-\; \frac{q}{c}A_{\mu}\right) \;-\; \frac{q\hbar}{2c}\sigma^{\mu\nu}F_{\mu\nu} \;-\; m^2c^2\right]\psi \;=\; 0$$

where $\sigma^{\mu\nu} = (i/2)[\gamma^{\mu}, \gamma^{\nu}]$ and $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the electromagnetic field tensor. Show that

$$\sigma^{\mu\nu}F_{\mu\nu} \;=\; 2i\left(\gamma^0\vec{\gamma}\cdot\vec{E} \;+\; i\vec{\Sigma}\cdot\vec{B}\right)$$