Physics 4674/5674 – Problem set 1

On this homework I’ll often use Roman indices $i, j$ instead of Greek indices $\mu, \nu$. This doesn’t mean anything, the indices can be written either way.

1. C 1.6 (C = Carroll)

2. Let $a_{ij}, a_{ijk}$ be indexed numbers, which are symmetric in their indices, show that
   (a) $\frac{\partial}{\partial x^i} \left( a_{jk} x^j x^k \right) = 2a_{ij} x^j$
   (b) $\frac{\partial^3}{\partial x^i \partial x^j \partial x^k} \left( a_{pqr} x^p x^q x^r \right) = 6a_{ijk}$

   Note the $a$’s are not quite tensors, since they are just indexed numbers, but they do give an excuse for doing tensor-type manipulations.

3. (a) If $a_{jik} = -a_{kji}$, show that $a_{ijk} = 0$.
   (b) If $a_{ijk} = a_{jik} = -a_{ikj}$, show that $a_{ijk} = 0$.

4. If $a_{ij} = -a_{ji}$, and $b^{ij} = +b^{ji}$, show that
   \[
   \left( \delta^i_k \delta^j_\ell + \delta^i_\ell \delta^j_k \right) a_{ij} = 0
   \]
   and that $a_{ij} b^{ij} = 0$.

5. Show that, although $T_{ij} = T_{(ij)} + T_{[ij]}$, it is not, in general, true that $T_{ijk} = T_{(ijk)} + T_{[ijk]}$.

6. Show that, if the skew symmetric covariant tensor $T$ of type (0,2) is such that
   \[
   T_{ij} = u_i v_j - u_j v_i
   \]
   for some covector components $u_i, v_j$, then
   \[
   T_{ij} T_{k\ell} + T_{ik} T_{\ell j} + T_{i\ell} T_{jk} = 0
   \]
   (These relations are known as the Plücker relations.)

7. Show that a general (2,0) tensor with components $a^{ij}$ cannot always be written $a^{ij} = u^i v^j$ for any $u^i, v^j$. (Hint: compare the number of components.)

8. If $V$ is a (0,1) tensor and $S$ a (2,0) tensor, show that $V_\mu S^{\mu\nu}$ transform as the components of a (1,0) tensor.