Both 4674 & 5674:

1. C 1.12

2. A particle moving in one dimension has the classical action

\[ S = \int dt \left( \frac{1}{2} m \dot{x}^2 + \lambda x (\ddot{x})^2 \right) \]

Use functional derivatives and Hamilton’s least-action principle to derive the classical equations of motion of this particle.

3. A real Klein-Gordon field with interactions is defined by the following action:

\[ S = \int d^4x \left( \partial_\mu \phi \partial^\mu \phi + \lambda \phi^4 + \mu \phi^6 \right) \]

Use functional derivatives and Hamilton’s least-action principle to derive the classical equations of motion of this field.

4. C 2.1

5. C 2.3

Only 5674:

6. Fermat’s principle of optics states that a light ray will follow the path for which

\[ \int n(x, y) ds \]

is a minimum, where \( n \) is the index of refraction, and the infinitesimal arc length

\[ ds = (\dot{x}^2 + \dot{y}^2)^{1/2} dt = (\dot{r}^2 + r^2 \dot{\theta}^2)^{1/2} dt \]

For simplicity let us restrict to motion in two dimensions. Suppose that \( n(r, \theta) = r^k \). Show that when \( k = -1 \), a light ray can travel in a circle about the origin.