Physics 4674/5674 – Problem set 1

On this homework I'll often use Roman indices i, j instead of Greek indices μ, ν. This doesn't mean anything, the indices can be written either way.

1. C 1.6 (C = Carroll)
2. Let a_{ij}, a_{ijk} be indexed numbers, which are symmetric in their indices, show that

   (a)
   $$\frac{\partial}{\partial x^i}\left(a_{jk}x^j x^k\right) \;=\; 2a_{ij}x^j$$

   (b)
   $$\frac{\partial^3}{\partial x^i \partial x^j \partial x^k}\left(a_{pqr}x^p x^q x^r\right) \;=\; 6a_{ijk}$$

   Note the a's are not quite tensors, since they are just indexed numbers, but they do give an excuse for doing tensor-type manipulations.
3. (a) If $a_{jik} = -a_{kji}$, show that $a_{ijk} = 0$.

   (b) If $a_{ijk} = a_{jik} = -a_{ikj}$, show that $a_{ijk} = 0$.
4. If $a_{ij} = -a_{ji}$, and $b^{ij} = +b^{ji}$, show that
   $$\left(\delta^i_k \delta^j_\ell \;+\; \delta^i_\ell \delta^j_k\right) a_{ij} \;=\; 0$$
   and that $a_{ij}b^{ij} = 0$.
5. Show that, although $T_{ij} = T_{(ij)} + T_{[ij]}$, it is *not*, in general, true that $T_{ijk} = T_{(ijk)} + T_{[ijk]}$.
6. Show that, if the skew symmetric covariant tensor T of type (0,2) is such that
   $$T_{ij} \;=\; u_i v_j \;-\; u_j v_i$$
   for some covector components u_i, v_j, then
   $$T_{ij}T_{k\ell} \;+\; T_{ik}T_{\ell j} \;+\; T_{i\ell}T_{jk} \;=\; 0$$
   (These relations are known as the Plücker relations.)
7. Show that a general (2,0) tensor with components a^{ij} can*not* always be written $a^{ij} = u^i v^j$ for any u^i, v^j. (Hint: compare the number of components.)
8. If V is a (0,1) tensor and S a (2,0) tensor, show that $V_\mu S^{\mu\nu}$ transform as the components of a (1,0) tensor.