Quantization of Fayet-Iliopoulos parameters in supergravity

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joint w/ J Distler arXiv: 1008.0419,
and w/ S Hellerman, arXiv: 1012.5999

Also: N Seiberg, 1005.0002; Banks, Seiberg 1011.5120

A brief partial history of FI in $\left({}^{\text{old}}_{\text{minimal}}\right)$ supergravity

(Dienes, Thomas, 0911.0677)

Old & complex literature on FI terms; some root issues:

* Any gauge group must be combined with the R symmetry; the FI term contributes to the charges of the gravitino, etc

(Freedman '77, Stelle-West '78, Barbieri et al '82)

which violates electric charge quantization.

(Witten, ``New issues...'', '86, footnote p 85)

This led to the lore that FI terms couldn't exist in 4d N=1 sugrav; however, recently, ...

* Sol'n: quantize the FI term.

(Seiberg, '10)

Seiberg worked w/ linearly-realized gp actions;
I'll describe today how to generalize, in classical theory.

Disambiguation:

* In 2009, Komargodski-Seiberg considered sugrav theories obtained by coupling rigid theories to gravity. (Not all sugrav's of this form.)

Found that Kahler form on sugrav moduli space must be exact, and FI parameter must vanish.

* Seiberg's 2010 paper concerns more general sugrav's, not given by coupling a rigid theory.

It's in these more general theories (inc. string compactifications) that FI can be nonzero, and this is what I will focus on today.

Outline:

* review Bagger-Witten

* quantization of FI parameters in sugrav
when sugrav moduli space is a **space**

But sometimes it's a ``stack''....

* review stacks and gerbes (= special stacks)
-- discrete symmetries in string theory

* exs & prop's of gerby moduli spaces
in field and string theory

* Bagger-Witten, FI quantization
when moduli space is a gerbe

Review of Bagger-Witten:

Bagger-Witten's pertinent paper studied
N=1 sugrav in 4d.

Now, as a sugrav theory, it contains a
(low-energy effective) 4d NLSM on a space,
the supergravity moduli space.

They derived a constraint on the metric on that
moduli space
(assuming the moduli space is a smooth manifold).

Review of Bagger-Witten:

Briefly, the supergravity moduli space $\mathcal{M}$
(the target space of a 4d NLSM)
comes with a natural line bundle $\mathcal{L}^{\otimes 2}$,
whose c_1 = Kahler form.
(hence quantized)

Across coordinate patches,

$$K \mapsto K + f + \overline{f}$$

$$\chi^i \mapsto \exp\left(+\frac{i}{2}\mathrm{Im}\, f\right)\chi^i, \;\; \psi_\mu \mapsto \exp\left(-\frac{i}{2}\mathrm{Im}\, f\right)\psi_\mu$$

$$\chi^i \in \Gamma\left(\phi^*\left(T\mathcal{M}\otimes\mathcal{L}\right)\right), \quad \psi_\mu \in \Gamma\left(TX\otimes\phi^*\mathcal{L}^{-1}\right)$$

Bagger-Witten dates to early ‘80s.

Very recently, there has been progress on
FI terms in 4d sugrav.

Seiberg in May 2010 argued that,
in the special case that the group action on the
sugrav moduli space is realized linearly
(ie, moduli space = vector space V,
group acting as a subgp of GL(V)),
the FI term exists & is quantized.

I’ll discuss generalization to nonlinear realizations
(ie, gen’l Kahler moduli spaces) today.

Quick & dirty argument for FI quantization:

Continuously varying the FI term,
continuously varies the symplectic form on the quotient space.

But that symplectic form = Kahler form,
& Bagger-Witten says is quantized.

Consistency requires FI term be quantized too.

Problem:

-- IR limit not same as NLSM, so irrelevant to B-W

Nice intuition, but need to work harder.

To gain a more complete understanding,
let's consider gauging the Bagger-Witten story.

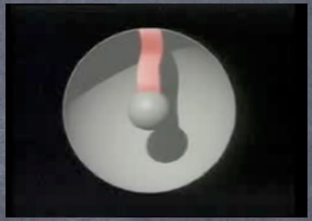
Have:

* sugrav moduli space $\mathcal{M}$
* line bundle $\mathcal{L}$
* group action on moduli space $\mathcal{M}$

Need to specify how group acts on $\mathcal{L}$ .....

In principle, if we now wish to gauge a group action on the supergravity moduli space $\mathcal{M}$,
then we need to specify the group action on $\mathcal{L}$.

* **not** always possible:
group actions on spaces do not always lift to bundles
Ex: spinors under rotations;
rotate 4π instead of 2π.



-- classical constraint on sugrav theories....

* **not** unique:
when they do lift, there are multiple lifts
(These will be the FI parameters.)

We'll see FI as a choice of group action on the Bagger-Witten line bundle directly in sugrav.

First: what is D?

For linearly realized group action,

If scalars ϕ_i have charges q_i w.r.t. U(1), then

$$D = \sum_i q_i |\phi_i|^2$$

up to additive shift (by Fayet-Iliopoulos parameter).

How to describe D more generally?

Def'n of D more generally:

$$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$$ inf' gp action on $\mathcal{M}$

where $$X^{(a)} = X^{(a)i} \frac{\partial}{\partial \phi^i}$$ ``holomorphic Killing vector''

`Killing' implies
$$\nabla_i X_j^{(a)} + \nabla_j X_i^{(a)} = 0$$
$$\nabla_{\bar\imath} X_j^{(a)} + \nabla_j X_{\bar\imath}^{(a)} = 0$$

which implies $$g_{i\bar\jmath} X^{(a)\bar\jmath} = i \frac{\partial}{\partial \phi^i} D^{(a)}$$

$$g_{i\bar\jmath} X^{(a)i} = -i \frac{\partial}{\partial \phi^{\bar\jmath}} D^{(a)}$$

for some $D^{(a)}$ -- defines $D^{(a)}$ up to additive shift (FI)

Closer examination of the supergravity:

$$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$$ inf' gp action on $\mathcal{M}$

$$\delta A_\mu^{(a)} = \partial_\mu \epsilon^{(a)} + f^{abc}\epsilon^{(b)} A_\mu^{(c)}$$

$$\delta K = \epsilon^{(a)} F^{(a)} + \epsilon^{(a)} \overline{F}^{(a)}$$

where $F^{(a)} = X^{(a)} K + i D^{(a)}$

Recall $K \mapsto K + f + \overline{f}$ implies

$$\chi^i \mapsto \exp\left(+\frac{i}{2}\mathrm{Im}\, f\right)\chi^i, \;\; \psi_\mu \mapsto \exp\left(-\frac{i}{2}\mathrm{Im}\, f\right)\psi_\mu$$

Hence * gp action on χ^i, ψ_μ includes $\mathrm{Im}\, F^{(a)}$ terms

* This will be gp action on $\mathcal{L}$

Indeed:

$$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$$ inf' gp action on $\mathcal{M}$

$$\delta A_\mu^{(a)} = \partial_\mu \epsilon^{(a)} + f^{abc}\epsilon^{(b)} A_\mu^{(c)}$$

$$\delta K = \epsilon^{(a)} F^{(a)} + \epsilon^{(a)} \overline{F}^{(a)}$$

where $F^{(a)} = X^{(a)} K + i D^{(a)}$

$$\delta\lambda^{(a)} = f^{abc}\epsilon^{(b)}\lambda^{(c)} - \underline{\frac{i}{2}\epsilon^{(a)} \mathrm{Im}\, F^{(a)} \lambda^{(a)}}$$

$$\delta\chi^i = \epsilon^{(a)} \left(\frac{\partial X^{(a)i}}{\partial \phi^j}\chi^j + \underline{\frac{i}{2} \mathrm{Im}\, F^{(a)} \chi^i} \right)$$

$$\delta\psi_\mu = \underline{-\frac{i}{2}\epsilon^{(a)} \mathrm{Im}\, F^{(a)} \psi_\mu}$$

Encode infinitesimal action on $\mathcal{L}$

We need the group to be represented faithfully.

Infinitesimally, the D's can be chosen to obey

$$\left(X^{(a)i}\partial_i + X^{(a)\overline{\imath}}\overline{\partial}_{\overline{\imath}}\right) D^{(b)} = -f^{abc}D^{(c)}$$

and then

$$\delta^{(b)}\epsilon^{(a)}\mathrm{Im}\,F^{(a)} - \delta^{(a)}\epsilon^{(b)}\mathrm{Im}\,F^{(b)} = -\epsilon^{(a)}\epsilon^{(b)}f^{abc}\mathrm{Im}\,F^{(c)}$$

If the group is semisimple,
the constraints above will fix D.
If there are U(1) factors, must work harder...

Next: constraints from representing **group** ....

An infinitesimal action is not enough.

Need an action of the **group** on $\mathcal{L}$,
not just its Lie algebra.

Lift of $g = \exp\left(i\epsilon^{(a)}T^a\right)$

is $\tilde{g} = \exp\left(\frac{i}{2}\epsilon^{(a)}\mathrm{Im}\,F^{(a)}\right)$

Require $\tilde{g}\tilde{h} = \widetilde{gh}$

so that the group is honestly represented.

(This is the part that can't always be done.)

The lifts $\tilde{g}$ might not obey $\tilde{g}\tilde{h} = \widetilde{gh}$ initially, but we can try to adjust them:

Since $F^{(a)} = X^{(a)}K + iD^{(a)}$,

shifting the D-term $D^{(a)}$

is equivalent to adding a phase to $\tilde{g}$:

$$\tilde{g} \equiv \exp\left(\frac{i}{2}\epsilon^{(a)}\mathrm{Im}\,F^{(a)}\right) \mapsto \tilde{g}\exp\left(i\theta_g\right)$$

for some θ_g encoding the shift in $D^{(a)}$.

If the lifts $\tilde{g}$ do not obey $\tilde{g}\tilde{h} = \widetilde{gh}$,

then we can shift $D^{(a)}$ to add phases:

$$\tilde{g} \mapsto \tilde{g} \exp\left(i\theta_g\right)$$

That *might* fix the problem, maybe.

Globally, the group $\tilde{G}$ formed by the $\tilde{g}$ is an extension

$$1 \longrightarrow U(1) \longrightarrow \tilde{G} \longrightarrow G \longrightarrow 1$$

If that extension splits, we can fix the problem;
if not, we're stuck -- cannot gauge G, not even classically.

(new consistency condition on classical sugrav)

Let's assume the extension splits,
so we can fix the problem and gauge G (classically).

In this case, there are multiple $\{\tilde{g}\}$'s, differing by phases.

Those different possibilities correspond to the different possible FI parameters
-- remember, the phases originate as shifts of $D^{(a)}$.

Let's count them.
We'll see they're quantized.

Count set of possible lifts $\{\tilde{g}\}$:

Start with one set of consistent lifts $\tilde{g}$, meaning they obey $\tilde{g}\tilde{h} = \widetilde{gh}$

Shift the D-terms: $\tilde{g} \mapsto \tilde{g}' \equiv \tilde{g}\exp(i\theta_g)$

Demand $\tilde{g}'\tilde{h}' = \widetilde{gh}'$

Implies $\theta_g + \theta_h = \theta_{gh}$

Result: Set of lifts is $\mathrm{Hom}(G, U(1))$

(= set of FI parameters)

So far: set of possible lifts is $\mathrm{Hom}(G, U(1))$

* this is a standard math result
for lifts of group actions to line bundles.
(though the sugrav realization is novel)

* Lifts = FI parameters,
so we see that FI parameters quantized.

Ex: $G = U(1)$ $\mathrm{Hom}(G, U(1)) = \mathbf{Z}$
-- integrally many lifts / FI parameters

Ex: G semisimple $\mathrm{Hom}(G, U(1)) = 0$
-- only one lift / FI parameter

D-terms:

Although the $D^{(a)}$ were only defined up to const' shift:

$$g_{i\bar{\jmath}} X^{(a)\bar{\jmath}} = i \frac{\partial}{\partial \phi^i} D^{(a)}$$

the constraint $\tilde{g}\tilde{h} = \widetilde{gh}$

determines their values up to a (quantized) shift by elements of Hom(G, U(1))

Supersymmetry breaking:

Is sometimes forced upon us.

If the FI parameters could be varied continuously, then we could always solve D=0 just by suitable choices.

Since the FI parameters are quantized, sometimes cannot solve D=0 for any available FI parameter.

Supersymmetry breaking:

Example: $\mathcal{M} = \mathbf{P}^1$ $G = SU(2)$

(Bagger, 1983)

$$\mathrm{Hom}(SU(2), U(1)) = 0$$

so equivariant lift unique

For Bagger-Witten $\mathcal{L} = \mathcal{O}(-n)$

$$(D^{(1)})^2 + (D^{(2)})^2 + (D^{(3)})^2 = \left(\frac{n}{2\pi} \right)^2$$

[Use $D^a = \phi T^a \phi$ on $\mathbf{P}^1$, plus fact that D's obey Lie algebra rel'ns to fix the value above.]

susy always broken

Math interpretation:

* In rigid susy, gauging ~ symplectic reduction

* Symplectic quotients do **not** have a restriction to integral Kahler classes;

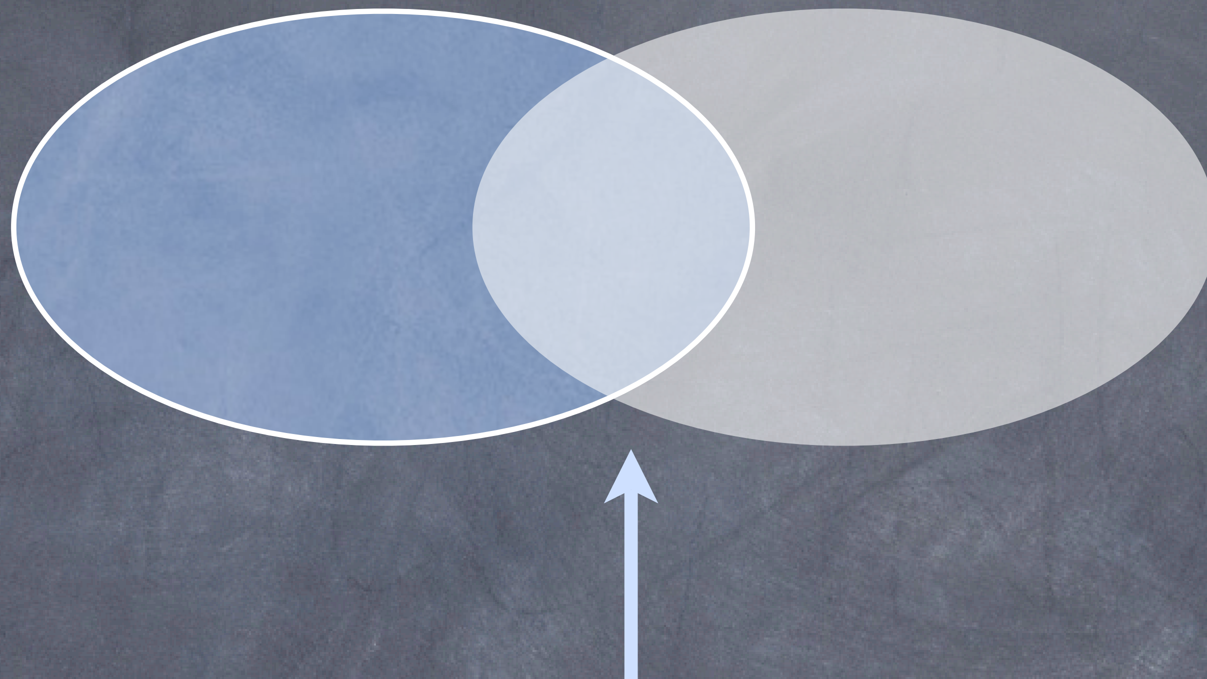
this can**not** be a symplectic quotient.

* Instead, propose: **GIT quotients**.

* Symplectic/GIT sometimes used interchangeably; however, GIT quotients restrict to integral classes.

Symplectic quotients

GIT quotients



complex Kahler manifolds,

integral Kahler forms

Why should GIT be relevant ?

* 1st, to specify GIT,
need to give an ample line bundle on original space,
that determines a projective embedding.
(= Bagger-Witten line bundle)

* 2nd, must specify a group action on that line bundle;
Kahler class ultimately determined by that group action.

Same structure as here: thus, sugrav = GIT

So far: quantization of FI parameters
when moduli space is an ordinary manifold

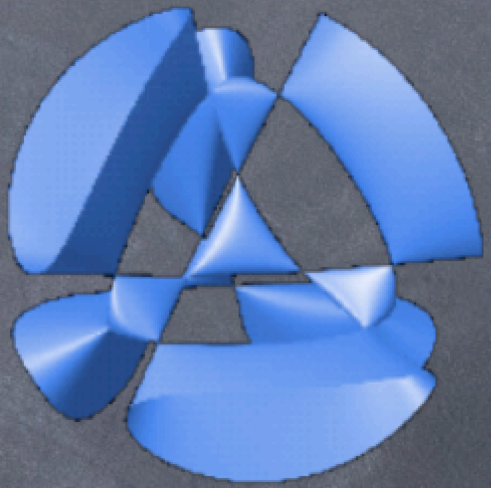
As a practical matter, this is never the case:
singularities, stack structures.

Next: outline what happens when
the moduli `space' is a smooth **stack**.

This will bear on NLSM's with restrictions on
nonperturbative sectors,
and on discrete symmetries in sugrav, strings ....

NLSM on a stack

A stack is a generalization of a space.



Idea: defined by incoming maps.

(and so nicely suited for NLSM's;
just have path integral sum over what
the def'n gives you)

Most moduli `spaces' are really stacks;
thus, to understand sugrav, need to understand stacks
as targets of 4d NLSM's.

Example: A space X as a stack

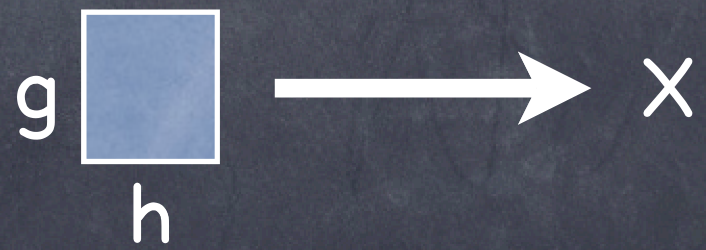
For every other space Y, associate to Y the set of continuous maps Y ---> X

Example: A quotient stack [X/G]

Maps Y ---> [X/G] are pairs

(principal G bundle (w/ connection) E on Y, G-equivariant map E --> X)

If Y = T^2 & G finite,



= twisted sector maps in string orbifold

All smooth `Deligne-Mumford' stacks (over **C**)
can be described as [X/G]
for some X, some G

(G not nec' finite, not nec' effectively-acting
-- these are not all orbifolds)

Program:

A NLSM on a stack
is a G-gauged sigma model on X

Problem: such presentations not unique

Potential fix: RG flow

Does RG flow wash out presentation-dependence, giving physics that only depends on the stack, and not on the choice of X, G?

Two dimensions: Yes

Numerous checks & work by myself, T Pantev, J Distler, S Hellerman, A Caldararu, and others in physics; extensive math literature on Gromov-Witten

Four dimensions: No

-- the stack does not determine gauge coupling

-- in low energy effective field theory, W bosons generate effects that can swamp NLSM interp'

Can associate stack to physics, but not physics to stack.

Let's consider a particularly interesting kind of stack.

Consider NLSM's in which the sum over nonperturbative sectors has been restricted;
only sum over maps of degree obeying divisibility prop'.

Since stacks describe, in essence, all possible NLSM's, naturally this is a kind of stack.

Specifically, this sort of stack is known as a gerbe.

(Also describes extra discrete symmetry present everywhere on space.)

(Seiberg, Banks-Seiberg '10; ES, Distler, Pantev, Hellerman, GW, ...)

Example: Gerby susy $\mathbf{CP}^N$ model in two dim's

* 2d U(1) susy gauge theory

* N+1 chiral superfields, charge k
-- nonminimal charges

(Global unbroken $\mathbf{Z}_k$)

How can this differ from ordinary susy $\mathbf{CP}^N$ model?

Answer: nonperturbative effects

The difference lies in nonperturbative effects. (Perturbatively, having nonminimal charges makes no difference.)

2d: Argument for compact worldsheet:

To specify Higgs fields completely, need to specify what bundle they couple to.

If the gauge field $\sim L$
then Φ charge Q implies
$\Phi \in \Gamma(L^{\otimes Q})$

Different bundles => different zero modes
=> different anomalies => different physics

Argument for noncompact worldsheet:

Utilize the fact that in 2d,
theta angle acts as electric field.

Want Higgs fields to have charge k
at same time that instanton number is integral.

Latter is correlated to periodicity of theta angle;
can fix to desired value by adding massive charge 1,
-1 fields -- for large enough sep', can excite, and that
sets periodicity.

(J Distler, R Plesser, Aspen 2004 & hepth/0502027, 0502044, 0502053;
N Seiberg, Banks-Seiberg, 2010)

So far, only discussed 2d case.

There is a closely analogous argument in analogous four-dimensional models coupled to gravity.

Instead of theta angle,
use Reissner-Nordstrom black holes.

Idea: if all states in the theory have charge a multiple of k, then, gerbe theory is same as ordinary one.

However, if have massive minimally-charged fields, then a RN BH can Hawking radiate down to charge 1, and so can sense fields with mass > cutoff.

(J Distler, private communication)

Return to the 2d gerby $\mathbf{CP}^N$ example:

Example: Anomalous global U(1)'s

$$\mathbf{P}^{N-1}: \quad U(1)_A \mapsto \mathbf{Z}_{2N}$$

$$\text{Here}: \quad U(1)_A \mapsto \mathbf{Z}_{2kN}$$

Example: A model correlation functions

$$\mathbf{P}^{N-1}: \quad < X^{N(d+1)-1} > = q^d$$

$$\text{Here}: \quad < X^{N(kd+1)-1} > = q^d$$

Example: quantum cohomology

$$\mathbf{P}^{N-1}: \quad \mathbf{C}[x]/(x^N - q)$$

$$\text{Here}: \quad \mathbf{C}[x]/(x^{kN} - q)$$

Different physics

More generally, for 2d gerbe theories, there are somewhat extensive results.

-- quantum cohomology rings

-- mirror symmetry, inc. Toda duals

-- decomposition conjecture for (2,2) susy theories....

(will describe next)

Decomposition conjecture

(Hellerman, Henriques, Pantev, Sharpe, etc)

In the special case of `banded' gerbes, the decomposition conjecture says

$$\mathrm{CFT}(G - \mathrm{gerbe\ on\ } X) \;=\; \mathrm{CFT}\left(\coprod_{\hat{G}} (X, B) \right)$$

where the B field is determined by the image of

$$H^2(X, Z(G)) \stackrel{Z(G) \to U(1)}{\longrightarrow} H^2(X, U(1))$$

More gen'ly, disjoint union of **different** spaces.

Example:

Consider $[X/D_4]$ where the center acts trivially.

$$1 \longrightarrow \mathbf{Z}_2 \longrightarrow D_4 \longrightarrow \mathbf{Z}_2 \times \mathbf{Z}_2 \longrightarrow 1$$

Can show this is physically distinct from $[X/\mathbf{Z}_2 \times \mathbf{Z}_2]$;
for example,

$$Z([X/D_4]) = Z\left([X/\mathbf{Z}_2 \times \mathbf{Z}_2] \coprod [X/\mathbf{Z}_2 \times \mathbf{Z}_2]_{\mathrm{d.t.}} \right)$$

Example:

Consider $[X/H]$ where <i> acts trivially:

$$1 \longrightarrow \langle i \rangle (= \mathbf{Z}_4) \longrightarrow H \longrightarrow \mathbf{Z}_2 \longrightarrow 1$$

Can show this is physically distinct from [X/$\mathbf{Z}_2$]; for example,

$$Z([X/H]) = Z\left([X/\mathbf{Z}_2] \coprod [X/\mathbf{Z}_2] \coprod X \right)$$

Suffice it to say,
there's been considerable work done on the 2d case.

Pertinent here: 4d case.

Specifically,
4d NLSM on sugrav moduli `space'.

Far less work done;
I'll outline some results and issues.

Four dimensions

Example:

Consider a U(1) susy gauge theory on in 4d, with N (massless) chiral superfields of charge k, N of charge -k.

To be different physically from charge 1 case, need either:

-- topologically nontrivial 4d spacetime (so that there are U(1) instantons)

-- massive fields of charge +1, -1

(parallels 2d case)

Gerby moduli spaces in string theory:

Consider toroidally-compactified Spin(32)/$\mathbf{Z}_2$ heterotic string.

Low-energy theory has only adjoints,
hence all invariant under $\mathbf{Z}_2$ center of Spin(32)/$\mathbf{Z}_2$

But, there are massive states that do see the center.

Math'ly, equivalent observation is that the moduli space of flat Spin(32)/$\mathbf{Z}_2$ connections has $\mathbf{Z}_2$ gerbe structure.

One can get enhanced gerbe structures along various strata.

Ex: toroidally-compactified $E_8 \times E_8$ heterotic string

-- no center, so no gerbe structure globally

-- but, over stratum where $E_8 \times E_8$ broken to $Spin(16)/\mathbf{Z}_2 \times Spin(16)/\mathbf{Z}_2$,
there is a $\mathbf{Z}_2 \times \mathbf{Z}_2$ gerbe structure,
matching the corresponding $Spin(32)/\mathbf{Z}_2$ compactification

Examples in Seiberg duality:

Several years ago, Matt Strassler was very interested in Spin/SO Seiberg duals.

hepth/9507018, 9510228, 9709081, 9808073

Prototypical example:

* Spin(8) gauge theory with N_f fields in $\mathbf{8}_V$,

and one massive $\mathbf{8}_S$

Seiberg dual to

* SO(N_f - 4) gauge theory with N_f vectors (from Higgsing SU(N_f-4) theory)

massive $\mathbf{8}_S$ <--> $\mathbf{Z}_2$ monopole

$$\pi_2(SU(N_f-4)/SO(N_f-4)) = \mathbf{Z}_2$$

* Spin(8) gauge theory with N_f fields in $\mathbf{8}_V$,

and one massive $\mathbf{8}_S$

Seiberg dual to

* SO(N_f - 4) gauge theory with N_f vectors
(from Higgsing SU(N_f-4) theory)

massive $\mathbf{8}_S$ <--> $\mathbf{Z}_2$ monopole

Important for his analysis that a $\mathbf{Z}_2$ center of Spin(8) acted trivially on massless matter, but nontrivially on the massive $\mathbf{8}_S$

-- so $\mathbf{Z}_2$ gerbe structure on moduli space on one side

Apply to quantization of FI parameters:

For a simple example,
consider the (anomalous) 4d gerby $\mathbf{CP}^N$ model:

* U(1) gauge theory
* N+1 chiral superfields charge k

now in supergravity

(The anomaly is irrelevant;
more complicated anomaly-free exs exist.)

* U(1) gauge theory

* N+1 chiral superfields charge k

D-terms: $$\sum_i k|\phi_i|^2 = r$$

But r is an integer, so this is same as

$$\sum_i |\phi_i|^2 = r/k$$

-- looks like ordinary **CP**^{N} model,
but now with fractional FI term.

Interpretation ??

We've argued that FI integrality in sugrav follows b/c FI term is a choice of equivariant structure on the Bagger-Witten line bdle.

Over a gerbe, there are `fractional' line bundles.

Ex: gerbe on **CP**^{N}

$$[x_0, \cdots, x_N] \cong [\lambda^k x_0, \cdots, \lambda^k x_N]$$

Can define a line bundle L by $y \mapsto \lambda^n y$

Call it $\mathcal{O}(n/k)$

Let's redo Bagger-Witten,
when the sugrav moduli space is a gerbe.

For same reasons as ordinary case,

$$\chi^i \in \Gamma\left(\phi^*\left(T\mathcal{M} \otimes \mathcal{L}\right)\right), \quad \psi_\mu \in \Gamma\left(TX \otimes \phi^* \mathcal{L}^{-1}\right)$$

Now, however, $\mathcal{M}$ is a gerbe.

Gerbes have more (ie fractional) bundles
than their underlying spaces,
so $\mathcal{L}$ can be fractional.

-- that's what's happening in the previous r/k ex.

Potential issue:

* Fractional line bundles have no smooth sections,
only multisections w/ branch cuts.

However, all maps into gerbes
= maps into spaces w/ divisibility constraint,
which turns out to
ensure pullback bundles are honest bundles.

so even if $\mathcal{L}$ is fractional,
$\phi^* \mathcal{L}$ is an honest bundle,
and so no branch cuts in χ^i, ψ_μ .

Have we missed any subtleties?

Maybe:

The 2d analogue is heterotic string on a gerbe (gauge group acts trivially on space, nontrivially on bundle).

These **sometimes** break modular invariance; details not well understood.

Speculation: possible that 4d examples suffer from (discrete) anomalies.

Is there a loophole in Bagger-Witten?

Naively, that's what's suggested by the gerby $\mathbf{P}^N$ model.

However, it can be argued that's not what's going on here.

Briefly,

-- there **is** a fractional quantization condition

-- but it does **not** apply to Bagger-Witten's theories; these are, effectively, different 4d theories.

Conclusion: no loophole

Summary:

* reviewed Bagger-Witten

* quantization of FI parameters in sugrav when moduli space is a space

* review stacks

* exs & prop's of gerby moduli spaces in field and string theory; discrete symmetries....

* Bagger-Witten, FI quantization when moduli space is a gerbe