A proposal for nonabelian mirrors

Eric Sharpe
Virginia Tech

W. Gu, ES, arXiv:1806.04678

Z. Chen, W. Gu, H. Parsian, ES, to appear

This talk will concern **mirror symmetry**,
an old example of a duality in string theory,
which has taken a variety of forms over the years.

Originally, mirror symmetry was a relation between Calabi-Yau manifolds, interpreted as a duality of 2d supersymmetric sigma model.

Two Calabi-Yau manifolds are said to be mirror if the two SCFT's are isomorphic, related ultimately by flipping a left $U(1)_R$ sign convention.

Implications:

- Hodge diamonds rotated:

  If X, Y are mirror CYs, then dim X = dim Y (=n) and $h^{p,q}(X) = h^{n-p,q}(Y)$.

- TFT's interchanged:

  A model on X = B model on Y

- Quantum physics of one = classical physics of other

Example of a mirror: T^2

T^2 is self-mirror;
mirror symmetry ~ T-duality.

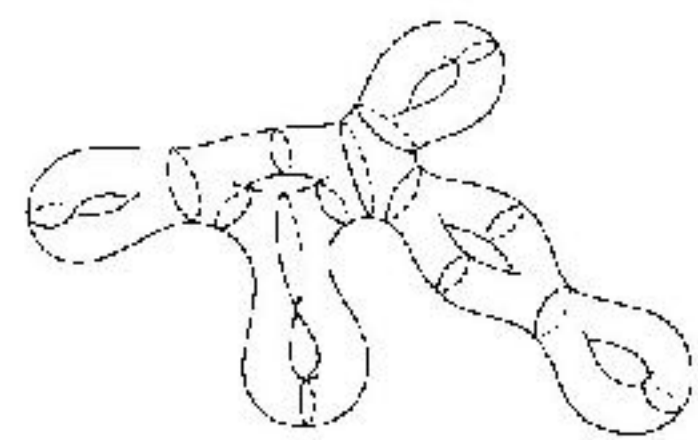
Hodge diamond:

$$\begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 1 & \end{array}$$

— symmetric under rotation

This symmetry is
specific to 2d manifolds
with 1 handle;
for g handles:

$$\begin{array}{ccc} & 1 & \\ g & & g \\ & 1 & \end{array}$$



Example of a mirror: K3 manifold

K3 is also self-mirror;
complex, Kahler structures interchanged

$h^{1,1}$ $\qquad$ $h^{1,1}$

Hodge diamond:

$$\begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array}$$

Kummer surface

$$(x^2+y^2+z^2-aw^2)^2-\left(\frac{3a-1}{3-a}\right)pqts=0$$
$$p=w-z-\sqrt{2}x$$
$$q=w-z+\sqrt{2}x$$
$$t=w+z+\sqrt{2}y$$
$$s=w+z-\sqrt{2}y$$
$$a=1.5$$
$$w=1$$

Example of a mirror: quintic 3-fold

The quintic (degree 5 hypersurface in $\mathbb{P}^4$) is a nontrivial CY which is not self-mirror.

Hodge diamonds:

Quintic

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 1 & & 0 & \\
1 & & 101 & & 101 & & 1 \\
 & 0 & & 1 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Mirror

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 101 & & 0 & \\
1 & & 1 & & 1 & & 1 \\
 & 0 & & 101 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Mirror symmetry for non-CY spaces

Mirror symmetry has also been defined for non-Calabi-Yau spaces.

Here, the mirror is a Landau-Ginzburg theory
(meaning, typically, free fields + superpotential).

A model on space = B-twisted LG theory

If the space is not CY (up to 2-torsion),
then no B-twist exists,
so the statement above is the best possible rel'n between TFTs.

Example: Mirror of $\mathbb{P}^n$ is LG model with n chiral superfields

and superpotential

$$W = \exp(-Y_1) + \cdots + \exp(-Y_n) + q \prod_{i=1}^{n} \exp(+Y_i)$$

This will be the prototype for the mirrors we will construct in this paper,
so let's take a little time to analyze in detail….

Example: Mirror of $\mathbb{P}^n$ is LG model with n chiral superfields and superpotential

$$W = \exp(-Y_1) + \cdots + \exp(-Y_n) + q\prod_{i=1}^{n}\exp(+Y_i)$$

In the B model, correlation functions are classical, determined by the critical locus, meaning, solutions of $dW = 0$.

(Reason: bosonic potential = $|dW|^2$)

Here, critical locus is given as follows:

$$\frac{\partial W}{\partial Y_j} = -\exp\left(-Y_j\right) + q\prod_{i=1}^{n}\exp\left(+Y_i\right)$$

$$= 0 \Rightarrow \quad \exp\left(-Y_j\right) = q\prod_{i=1}^{n}\exp\left(+Y_i\right)$$ independent of j

Define $\sigma \equiv \exp\left(-Y_j\right)$ then on the critical locus,

$$\exp\left(-Y_j\right) = q\prod_{i=1}^{n}\exp\left(+Y_i\right) \quad \Rightarrow \quad \sigma = q\frac{1}{\sigma^n}$$ or more simply, $\underline{\sigma^{n+1} = q}$

This matches the quantum cohomology relation of the A model on $\mathbb{P}^n$.

Critical locus of B model mirror ~ quantum cohomology of A model

Example: Mirror of $\mathbb{P}^n$ is LG model with n chiral superfields and superpotential

$$W = \exp(-Y_1) + \cdots + \exp(-Y_n) + q \prod_{i=1}^{n} \exp(+Y_i)$$

Now, let's compare correlation functions. On S^2, in the B model,

$$\langle f \rangle = \sum_{\text{vacua}} \frac{f}{H} \quad \text{where} \quad H = \det \partial^2 W$$

Here, can show $H = (n+1)\sigma^n$ on the critical locus

$$\langle \sigma^k \rangle = \sum_{\sigma^{n+1}=q} \frac{\sigma^k}{(n+1)\sigma^n}$$

~ sum over (n+1)th roots of unity (well, q)

so = 0 if k-n not divisible by n+1.

Nonzero B model correlation functions: $\langle \sigma^{n+d(n+1)} \rangle = q^d$

Example: Mirror of $\mathbb{P}^n$ is LG model with n chiral superfields and superpotential

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Compare results for A model on $\mathbb{P}^n$:

σ ~ generator of $H^2(\mathbb{P}^n)$

$$\langle \sigma^k \rangle \sim \sum_d q^d \int_{\mathcal{M}_d} \sigma^k \qquad \mathcal{M}_0 = \mathbb{P}^n \qquad \mathcal{M}_d = \mathbb{P}^{(n+1)(d+1)-1} = \mathbb{P}^{n+d(n+1)}$$

The integral is nonzero only if σ^k is a top-form,
hence,
the nonzero A model correlation functions are

$$\langle \sigma^{n+d(n+1)} \rangle = q^d$$

Matches B model result, as expected.

Mirrors of this form can be efficiently computed using
Hori-Vafa's construction of abelian duality. (Hori, Vafa hepth/0002222)

They have a general prescription for mapping
2d susy abelian gauge theories to LG models.

(Today's talk will describe a generalization to nonabelian 2d theories.)

For $\mathbb{P}^n$, for example,
we describe it as a U(1) gauge theory with n+1 chiral superfields of charge +1.

Hori-Vafa prescription gives

$$W = \sigma\left(\sum_{i=1}^{n+1} Y_i - t\right) + \sum_{i=1}^{n+1} \exp\left(-Y_i\right)$$

Integrate out σ to get the constraint

$$\sum_{i=1}^{n+1} Y_i = t$$

Eliminate Y_{n+1}:

$$Y_{n+1} = t - \sum_{i=1}^{n} Y_i$$

Plug back in:

$$W = \sum_{i=1}^{n} \exp\left(-Y_i\right) + q\prod_{i=1}^{n} \exp\left(+Y_i\right)$$

as used previously.

General case next….

Hori-Vafa abelian duality

(Hori, Vafa hepth/0002222)

$U(1)^r$ gauge theory with matter multiplets of charges ρ_i^a

Mirror:

Fields σ_a $\quad a \in \{1, \cdots, r\}$ $\quad \sigma_a = \overline{D}_+ D_- V_a$

Y^i mirror to matter fields

Superpotential:

$$W = \sum_a \sigma_a \left(\sum_i \rho_i^a Y^i - t_a \right) + \sum_i \exp\left(-Y^i\right)$$

Periodicities:

$$Y^i \sim Y^i + 2\pi i \qquad \theta \sim \theta + 2\pi\rho$$

After all, in 2d, the theta angle acts like an electric field, and periodicity on a noncompact space is determined by screening by matter fields.

Proposed nonabelian generalization….

Our nonabelian proposal

For a G gauge theory, pick a Cartan torus $U(1)^r \subset G$.
Matter multiplets in representation ρ.

Mirror: Weyl-group orbifold of the following LG model:

Fields	σ_a	$a \in \{1, \cdots, r\}$	$\sigma_a = \overline{D}_+ D_- V_a$
	Y^i	mirror to matter fields	
	$X_{\tilde{\mu}}$	corresponding to nonzero roots of $\mathfrak{g}$	

Superpotential:

$$W = \sum_a \sigma_a \left(\sum_i \rho_i^a Y^i - \sum_{\tilde{\mu}} \alpha_{\tilde{\mu}}^a \ln X_{\tilde{\mu}} - t_a \right) + \sum_i \exp\left(-Y^i\right) + \sum_{\tilde{\mu}} X_{\tilde{\mu}}$$

$\rho_i =$ weight vector
$\alpha_{\tilde{\mu}} =$ root vector

Idea: "Abelian duality in Cartan torus, at generic pt on Coulomb branch"

Periodicities:

$$Y^i \sim Y^i + 2\pi i \qquad \theta \sim \theta + 2\pi M$$

for M the lattice gen' by weights of matter fields.

After all, in 2d, the theta angle acts like an electric field, and periodicity on a noncompact space is determined by screening by matter fields.

Our nonabelian proposal

Fields

σ_a — $a \in \{1, \cdots, r\}$ — $\sigma_a = \overline{D}_+ D_- V_a$

Y^i — mirror to matter fields

$X_{\tilde{\mu}}$ — corresponding to nonzero roots of $\mathfrak{g}$

Superpotential:

$$W = \sum_a \sigma_a \left(\sum_i \rho_i^a Y^i - \sum_{\tilde{\mu}} \alpha_{\tilde{\mu}}^a \ln X_{\tilde{\mu}} - t_a \right) + \sum_i \exp\left(-Y^i\right) + \sum_{\tilde{\mu}} X_{\tilde{\mu}}$$

$\rho_i =$ weight vector
$\alpha_{\tilde{\mu}} =$ root vector

Weyl-group orbifold:

The Weyl orbifold maps weights to weights

$$Y^i \mapsto Y^j \qquad \sum_a \sigma_a \rho_i^a \mapsto \sum_a \sigma_a \rho_j^a$$

and roots to roots

$$X_{\tilde{\mu}} \mapsto X_{\tilde{\nu}} \qquad \sum_a \sigma_a \alpha_{\tilde{\mu}}^a \mapsto \sum_a \sigma_a \alpha_{\tilde{\nu}}^a$$

and so manifestly preserves the superpotential.

Weyl-group orbifold:

The Weyl orbifold maps weights to weights $Y^i \mapsto Y^j$ $\quad \sum_a \sigma_a \rho_i^a \mapsto \sum_a \sigma_a \rho_j^a$

and roots to roots $X_{\tilde{\mu}} \mapsto X_{\tilde{\nu}}$ $\quad \sum_a \sigma_a \alpha_{\tilde{\mu}}^a \mapsto \sum_a \sigma_a \alpha_{\tilde{\nu}}^a$

Existence of B twist:

Mirror symmetry should map the original A-twisted gauge theory to a B-twist of the Landau-Ginzburg orbifold.

For the closed string B model to exist, the orbifold must preserve the hol' top form *up to a sign*.

(ES, hepth/0605005)

The Weyl group orbifold satisfies this property:

Each Weyl reflection interchanges Ys with Ys and Xs with Xs, so as a result, for example,

$$dX_1 \wedge \cdots \wedge dX_n \mapsto \pm\, dX_1 \wedge \cdots \wedge dX_n$$

and so the holomorphic top-form changes by at most a sign.

— so, the proposal is compatible with existence of B twist.

Twisted masses:

If the original gauge theory has twisted masses,
they can be incorporated into the mirror by adding a term to the mirror superpotential:

$$W = \sum_a \sigma_a \left(\sum_i \rho_i^a Y^i - \sum_{\tilde{\mu}} \alpha_{\tilde{\mu}}^a \ln X_{\tilde{\mu}} - t_a \right) - \sum_i \tilde{m}_i Y^i + \sum_i \exp\left(-Y^i\right) + \sum_{\tilde{\mu}} X_{\tilde{\mu}}$$

R charges:

Note:

- only integral R charges are A-twistable
- positivity of bosonic potentials constrains values

Result: R charge $\in \{0,1,2\}$

Mirror to a field with nonzero R charge: fundamental field is exp(- (r/2) Y)

Example: X fields above are (morally) mirror to fields of R charge 2

Operator mirror map:

To make this useful, we need to relate correlators in the original gauge theory to correlators in the mirror Landau-Ginzburg orbifold.

We can derive such a map from the critical locus of the mirror superpotential:

$$W = \sum_a \sigma_a \left(\sum_i \rho_i^a Y^i - \sum_{\tilde{\mu}} \alpha_{\tilde{\mu}}^a \ln X_{\tilde{\mu}} - t_a \right) - \sum_i \tilde{m}_i Y^i + \sum_i \exp\left(-Y^i\right) + \sum_{\tilde{\mu}} X_{\tilde{\mu}}$$

$$\frac{\partial W}{\partial X_{\tilde{\mu}}} = -\frac{\sum_a \sigma_a \alpha_{\tilde{\mu}}^a}{X_{\tilde{\mu}}} + 1 = 0 \qquad \Rightarrow \underline{X_{\tilde{\mu}} = \sum_a \sigma_a \alpha_{\tilde{\mu}}^a}$$

$$\frac{\partial W}{\partial Y^i} = \sum_a \sigma_a \rho_i^a - \tilde{m}_i - \exp\left(-Y^i\right) = 0 \qquad \Rightarrow \underline{\exp\left(-Y^i\right) + \tilde{m}_i = \sum_a \sigma_a \rho_i^a}$$

In this fashion, we can match correlation functions in the original (A-twisted) gauge theories with correlation functions in the mirror (B-twisted) Landau-Ginzburg orbifolds.

Correlation functions:

Here's a formal argument comparing correlation functions in the proposed mirror to those in the original A model.

Briefly, $\langle f \rangle = \sum_{\text{critical loci}} \frac{f}{H^{1-g}}$ on a genus g worldsheet

where H is determinant of matrix of second derivatives on the critical locus:

$$H = \det \begin{array}{cc} \begin{array}{cc} X,Y & \sigma \end{array} & \\ \left[\begin{array}{cc} A & B \\ C & D \end{array} \right] & \begin{array}{c} X,Y \\ \sigma \end{array} \end{array}$$

$$A \sim \quad \frac{\partial^2 W}{\partial X_{\tilde{\mu}} \partial X_{\tilde{\nu}}} = \delta_{\tilde{\mu}\tilde{\nu}} \left(\sum_c \sigma_c \alpha^c_{\tilde{\mu}} \right)^{-1} \qquad \frac{\partial^2 W}{\partial Y^i \partial Y^j} = \delta^{ij} \left(\sum_c \sigma_c \rho^c_i - \tilde{m}_i \right) \qquad \frac{\partial^2 W}{\partial X_{\tilde{\mu}} \partial Y^i} = 0$$

$$B = C^T \sim \quad \frac{\partial^2 W}{\partial X_{\tilde{\mu}} \partial \sigma_a} = - \alpha^a_{\tilde{\mu}} \left(\sum_c \sigma_a \alpha^c_{\tilde{\mu}} \right)^{-1} \qquad \frac{\partial^2 W}{\partial Y^i \partial \sigma_a} = \rho^a_i$$

$$D \sim \quad \frac{\partial^2 W}{\partial \sigma_a \partial \sigma_b} = 0 \qquad \Rightarrow D = 0$$

$$H = (\det A) \det(D - CA^{-1}B) = (\det A) \det(-CA^{-1}B)$$

Correlation functions:

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$$H = (\det A)\det(D - CA^{-1}B) = (\det A)\det(-CA^{-1}B)$$

$$\det A = \left[\prod_{\tilde{\mu}} \left(\sum_c \sigma_c \alpha^c_{\tilde{\mu}} \right) \right]^{-1} \left[\prod_i \left(\sum_c \sigma_c \rho^c_i - \tilde{m}_i \right) \right] \qquad (-CA^{-1}B)_{ab} = \frac{\partial^2 W_{\text{eff}}}{\partial \sigma_a \partial \sigma_b}$$

where

$$W_{\text{eff}} = -\sum_a \sum_i \sigma_a \rho^a_i \ln\left(\sum_b \sigma_b \rho^b_i - \tilde{m}_i \right) + \sum_a \sum_i \sigma_a \rho^a_i - \sum_a \sigma_a t_a$$
$$- \sum_{\text{pos}'} i\pi \alpha^a_{\tilde{\mu}} \sigma_a + \sum_i \tilde{m}_i \ln\left(\sum_b \sigma_b \rho^b_i - \tilde{m}_i \right)$$

— formally matches known exact 1-loop results

(Doesn't mention orb' twisted sectors — we'll see later there aren't any contributions.)

We have checked this proposal extensively in examples.
In the rest of this talk, we will outline a few particular ones.

- Grassmannians G(k,n)

  U(k) gauge theory with n fundamentals

- SO(2k) gauge theory with n vectors + twisted masses

- Pure 2d susy gauge theories & IR behavior

Example: Grassmannian G(k,n)

The Grassmannian G(k,n) is described by a 2d U(k) theory with n fundamentals — generalizes the CP^{n-1} model.

A-twisted gauge theory results:

Coulomb branch: S_k orbifold of $\sigma_1, \cdots, \sigma_k$

Vacua: $\sigma_a \neq \sigma_b$ if $a \neq b$

$$\left(\sigma_a\right)^n = (-)^{k-1} q \quad \Rightarrow \text{ quantum cohomology ring}$$

So, choose k distinct & unordered (S_k orb') values amongst n roots of equation above.

Total number of vacua = $\begin{pmatrix} n \\ k \end{pmatrix}$

Example: Grassmannian G(k,n)

A-twisted gauge theory results:

$$\sigma_a \neq \sigma_b \qquad \text{if} \qquad a \neq b$$

Question: How can the condition above be realized in the mirror?

This condition describes an open set.

We're all familiar with how one describes a *closed* set
— as the critical locus of a superpotential,
which is how e.g. GLSMs describe hypersurfaces —
but how do you realize an *open* set?

In susy localization, expressions for correlation functions have factors multiplying integration measures which are of the form

$$\prod_{a<b} \left(\sigma_a - \sigma_b\right)^2$$

so that there is no contribution from points where σs collide.

We'll see that the mirror superpotential has poles at the corresponding points, dynamically excluding them.

Example: Grassmannian G(k,n)

Now, let's study the proposed mirror.

Let's modify the notation to be more convenient:

$$Y^i \mapsto Y^{ia} \qquad X_{\tilde{\mu}} \mapsto X_{\mu\nu} \quad (\mu \neq \nu)$$

i = flavor index a, μ = color index

Proposed mirror: S_k orbifold of

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{ib} \rho^a_{ib} Y^{ib} - \sum_{\mu\neq\nu} \alpha^a_{\mu\nu} \ln X_{\mu\nu} - t \right) + \sum_{ia} \exp\left(-Y^{ia}\right) + \sum_{\mu\neq\nu} X_{\mu\nu}$$

where $\rho^a_{ib} = \delta^a_b, \quad \alpha^a_{\mu\nu} = -\delta^a_\mu + \delta^a_\nu$

The orbifold acts by permuting σs, and similarly on Ys, Xs.

Simplify:

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{i} Y^{ia} + \sum_{\nu\neq a} \ln\left(\frac{X_{a\nu}}{X_{\nu a}}\right) - t \right) + \sum_{ia} \exp\left(-Y^{ia}\right) + \sum_{\mu\neq\nu} X_{\mu\nu}$$

Example: Grassmannian G(k,n)

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_i Y^{ia} + \sum_{\nu \neq a} \ln\left(\frac{X_{a\nu}}{X_{\nu a}}\right) - t \right) + \sum_{ia} \exp\left(-Y^{ia}\right) + \sum_{\mu \neq \nu} X_{\mu\nu}$$

Integrate out σ s:

Constraint $$\sum_i Y^{ia} + \sum_{\nu \neq a} \ln\left(\frac{X_{a\nu}}{X_{\nu a}}\right) - t = 0$$

Eliminate Y^{na}: $$Y^{na} = -\sum_{i=1}^{n-1} Y^{ia} - \sum_{\nu \neq a} \ln\left(\frac{X_{a\nu}}{X_{\nu a}}\right) + t$$

Define $$\Pi_a = \exp\left(-Y^{na}\right)$$

$$= q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right)\right)\left(\prod_{\nu \neq a} \frac{X_{a\nu}}{X_{\nu a}}\right)$$

Then the superpotential becomes

$$W = \sum_{i=1}^{n-1} \sum_{a=1}^{k} \exp\left(-Y^{ia}\right) + \sum_{\mu \neq \nu} X_{\mu\nu} + \sum_{a=1}^{k} \Pi_a$$

Example: Grassmannian G(k,n)

$$W \;=\; \sum_{i=1}^{n-1}\sum_{a=1}^{k} \exp\left(-Y^{ia}\right) \;+\; \sum_{\mu\neq\nu} X_{\mu\nu} \;+\; \sum_{a=1}^{k} \Pi_a$$

where $\quad \Pi_a \;=\; q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right)\right)\left(\prod_{\nu\neq a}\frac{X_{a\nu}}{X_{\nu a}}\right)$

In passing, for ordinary projective spaces $\mathbb{P}^{n-1}$,
following the same procedure,
Hori-Vafa obtained

$$W \;=\; \sum_{i=1}^{n-1} \exp\left(-Y^{i}\right) \;+\; q\prod_{i=1}^{n-1} \exp\left(+Y^{i}\right)$$

— clearly, this is a special case of the result above.

Example: Grassmannian G(k,n)

$$W = \sum_{i=1}^{n-1} \sum_{a=1}^{k} \exp\left(-Y^{ia}\right) + \sum_{\mu \neq \nu} X_{\mu\nu} + \sum_{a=1}^{k} \Pi_a$$

$$\text{where} \quad \Pi_a = q \left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right) \right) \left(\prod_{\nu \neq a} \frac{X_{a\nu}}{X_{\nu a}} \right)$$

We'll compute vacua & correlation functions,
but first,
some general observations.

- The superpotential above has poles at $X_{\mu\nu} = 0$.

This will be a generic feature of these nonabelian mirrors.

Operator mirror map: $X_{\mu\nu} = \sum_a \sigma_a \alpha^a_{\mu\nu} = -\sigma_\mu + \sigma_\nu$

So we see that the poles above are mirror to places where σs collide

- Nonabelian enhancement in original gauge theory
- Excluded in A model
- Excluded here b/c bosonic potential diverges

Example: Grassmannian G(k,n)

$$W = \sum_{i=1}^{n-1} \sum_{a=1}^{k} \exp\left(-Y^{ia}\right) + \sum_{\mu \neq \nu} X_{\mu\nu} + \sum_{a=1}^{k} \Pi_a$$

$$\text{where} \quad \Pi_a = q \left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right) \right) \left(\prod_{\nu \neq a} \frac{X_{a\nu}}{X_{\nu a}} \right)$$

- Strange behavior at nongeneric points where multiple $X_{\mu\nu} = 0$:

We excluded generic loci where any one X vanishes,
but something more subtle happens when multiple X's vanish.

The ratio $\frac{X_+}{X_-}$ is not *continuous* at $X_+ = X_- = 0$

- Presumably reflects missing physics at these nongeneric loci.
- *Might* be possible to regularize.

For example, a blowup will separate the divisors of zeroes & poles,
but unfortunately not compatible with B twist.

- Generic paths to these points break susy in limit.
- In any event, will see later that do not contribute to correlation functions.

More detailed understanding left for future work.

Example: Grassmannian G(k,n)

$$W \;=\; \sum_{i=1}^{n-1}\sum_{a=1}^{k} \exp\left(-Y^{ia}\right) \;+\; \sum_{\mu\neq\nu} X_{\mu\nu} \;+\; \sum_{a=1}^{k} \Pi_a$$

where $\quad \Pi_a \;=\; q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right)\right)\left(\prod_{\nu\neq a}\frac{X_{a\nu}}{X_{\nu a}}\right)$

Compute critical loci:

$$\frac{\partial W}{\partial Y^{ia}} = -\exp\left(-Y^{ia}\right) \;+\; \Pi_a \;=\; 0 \quad\Rightarrow\quad \exp\left(-Y^{ia}\right) \;=\; \Pi_a$$

independent of i

$$\frac{\partial W}{\partial X_{\mu\nu}} \;=\; 1 \;+\; \frac{\Pi_\mu - \Pi_\nu}{X_{\mu\nu}} \;=\; 0 \qquad\Rightarrow\qquad X_{\mu\nu} = -\Pi_\mu + \Pi_\nu$$

Hence on the critical locus,

$$\prod_{\nu\neq a}\frac{X_{a\nu}}{X_{\nu a}} \;=\; (-)^{k-1}$$

$$\Pi_a \;=\; (-)^{k-1} q \left(\prod_{i=1}^{n-1} \Pi_a^{-1}\right) \qquad\Rightarrow\qquad \left(\Pi_a\right)^n \;=\; (-)^{k-1} q$$

Example: Grassmannian G(k,n)

$$W \;=\; \sum_{i=1}^{n-1}\sum_{a=1}^{k} \exp\left(-Y^{ia}\right) \;+\; \sum_{\mu\neq\nu} X_{\mu\nu} \;+\; \sum_{a=1}^{k} \Pi_a$$

where $$\Pi_a \;=\; q\left(\prod_{i=1}^{n-1}\exp\left(+Y^{ia}\right)\right)\left(\prod_{\nu\neq a}\frac{X_{a\nu}}{X_{\nu a}}\right)$$

Critical loci:

$$\exp\left(-Y^{ia}\right) \;=\; \Pi_a \qquad X_{\mu\nu} \;=-\;\Pi_\mu + \Pi_\nu \qquad \left(\Pi_a\right)^n \;=\; (-)^{k-1} q$$

Excluded locus: $X_{\mu\nu} \neq 0 \quad \Rightarrow \quad \Pi_\mu \neq \Pi_\nu$ for $\mu \neq \nu$

This is starting to look like the equations satisfied by A model vacua….

Recall operator mirror map:

$$\left.\begin{aligned} \exp\left(-Y^{ia}\right) \;&=\; \sum_a \sigma_b \rho^b_{ia} \;=\; \sigma_a \\ X_{\mu\nu} \;&=\; \sum_a \sigma_a \alpha^a_{\mu\nu} \;=-\; \sigma_\mu + \sigma_\nu \end{aligned}\right\} \;\Rightarrow\; \Pi_\mu \;=\; \sigma_\mu$$

which is the good reason why the equations for Π s match those for σ s.

Example: Grassmannian G(k,n)

$$W = \sum_{i=1}^{n-1}\sum_{a=1}^{k} \exp\left(-Y^{ia}\right) + \sum_{\mu\neq\nu} X_{\mu\nu} + \sum_{a=1}^{k} \Pi_a$$

$$\text{where} \quad \Pi_a = q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right)\right)\left(\prod_{\nu\neq a}\frac{X_{a\nu}}{X_{\nu a}}\right)$$

Critical loci:

$$\exp\left(-Y^{ia}\right) = \Pi_a \qquad X_{\mu\nu} = -\Pi_\mu + \Pi_\nu \qquad \left(\Pi_a\right)^n = (-)^{k-1} q$$

Excluded locus: $X_{\mu\nu} \neq 0 \quad \Rightarrow \quad \Pi_\mu \neq \Pi_\nu$ for $\mu \neq \nu$

Operator mirror map: $\Pi_\mu = \sigma_\mu$

Compare A model: $\sigma_a \neq \sigma_b$ if $a \neq b$ $\qquad \left(\sigma_a\right)^n = (-)^{k-1} q$

Same equations, same solutions

— Same number of vacua, plus,
quantum cohomology derived as critical locus equations in B model

Example: Grassmannian G(k,n)

$$W = \sum_{i=1}^{n-1}\sum_{a=1}^{k} \exp\left(-Y^{ia}\right) + \sum_{\mu\neq\nu} X_{\mu\nu} + \sum_{a=1}^{k} \Pi_a$$

where $\Pi_a = q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right)\right)\left(\prod_{\nu\neq a}\frac{X_{a\nu}}{X_{\nu a}}\right)$

Critical loci:

$$\exp\left(-Y^{ia}\right) = \Pi_a \qquad X_{\mu\nu} = -\Pi_\mu + \Pi_\nu \qquad \left(\Pi_a\right)^n = (-)^{k-1}q$$

Excluded locus: $X_{\mu\nu} \neq 0 \quad \Rightarrow \quad \Pi_\mu \neq \Pi_\nu$ for $\mu \neq \nu$

- Orbifold fixed points

What about that Weyl group orbifold and its twisted sectors?

The Weyl group S_n acts as $\Pi_\mu \leftrightarrow \Pi_\nu \quad Y^{ia} \leftrightarrow Y^{ib} \quad X_{\mu\nu} \leftrightarrow X_{\mu'\nu'}$

Fixed-point locus of the orbifold at e.g. $\Pi_\mu = \Pi_\nu \quad Y^{ia} = Y^{ib} \quad X_{\mu\nu} = X_{\mu'\nu'}$

Fixed-point locus intersects critical locus along excluded locus.

— so, expect no contribution from twisted sectors

Example: Grassmannian G(k,n)

$$W \;=\; \sum_{i=1}^{n-1}\sum_{a=1}^{k} \exp\left(-Y^{ia}\right) \;+\; \sum_{\mu\neq\nu} X_{\mu\nu} \;+\; \sum_{a=1}^{k} \Pi_a$$

$$\text{where} \quad \Pi_a \;=\; q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{ia}\right)\right)\left(\prod_{\nu\neq a} \frac{X_{a\nu}}{X_{\nu a}}\right)$$

Critical loci:

$$\exp\left(-Y^{ia}\right) \;=\; \Pi_a \qquad X_{\mu\nu} \;=\, -\,\Pi_\mu + \Pi_\nu \qquad \left(\Pi_a\right)^n \;=\; (\,-\,)^{k-1} q$$

Excluded locus: $\quad X_{\mu\nu} \neq 0 \quad \Rightarrow \quad \Pi_\mu \neq \Pi_\nu \quad$ for $\quad \mu \neq \nu$

Now, let's compute correlation functions.

In B model, for isolated vacua = critical loci, and worldsheet S^2,

$$\langle f \rangle \;=\; \frac{1}{|\,S_k\,|} \sum_{\text{vacua}} \frac{f}{H} \quad \text{where} \quad H \;=\; \det \partial^2 W$$

The first factor is a remnant of the Weyl-group orbifold.

Example: Grassmannian G(k,n)

Critical loci:

$$\exp\left(-Y^{ia}\right) \;=\; \Pi_a \qquad\qquad X_{\mu\nu} \;= -\,\Pi_\mu + \Pi_\nu \qquad\qquad \left(\Pi_a\right)^n \;=\; (\,-\,)^{k-1} q$$

Excluded locus: $X_{\mu\nu} \neq 0 \quad \Rightarrow \quad \Pi_\mu \neq \Pi_\nu$ for $\mu \neq \nu$

Correlation functions: $\langle f \rangle \;=\; \dfrac{1}{|S_k|} \displaystyle\sum_{\text{vacua}} \frac{f}{H}$ where $H \;=\; \det \partial^2 W$

For G(2,n), can show $H = -\,n^2 \dfrac{\left(\Pi_1\right)^{n-1} \left(\Pi_2\right)^{n-1}}{\left(\Pi_1 - \Pi_2\right)^2}$

& from summing over vacua, the nonzero correlation functions of deg 2n-4 are

$$\langle \Pi_1^{n-1} \Pi_2^{n-3} \rangle \;= -\,\frac{1}{2!} \;=\; \langle \Pi_1^{n-3} \Pi_2^{n-1} \rangle \qquad\qquad \langle \Pi_1^{n-2} \Pi_2^{n-2} \rangle \;= +\,\frac{2}{2!}$$

Compare A model results….

Example: Grassmannian G(k,n)

Correlation functions: In the LG orbifold mirror to G(2,n), we computed

$$\langle f \rangle = \frac{1}{|S_k|} \sum_{\text{vacua}} \frac{f}{H} = -\frac{1}{2!n^2} \sum_{\text{vacua}} \frac{\left(\Pi_1 - \Pi_2\right)^2}{\left(\Pi_1\right)^{n-1}\left(\Pi_2\right)^{n-1}} f$$

Nonzero correlators of deg 2n-4:

$$\langle \Pi_1^{n-1}\Pi_2^{n-3} \rangle = -\frac{1}{2!} = \langle \Pi_1^{n-3}\Pi_2^{n-1} \rangle \qquad \langle \Pi_1^{n-2}\Pi_2^{n-2} \rangle = +\frac{2}{2!}$$

Compare original A-twisted gauge theory results for G(2,n): (Guo, Lu, ES, 1512.08586)

$$\langle f(\sigma) \rangle = -\frac{1}{2!} \mathrm{JK-Res} \left[\frac{\left(\sigma_1 - \sigma_2\right)^2}{\sigma_1^n \sigma_2^n} f(\sigma) \right]$$

Nonzero correlators of deg 2n-4:

$$\langle \sigma_1^{n-1}\sigma_2^{n-3} \rangle = -\frac{1}{2!} = \langle \sigma_1^{n-3}\sigma_2^{n-1} \rangle \qquad \langle \sigma_1^{n-2}\sigma_2^{n-2} \rangle = +\frac{2}{2!}$$

(for the cases we've checked: n=3, 4, 5)

Recall operator mirror map relates $\Pi_\mu \leftrightarrow \sigma_\mu$

Perfect match!

Aside:

If one integrates out the X fields, the effect is to add factors of

$$\prod_{a<b} \left(\sigma_a - \sigma_b\right)^2$$

to the integration measure, and generate a superpotential of the form

$$W_{\rm eff} \;=\; \sum_a \sigma_a \left[\sum_i Y_{ia} - \tilde{t} \right] \;+\; \sum_i \exp\left(-Y_i\right)$$

Furthermore, working in the untwisted sector of the orbifold, we restrict to S_k-invariant field combinations.

— matches Hori-Vafa (hep-th/0002222) appendix A,
Gomis-Lee (1210.6022)
proposals for Grassmannian mirrors

Next, let's consider the mirror of an SO(2k) gauge theory with n vectors.

This 2d gauge theory was previously studied by Hori in 2011,
so we can compare our proposed mirror's results to what he obtained.

Hori computed: (Hori, 1104.2853)

A model excluded locus $\sigma_a \neq \pm \tilde{m}_i \qquad \sigma_a \neq \pm \sigma_b$

& Coulomb branch relation $$\prod_{i=1}^{n} \left(\sigma_a - \tilde{m}_i\right) = q \prod_{i=1}^{n} \left(-\sigma_a - \tilde{m}_i\right)$$

(an analogue of quantum cohomology, except that q is not a continuous parameter,
and there's no geometric limit)

from which he derived various properties of these theories.

We'll recover the same excluded locus and Coulomb branch relation from the proposed mirror.

Example: SO(2k) gauge theory with n chirals in the vector representation

Proposed mirror: Weyl-group orbifold of

Fields:
- σ_a $a \in \{1, \cdots, k\}$
- $Y^{i\alpha}$ i flavor index, $i \in \{1, \cdots, n\}$ α vector index, $\alpha \in \{1, \cdots, 2k\}$
  mirror to matter fields
- $X_{\mu\nu} = X_{\nu\mu}^{-1}$ $\mu, \nu \in \{1, \cdots, 2k\}$ excluding $X_{2a-1,2a}$ (corresponding to Cartan)
  — Lie algebra is imaginary antisymm matrices, & we're dropping i's.

Superpotential:

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{i\alpha\beta} \rho^a_{i\alpha\beta} Y^{i\beta} - \sum_{\mu<\nu;\mu',\nu'} \alpha^a_{\mu\nu,\mu'\nu'} \ln X_{\mu'\nu'} - t \right) + \sum_{i\alpha} \exp\left(-Y^{i\alpha}\right) + \sum_{\mu<\nu} X_{\mu\nu} - \sum_{i\alpha} \tilde{m}_i Y^{i\alpha}$$

— Following Georgi, represented Cartan by block-diagonals with Paul σ_2 on diagonal

— hence ρ, α above represent commutators

$$\rho^a_{i\alpha\beta} = \delta^{2a-1}_{\alpha} \delta^{2a}_{\beta} - \delta^{2a-1}_{\beta} \delta^{2a}_{\alpha}$$

$$\alpha^a_{\mu\nu,\mu'\nu'} = \delta_{\nu\nu'} \left(\delta^{2a-1}_{\mu} \delta^{2a}_{\mu'} - \delta^{2a}_{\mu} \delta^{2a-1}_{\mu'} \right) + \delta_{\mu\mu'} \left(\delta^{2a-1}_{\nu} \delta^{2a}_{\nu'} - \delta^{2a}_{\nu} \delta^{2a-1}_{\nu'} \right)$$

Example: SO(2k) gauge theory with n chirals in the vector representation

Proposed mirror: Weyl-group orbifold of

Fields:
- $\sigma_a \quad a \in \{1, \cdots, k\}$
- $Y^{i\alpha}$ i flavor index, $i \in \{1, \cdots, n\}$ α vector index, $\alpha \in \{1, \cdots, 2k\}$
- $X_{\mu\nu} = X_{\nu\mu}^{-1} \quad \mu, \nu \in \{1, \cdots, 2k\}$ excluding $X_{2a-1,2a}$ (corresponding to Cartan)

Superpotential:

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{i\alpha\beta} \rho^a_{i\alpha\beta} Y^{i\beta} - \sum_{\mu<\nu;\mu',\nu'} \alpha^a_{\mu\nu,\mu'\nu'} \ln X_{\mu'\nu'} - t \right) + \sum_{i\alpha} \exp\left(-Y^{i\alpha}\right) + \sum_{\mu<\nu} X_{\mu\nu} - \sum_{i\alpha} \tilde{m}_i Y^{i\alpha}$$

$$\rho^a_{i\alpha\beta} = \delta^{2a-1}_{\alpha} \delta^{2a}_{\beta} - \delta^{2a-1}_{\beta} \delta^{2a}_{\alpha}$$

$$\alpha^a_{\mu\nu,\mu'\nu'} = \delta_{\nu\nu'} \left(\delta^{2a-1}_{\mu} \delta^{2a}_{\mu'} - \delta^{2a}_{\mu} \delta^{2a-1}_{\mu'} \right) + \delta_{\mu\mu'} \left(\delta^{2a-1}_{\nu} \delta^{2a}_{\nu'} - \delta^{2a}_{\nu} \delta^{2a-1}_{\nu'} \right)$$

$t =$ discrete theta angle

This is not an ordinary theta angle.

Only takes values in Weyl-invariant constants: $0, \pi i$

Distinguishes two different 2d SO(2k) theories.

Example: SO(2k) gauge theory with n chirals in the vector representation

Proposed mirror: Weyl-group orbifold of

Fields:
- $\sigma_a \quad a \in \{1,\cdots,k\}$
- $Y^{i\alpha}$ i flavor index, $i \in \{1,\cdots,n\}$ α vector index, $\alpha \in \{1,\cdots,2k\}$
- $X_{\mu\nu} = X_{\nu\mu}^{-1} \quad \mu,\nu \in \{1,\cdots,2k\}$ excluding $X_{2a-1,2a}$ (corresponding to Cartan)

Superpotential:

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{i\alpha\beta} \rho^a_{i\alpha\beta} Y^{i\beta} - \sum_{\mu<\nu;\mu',\nu'} \alpha^a_{\mu\nu,\mu'\nu'} \ln X_{\mu'\nu'} - t \right) + \sum_{i\alpha} \exp\left(-Y^{i\alpha}\right) + \sum_{\mu<\nu} X_{\mu\nu} - \sum_{i\alpha} \tilde{m}_i Y^{i\alpha}$$

$$\rho^a_{i\alpha\beta} = \delta^{2a-1}_{\alpha}\delta^{2a}_{\beta} - \delta^{2a-1}_{\beta}\delta^{2a}_{\alpha}$$

$$\alpha^a_{\mu\nu,\mu'\nu'} = \delta_{\nu\nu'}\left(\delta^{2a-1}_{\mu}\delta^{2a}_{\mu'} - \delta^{2a}_{\mu}\delta^{2a-1}_{\mu'}\right) + \delta_{\mu\mu'}\left(\delta^{2a-1}_{\nu}\delta^{2a}_{\nu'} - \delta^{2a}_{\nu}\delta^{2a-1}_{\nu'}\right)$$

Let’s simplify before discussing Weyl group action.

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{i=1}^{n} \left(Y^{i,2a} - Y^{i,2a-1}\right) - \sum_{\mu<2a-1} \ln\left(\frac{X_{\mu,2a}}{X_{\mu,2a-1}}\right) - \sum_{\mu>2a} \ln\left(\frac{X_{2a,\mu}}{X_{2a-1,\mu}}\right) - t \right)$$

$$+ \sum_{i\alpha} \exp\left(-Y^{i\alpha}\right) + \sum_{\mu<\nu} X_{\mu\nu} - \sum_{i\alpha} \tilde{m}_i Y^{i\alpha}$$

Example: SO(2k) gauge theory with n chirals in the vector representation

Superpotential:

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{i=1}^{n} \left(Y^{i,2a} - Y^{i,2a-1} \right) - \sum_{\mu<2a-1} \ln\left(\frac{X_{\mu,2a}}{X_{\mu,2a-1}} \right) - \sum_{\mu>2a} \ln\left(\frac{X_{2a,\mu}}{X_{2a-1,\mu}} \right) - t \right)$$

$$+ \sum_{i\alpha} \exp\left(-Y^{i\alpha}\right) + \sum_{\mu<\nu} X_{\mu\nu} - \sum_{i\alpha} \tilde{m}_i Y^{i\alpha}$$

Weyl group W: $1 \longrightarrow K \longrightarrow W \longrightarrow S_k \longrightarrow 1$

S_k acts by interchanging σs and blocks of Ys, Xs

$K \subset (\mathbb{Z}_2)^k$ is the subgroup with an even number of nontriv' factors

Each $\mathbb{Z}_2$ factor acts (for one index a) as

$$\sigma_a \mapsto -\sigma_a$$

$$Y^{i,2a} \leftrightarrow Y^{i,2a-1}$$

$$X_{\mu,2a} \leftrightarrow X_{\mu,2a-1} \qquad X_{2a,\nu} \leftrightarrow X_{2a-1,\nu}$$

Straightforward to see that superpotential is invariant.

Example: SO(2k) gauge theory with n chirals in the vector representation

Superpotential:

$$W = \sum_{a=1}^{k} \sigma_a \left(\sum_{i=1}^{n} \left(Y^{i,2a} - Y^{i,2a-1}\right) - \sum_{\mu<2a-1} \ln\left(\frac{X_{\mu,2a}}{X_{\mu,2a-1}}\right) - \sum_{\mu>2a} \ln\left(\frac{X_{2a,\mu}}{X_{2a-1,\mu}}\right) - t \right)$$

$$+ \sum_{i\alpha} \exp\left(-Y^{i\alpha}\right) + \sum_{\mu<\nu} X_{\mu\nu} - \sum_{i\alpha} \tilde{m}_i Y^{i\alpha}$$

Integrate out σs:

$$\sum_{i=1}^{n} \left(Y^{i,2a} - Y^{i,2a-1}\right) - \sum_{\mu<2a-1} \ln\left(\frac{X_{\mu,2a}}{X_{\mu,2a-1}}\right) - \sum_{\mu>2a} \ln\left(\frac{X_{2a,\mu}}{X_{2a-1,\mu}}\right) = t$$

Eliminate $Y^{n,2a}$:

$$Y^{n,2a} = t - \sum_{i=1}^{n-1} Y^{i,2a} + \sum_{i=1}^{n} Y^{i,2a-1} + \sum_{\mu<2a-1} \ln\left(\frac{X_{\mu,2a}}{X_{\mu,2a-1}}\right) + \sum_{\mu>2a} \ln\left(\frac{X_{2a,\mu}}{X_{2a-1,\mu}}\right)$$

Define $\Pi_a = \exp\left(-Y^{n,2a}\right)$

$$= q \left(\prod_{i=1}^{n-1} \exp\left(+Y^{i,2a}\right) \right) \left(\prod_{i=1}^{n} \exp\left(-Y^{i,2a-1}\right) \right) \left(\prod_{\mu<2a-1} \frac{X_{\mu,2a}}{X_{\mu,2a-1}} \right) \left(\prod_{\mu>2a} \frac{X_{2a,\mu}}{X_{2a-1,\mu}} \right)$$

Example: SO(2k) gauge theory with n chirals in the vector representation

$$W = \sum_{i=1}^{n-1}\sum_{a=1}^{k} \exp\left(-Y^{i,2a}\right) + \sum_{i=1}^{n}\sum_{a=1}^{k} \exp\left(-Y^{i,2a-1}\right) + \sum_{\mu<\nu} X_{\mu\nu} + \sum_{a=1}^{k} \Pi_a$$
$$- \sum_{i=1}^{n-1}\sum_{a=1}^{k} \tilde{m}_i Y^{i,2a} - \sum_{i=1}^{n}\sum_{a=1}^{k} \tilde{m}_i Y^{i,2a-1}$$
$$- \tilde{m}_n \sum_{a=1}^{k}\left(-\sum_{i=1}^{n-1} Y^{i,2a} + \sum_{i=1}^{n} Y^{i,2a-1} + \sum_{\mu<2a-1} \ln\left(\frac{X_{\mu,2a}}{X_{\mu,2a-1}}\right) + \sum_{\mu>2a} \ln\left(\frac{X_{2a,\mu}}{X_{2a-1,\mu}}\right)\right)$$

where

$$\Pi_a = q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{i,2a}\right)\right)\left(\prod_{i=1}^{n} \exp\left(-Y^{i,2a-1}\right)\right)\left(\prod_{\mu<2a-1} \frac{X_{\mu,2a}}{X_{\mu,2a-1}}\right)\left(\prod_{\mu>2a} \frac{X_{2a,\mu}}{X_{2a-1,\mu}}\right)$$

Critical loci:

$$\frac{\partial W}{\partial Y^{i,2a}}: \quad \exp\left(-Y^{i,2a}\right) = \Pi_a - \tilde{m}_i + \tilde{m}_n \qquad\qquad \frac{\partial W}{\partial Y^{i,2a-1}}: \quad \exp\left(-Y^{i,2a-1}\right) = -\Pi_a - \tilde{m}_i - \tilde{m}_n$$

$$\frac{\partial W}{\partial X_{\mu\nu}}: \quad \begin{array}{ll} X_{2a,2b} = \Pi_a + \Pi_b + 2\tilde{m}_n & X_{2a,2b-1} = \Pi_a - \Pi_b \\ X_{2a-1,2b-1} = -\Pi_a - \Pi_b - 2\tilde{m}_n & X_{2a-1,2b} = -\Pi_a + \Pi_b \end{array} \quad \text{for } a<b$$

or more simply, $X_{\mu\nu} = \sum_a \left(\Pi_a + \tilde{m}_n\right)\left(\delta_\mu^{2a} - \delta_\mu^{2a-1} + \delta_\nu^{2a} - \delta_\nu^{2a-1}\right)$ for $\mu<\nu$

Example: SO(2k) gauge theory with n chirals in the vector representation

Critical loci: Excluded locus:

$$\left.\begin{aligned} \exp\left(-Y^{i,2a}\right) &= \Pi_a - \tilde{m}_i + \tilde{m}_n \\ \exp\left(-Y^{i,2a-1}\right) &= -\Pi_a - \tilde{m}_i - \tilde{m}_n \end{aligned}\right\} \neq 0 \Rightarrow \quad \Pi_a + \tilde{m}_n \neq \pm \tilde{m}_i$$

$$\left.\begin{aligned} X_{2a,2b} &= \Pi_a + \Pi_b + 2\tilde{m}_n \\ X_{2a-1,2b-1} &= -\Pi_a - \Pi_b - 2\tilde{m}_n \\ X_{2a,2b-1} &= \Pi_a - \Pi_b \\ X_{2a-1,2b} &= -\Pi_a + \Pi_b \end{aligned}\right\} \neq 0 \Rightarrow \quad \Pi_a + \tilde{m}_n \neq \pm\left(\Pi_b + \tilde{m}_n\right)$$

Coulomb branch relation:

$$\Pi_a = q\left(\prod_{i=1}^{n-1} \exp\left(+Y^{i,2a}\right)\right)\left(\prod_{i=1}^{n} \exp\left(-Y^{i,2a-1}\right)\right)\left(\prod_{\mu<2a-1} \frac{X_{\mu,2a}}{X_{\mu,2a-1}}\right)\left(\prod_{\mu>2a} \frac{X_{2a,\mu}}{X_{2a-1,\mu}}\right)$$

On critical locus,

$$= q\left(\prod_{i=1}^{n-1} \frac{1}{\Pi_a - \tilde{m}_i + \tilde{m}_n}\right)\left(\prod_{i=1}^{n}\left(-\Pi_a - \tilde{m}_i - \tilde{m}_n\right)\right)\left(\left(-\right)^{2(k-1)}\right)$$

hence

$$\prod_{i=1}^{n}\left(\Pi_a - \tilde{m}_i + \tilde{m}_n\right) = q\prod_{i=1}^{n}\left(-\Pi_a - \tilde{m}_i - \tilde{m}_n\right)$$

Example: SO(2k) gauge theory with n chirals in the vector representation

Excluded locus: $\Pi_a + \tilde{m}_n \neq \pm \tilde{m}_i \qquad \Pi_a + \tilde{m}_n \neq \pm \left(\Pi_b + \tilde{m}_n\right)$

Coulomb branch relation: $\displaystyle\prod_{i=1}^{n} \left(\Pi_a - \tilde{m}_i + \tilde{m}_n\right) = q \prod_{i=1}^{n} \left(-\Pi_a - \tilde{m}_i - \tilde{m}_n\right)$

Operator mirror map:

$$
\begin{aligned}
\exp\left(-Y^{i\alpha}\right) &= -\tilde{m}_i + \sum_{a=1}^{k} \sum_{i\beta} \rho^{a}_{i\beta\alpha} \\
&= -\tilde{m}_i + \sum_{a} \sigma_a \left(\delta^{2a}_{\alpha} - \delta^{2a-1}_{\alpha}\right) \\
&= = \tilde{m}_i + \begin{cases} \sigma_a, & \alpha = 2a \\ -\sigma_a, & \alpha = 2a-1 \end{cases}
\end{aligned}
$$

Compare:

$$\exp\left(-Y^{i,2a}\right) = \Pi_a - \tilde{m}_i + \tilde{m}_n$$

$$\exp\left(-Y^{i,2a-1}\right) = -\Pi_a - \tilde{m}_i - \tilde{m}_n$$

$$
\begin{aligned}
X_{\mu\nu} &= \sum_{a=1}^{k} \sum_{\mu'<\nu'} \sigma_a \, \alpha^{a}_{\mu'\nu',\mu\nu} \qquad \text{for} \quad \mu < \nu \\
&= \sum_{a=1}^{k} \left(\delta^{2a}_{\mu} - \delta^{2a-1}_{\mu} + \delta^{2a}_{\nu} - \delta^{2a-1}_{\nu}\right) \sigma_a
\end{aligned}
$$

$$X_{\mu\nu} = \sum_{a} \left(\delta^{2a}_{\mu} - \delta^{2a-1}_{\mu} + \delta^{2a}_{\nu} - \delta^{2a-1}_{\nu}\right) \left(\Pi_a + \tilde{m}_n\right)$$

$$\Rightarrow \quad \underline{\sigma_a = \Pi_a + \tilde{m}_n}$$

Example: SO(2k) gauge theory with n chirals in the vector representation

Excluded locus: $\Pi_a + \tilde{m}_n \neq \pm \tilde{m}_i \qquad \Pi_a + \tilde{m}_n \neq \pm \left(\Pi_b + \tilde{m}_n \right)$

Coulomb branch relation: $$\prod_{i=1}^{n} \left(\Pi_a - \tilde{m}_i + \tilde{m}_n \right) = q \prod_{i=1}^{n} \left(-\Pi_a - \tilde{m}_i - \tilde{m}_n \right)$$

Operator mirror map: $\sigma_a = \Pi_a + \tilde{m}_n$

Predicts A model excluded locus $\sigma_a \neq \pm \tilde{m}_i \qquad \sigma_a \neq \pm \sigma_b$

& Coulomb branch relation $$\prod_{i=1}^{n} \left(\sigma_a - \tilde{m}_i \right) = q \prod_{i=1}^{n} \left(-\sigma_a - \tilde{m}_i \right)$$

which match known results for this theory. (Hori, 1104.2853)

Pure 2d (2,2) susy gauge theories

It has been argued (Aharony et al 1611.02763) that 2d (2,2) susy pure SU(k) gauge theories flow in the IR to a theory of k-1 free twisted chiral multiplets.

We can see this in the mirror, at least at the level of TFT computations.

Example: pure SU(2) theory

Mirror LG model:

$$W = 2\sigma_1 \ln\left(\frac{X_{12}}{X_{21}}\right) + X_{12} + X_{21}$$

Critical loci:

$$\frac{\partial W}{\partial \sigma_1}: \left(\frac{X_{12}}{X_{21}}\right)^2 = 1 \qquad \frac{\partial W}{\partial X_{12}}: X_{12} = -2\sigma_1 \qquad \frac{\partial W}{\partial X_{21}}: X_{21} = +2\sigma_1$$

Solved by $X_{12} = -X_{21}$ $\quad\sigma_1$ unconstrained

& W=0 along this locus.

Pure 2d (2,2) susy gauge theories

Example: pure SU(2) theory $$W \;=\; 2\sigma_1 \ln\left(\frac{X_{12}}{X_{21}}\right) \;+\; X_{12} \;+\; X_{21}$$

Example: pure SO(3) theory $$W \;=\; \sigma_1 \ln\left(\frac{X_{12}}{X_{21}}\right) \;+\; X_{12} \;+\; X_{21} \;+\; t\sigma_1$$

$t \in \{0, \pi i\}$ encodes discrete theta angle

(distinguishes $SO(3)_+$, $SO(3)_-$ theories)

Critical loci:

$$\frac{\partial W}{\partial \sigma_1} : \;\; \frac{X_{12}}{X_{21}} \;=\; \exp(-t) \qquad\qquad \frac{\partial W}{\partial X_{12}} : \;\; X_{12} \;= -\, \sigma_1 \qquad\qquad \frac{\partial W}{\partial X_{21}} : \;\; X_{21} \;= +\, \sigma_1$$

Cases:

$SO(3)_+$: $t = 0 :$ $\dfrac{X_{12}}{X_{21}} = +\,1 \;\;\&\;\; X_{12} = -\,X_{21}$ — inconsistent, no sol'ns, no vacua

susy broken

$SO(3)_-$: $t = \pi i :$ $\dfrac{X_{12}}{X_{21}} = -\,1 \;\;\&\;\; X_{12} = -\,X_{21}$ — consistent, σ_1 unconstrained

one free twisted chiral superfield in IR

Pure 2d (2,2) susy gauge theories

Example: pure SU(2) theory $$W = 2\sigma_1 \ln\left(\frac{X_{12}}{X_{21}}\right) + X_{12} + X_{21}$$

Example: pure SO(3) theory $$W = \sigma_1 \ln\left(\frac{X_{12}}{X_{21}}\right) + X_{12} + X_{21} + t\sigma_1$$

Summary:

SU(2), SO(3)$_-$ consistent w/ flow in IR to one free twisted chiral superfield

SO(3)$_+$ breaks susy

There's a relationship between these three….

Pure 2d (2,2) susy gauge theories

$SU(2)$, $SO(3)_-$ consistent w/ flow in IR to one free twisted chiral superfield

$SO(3)_+$ breaks susy

Decomposition:

If a 2d G-gauge theory has massless matter invariant under a finite subgp H of G,
or if there is no massless matter,
then in IR,

G-gauge theory = disjoint union of G/H gauge theories
w/ various discrete theta angles

(Hellerman et al, hepth/0606034)

(Equivalent to theories w/ restriction on topological sectors;
also, to theories `coupled to TFTs')

Here, schematically: $SU(2) = SO(3)_+ + SO(3)_-$

So if $SU(2)$ flows in IR to free theory w/ one superfield,
exactly one of $SO(3)_+$, $SO(3)_-$ will flow in IR to free theory,
& other cannot have susy vacua.

— consistent!

Pure 2d (2,2) susy gauge theories

$SU(2)$, $SO(3)_-$ consistent w/ flow in IR to one free twisted chiral superfield

$SO(3)_+$ breaks susy

$$SU(2) = SO(3)_+ + SO(3)_-$$

More generally, we find (TFT-level) evidence for:

$SU(k)$ flows in IR to k-1 free twisted chiral superfields

$SO(2k)_+$ flows in IR to k free twisted chiral superfields

$SO(2k)_-$ breaks susy

$SO(2k+1)_+$ breaks susy

$SO(2k+1)_-$ flows in IR to k free twisted chiral superfields

$Sp(2k)$ flows in IR to k free twisted chiral superfields

Another decomposition example: $SU(4) = SO(6)_+ + SO(6)_-$

3 free tw' chirals

breaks susy

Pure 2d (2,2) susy gauge theories

$SO(2k)_+$ flows in IR to k free twisted chiral superfields

$SO(2k)_-$ breaks susy

$SO(2k+1)_+$ breaks susy

$SO(2k+1)_-$ flows in IR to k free twisted chiral superfields

Another application of decomposition:

For pure gauge theories, $\text{Spin} = SO_+ + SO_-$

(In fact, since the center of Spin is either $\mathbf{Z}_4$ or $\mathbf{Z}_2 \times \mathbf{Z}_2$, a finer decomposition exists, but is not relevant here.)

Given the results for SO theories,
we conjecture that
a pure Spin theory flows in the IR to free twisted chiral multiplets
(as many as the rank)

Pure 2d (2,2) susy gauge theories

SU(k) flows in IR to k-1 free twisted chiral superfields

Spin(2k) flows in IR to k free twisted chiral superfields

Spin(2k+1) flows in IR to k free twisted chiral superfields

Sp(2k) flows in IR to k free twisted chiral superfields

In other work, (Chen, Parsian, ES, to appear),
we'll check — at same level of TFTs —
that pure G_2, F_4, E_6, E_7, E_8 theories flow to free twisted chiral superfields.

Conjecture: a pure 2d (2,2) susy G-gauge theory
(G connected, simply-connected, semisimple)
flows in the IR to a theory of free twisted chiral superfields,
as many as the rank.

Conjecture: a pure 2d (2,2) susy G/H -gauge theory,
for H a subgp of center of G,
for one discrete theta angle flows to free theory,
and for other discrete theta angles, has no susy vacua.

Summary

I've outlined a proposal for a generalization of Hori-Vafa mirrors
to 2d (2,2) susy *nonabelian* gauge theories,
yielding a Landau-Ginzburg orbifold whose classical physics
encodes the quantum physics of the 2d gauge theory.

- Checked for Grassmannians G(k,n)

- Checked for SO(2k) gauge theory with n vectors

- Discussed mirrors to pure 2d gauge theories

Thank you for your time!