Physics 4674/5674 – Problem set 3

Both 4674 & 5674:

1. C 1.12

2. A particle moving in one dimension has the classical action

$$S = \int dt \left(\frac{1}{2} m \dot{x}^2 + \lambda x \left(\ddot{x} \right)^2 \right)$$

Use functional derivatives and Hamilton's least-action principle to derive the classical equations of motion of this particle.

3. A real Klein-Gordon field with interactions is defined by the following action:

$$S = \int d^4x \left(\partial_\mu \phi \partial^\mu \phi + \lambda \phi^4 + \mu \phi^6 \right)$$

Use functional derivatives and Hamilton's least-action principle to derive the classsical equations of motion of this field.

4. C 2.1

5. C 2.3

Only 5674:

6. Fermat's principle of optics states that a light ray will follow the path for which

$$\int n(x, y) ds$$

is a minimum, where n is the index of refraction, and the infinitesimal arc length

$$ds = \left(\dot{x}^2 + \dot{y}^2 \right)^{1/2} dt = \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right)^{1/2} dt$$

For simplicity let us restrict to motion in two dimensions. Suppose that $n(r, \theta) = r^k$. Show that when $k = -1$, a light ray can travel in a circle about the origin.