Quantization of Fayet-Iliopoulos parameters in supergravity

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joint w/ J Distler arXiv: 1008.0419,
and w/ S Hellerman, arXiv: 1012.5999

Also: N Seiberg, 1005.0002; Banks, Seiberg 1011.5120

As this is perhaps a wide audience,
let me begin with a short overview.

`supergravity' = supersymmetric general relativity

graviton $\longleftrightarrow$ gravitino

In typical, `easy,' scenarios arising in string theory,
the 4d theory is, at high energies,
a supergravity theory.

Hence, understanding supergravity (abbr. **sugrav**)
is important for string theorists.

In supersymmetric gauge theories (w/ or w/o gravity), there is a parameter appearing in bosonic potentials, known as the **Fayet-Iliopoulos parameter**.

Example:

U(1) gauge theory, complex scalars ϕ_i of charge Q_i.

There is a ~ universal contribution to the bosonic potential, of the form

$$\left(\sum_i Q_i |\phi_i|^2 - r \right)^2$$

Fayet-Iliopoulos parameter

In supersymmetric theories not coupled to gravity, Fayet-Iliopoulos (FI) parameters are well-understood.

In supergravity theories, on the other hand, there's been debate in the literature regarding whether Fayet-Iliopoulos parameters even exist.

Today, I'll present a resolution of these issues.

Brief outline of literature on FI params in sugrav:

(Dienes, Thomas, 0911.0677)

* Any gauge group must be combined w/ U(1) symmetry that acts only on gaugino, gravitino (the ``R symmetry”).

Implies FI parameter contributes to the charges of the gravitino, etc

(Freedman ‘77, Stelle-West ‘78, Barbieri et al ‘82)

which, if parameter varies continuously, violates electric charge quantization.

(Witten, ``New issues...”, ‘86, footnote p 85)

* Solution: quantize the FI parameter.

(Seiberg; Distler-Sharpe; ‘10)

I’ll outline the general analysis.

The starting point for this discussion is another quantization condition on N=1 sugrav in 4d, worked out by Bagger-Witten in the early '80s.

N=1 sugrav in 4d contains a
(low-energy effective) 4d NLSM on a space $\mathcal{M}$,
namely the supergravity moduli space.

~ space of scalar field vevs

They derived a constraint on the metric on that moduli space $\mathcal{M}$
(assuming the moduli space is a smooth manifold).

Review of Bagger-Witten:

Briefly, the supergravity moduli space $\mathcal{M}$
(the target space of a 4d NLSM)
comes with a natural line bundle $\mathcal{L}^{\otimes 2}$,
whose c_1 = Kahler form (hence quantized)
$= g_{i\bar{\jmath}} dz^i \wedge d\overline{z}^{\bar{\jmath}}$

How to see this?

Start with the fact that
the moduli space $\mathcal{M}$ is constrained to be Kahler,
which means $g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K$ for some function K,
called the Kahler potential.

Bagger-Witten, cont'd

Across coordinate patches,

$$K \mapsto K + f + \overline{f}$$

In a supersymmetric theory not coupled to gravity, this is a symmetry of the action.

In N=1 sugrav, however, action only invariant if combine above with an action on fermions:

$$\chi^i \mapsto \exp\left(+\frac{i}{2}\mathrm{Im}\, f\right)\chi^i, \;\; \psi_\mu \mapsto \exp\left(-\frac{i}{2}\mathrm{Im}\, f\right)\psi_\mu$$

which implies existence of the B-W line bundle $\mathcal{L}$.

$$\chi^i \in \Gamma\left(\phi^*\left(T\mathcal{M} \otimes \mathcal{L}\right)\right), \quad \psi_\mu \in \Gamma\left(TX \otimes \phi^*\mathcal{L}^{-1}\right)$$

Quick & dirty argument for FI quantization:

Continuously varying the FI term, continuously varies the symplectic form on the quotient space.

But that symplectic form = Kahler form, & Bagger-Witten says is quantized.

Consistency requires FI term be quantized too.

Problem:

-- IR limit not same as NLSM, so irrelevant to B-W

Nice intuition, but need to work harder.

To gain a more complete understanding,
let's consider gauging the Bagger-Witten story.

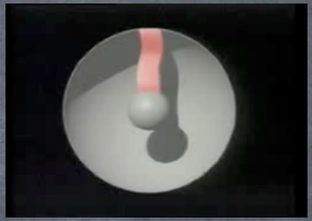
Have:

* sugrav moduli space $\mathcal{M}$
* line bundle $\mathcal{L}$
* group action on moduli space $\mathcal{M}$

Need to specify how group acts on $\mathcal{L}$ .....

In principle, if we now wish to gauge a group action on the supergravity moduli space $\mathcal{M}$, then we need to specify the group action on $\mathcal{L}$.

* **not** always possible:
group actions on spaces do not always lift to bundles
Ex: spinors under rotations;
rotate 4π instead of 2π.



-- classical constraint on sugrav theories....

* **not** unique:
when they do lift, there are multiple lifts
(These will be the FI parameters.)

We'll see FI as a choice of group action on the Bagger-Witten line bundle directly in sugrav.

First: what is D?

For linearly realized group action,

If scalars ϕ_i have charges q_i w.r.t. U(1),
then

$$D = \sum_i q_i |\phi_i|^2$$

up to additive shift (by Fayet-Iliopoulos parameter).

How to describe D more generally?

Def'n of D more generally:

$$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$$ inf' gp action on $\mathcal{M}$

where $$X^{(a)} = X^{(a)i} \frac{\partial}{\partial \phi^i}$$ "holomorphic Killing vector"

'Killing' implies
$$\nabla_i X_j^{(a)} + \nabla_j X_i^{(a)} = 0$$
$$\nabla_{\bar\imath} X_j^{(a)} + \nabla_j X_{\bar\imath}^{(a)} = 0$$

which implies $$g_{i\bar\jmath} X^{(a)\bar\jmath} = i \frac{\partial}{\partial \phi^i} D^{(a)}$$

$$g_{i\bar\jmath} X^{(a)i} = -i \frac{\partial}{\partial \phi^{\bar\jmath}} D^{(a)}$$

for some $D^{(a)}$ -- defines $D^{(a)}$ up to additive shift (FI)

Closer examination of the supergravity:

$$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$$ inf' gp action on $\mathcal{M}$

$$\delta A_\mu^{(a)} = \partial_\mu \epsilon^{(a)} + f^{abc}\epsilon^{(b)} A_\mu^{(c)}$$

$$\delta K = \epsilon^{(a)} F^{(a)} + \epsilon^{(a)} \overline{F}^{(a)}$$

where $F^{(a)} = X^{(a)} K + i D^{(a)}$

Recall $K \mapsto K + f + \overline{f}$ implies

$$\chi^i \mapsto \exp\left(+\frac{i}{2}\mathrm{Im}\, f\right)\chi^i, \;\; \psi_\mu \mapsto \exp\left(-\frac{i}{2}\mathrm{Im}\, f\right)\psi_\mu$$

Hence * gp action on χ^i, ψ_μ includes $\mathrm{Im}\, F^{(a)}$ terms

* This will be gp action on $\mathcal{L}$

Indeed:

$$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$$ inf' gp action on $\mathcal{M}$

$$\delta A_\mu^{(a)} = \partial_\mu \epsilon^{(a)} + f^{abc}\epsilon^{(b)} A_\mu^{(c)}$$

$$\delta K = \epsilon^{(a)} F^{(a)} + \epsilon^{(a)} \overline{F}^{(a)}$$

where $F^{(a)} = X^{(a)} K + i D^{(a)}$

$$\delta\lambda^{(a)} = f^{abc}\epsilon^{(b)}\lambda^{(c)} - \underline{\frac{i}{2}\epsilon^{(a)} \mathrm{Im}\, F^{(a)} \lambda^{(a)}}$$

$$\delta\chi^i = \epsilon^{(a)} \left(\frac{\partial X^{(a)i}}{\partial \phi^j}\chi^j + \underline{\frac{i}{2} \mathrm{Im}\, F^{(a)} \chi^i} \right)$$

$$\delta\psi_\mu = \underline{-\frac{i}{2}\epsilon^{(a)} \mathrm{Im}\, F^{(a)} \psi_\mu}$$

Encode infinitesimal action on $\mathcal{L}$

We need the group to be represented faithfully.

Infinitesimally, the D's can be chosen to obey

$$\left(X^{(a)i}\partial_i + X^{(a)\bar{\imath}}\bar{\partial}_{\bar{\imath}}\right) D^{(b)} = -f^{abc} D^{(c)}$$

and then

$$\delta^{(b)}\epsilon^{(a)}\mathrm{Im}\, F^{(a)} - \delta^{(a)}\epsilon^{(b)}\mathrm{Im}\, F^{(b)} = -\epsilon^{(a)}\epsilon^{(b)} f^{abc}\mathrm{Im}\, F^{(c)}$$

If the group is semisimple,
the constraints above will fix D.
If there are U(1) factors, must work harder...

Next: constraints from representing **group** ....

An infinitesimal action is not enough.

Need an action of the **group** on $\mathcal{L}$,
not just its Lie algebra.

Lift of $g = \exp\left(i\epsilon^{(a)}T^a\right)$

is $\tilde{g} = \exp\left(\frac{i}{2}\epsilon^{(a)}\mathrm{Im}\,F^{(a)}\right)$

Require $\tilde{g}\tilde{h} = \widetilde{gh}$

so that the group is honestly represented.

(This is the part that can't always be done.)

The lifts $\tilde{g}$ might not obey $\tilde{g}\tilde{h} = \widetilde{gh}$ initially, but we can try to adjust them:

Since $F^{(a)} = X^{(a)}K + iD^{(a)}$,

shifting the D-term $D^{(a)}$

is equivalent to adding a phase to $\tilde{g}$:

$$\tilde{g} \equiv \exp\left(\frac{i}{2}\epsilon^{(a)}\mathrm{Im}\,F^{(a)}\right) \mapsto \tilde{g}\exp\left(i\theta_g\right)$$

for some θ_g encoding the shift in $D^{(a)}$.

If the lifts $\tilde{g}$ do not obey $\tilde{g}\tilde{h} = \widetilde{gh}$,
then we can shift $D^{(a)}$ to add phases:

$$\tilde{g} \mapsto \tilde{g} \exp\left(i\theta_g\right)$$

That *might* fix the problem, maybe.

Globally, the group $\tilde{G}$ formed by the $\tilde{g}$ is an extension

$$1 \longrightarrow U(1) \longrightarrow \tilde{G} \longrightarrow G \longrightarrow 1$$

If that extension splits, we can fix the problem;
if not, we're stuck -- cannot gauge G, not even classically.

(new consistency condition on classical sugrav)

Let's assume the extension splits,
so we can fix the problem and gauge G (classically).

In this case, there are multiple $\{\tilde{g}\}$'s, differing by phases.

Those different possibilities correspond to the different possible FI parameters
-- remember, the phases originate as shifts of $D^{(a)}$.

Let's count them.
We'll see they're quantized.

Count set of possible lifts $\{\tilde{g}\}$:

Start with one set of consistent lifts $\tilde{g}$, meaning they obey $\tilde{g}\tilde{h} = \widetilde{gh}$

Shift the D-terms: $\tilde{g} \mapsto \tilde{g}' \equiv \tilde{g}\exp(i\theta_g)$

Demand $\tilde{g}'\tilde{h}' = \widetilde{gh}'$

Implies $\theta_g + \theta_h = \theta_{gh}$

Result: Set of lifts is $\mathrm{Hom}(G, U(1))$

(= set of FI parameters)

So far: set of possible lifts is $\mathrm{Hom}(G, U(1))$

* this is a standard math result
for lifts of group actions to line bundles.
(though the sugrav realization is novel)

* Lifts = FI parameters,
so we see that FI parameters quantized.

Ex: $G = U(1) \quad \mathrm{Hom}(G, U(1)) = \mathbf{Z}$
-- integrally many lifts / FI parameters

Ex: G semisimple $\quad \mathrm{Hom}(G, U(1)) = 0$
-- only one lift / FI parameter

D-terms:

Although the $D^{(a)}$ were only defined up to const' shift:

$$g_{i\bar{\jmath}} X^{(a)\bar{\jmath}} = i \frac{\partial}{\partial \phi^i} D^{(a)}$$

the constraint $\tilde{g}\tilde{h} = \widetilde{gh}$

determines their values up to a (quantized) shift by elements of Hom(G, U(1))

Supersymmetry breaking:

Is sometimes forced upon us.

If the FI parameters could be varied continuously, then we could always solve $D=0$ just by suitable choices.

Since the FI parameters are quantized, sometimes cannot solve $D=0$ for any available FI parameter.

Supersymmetry breaking:

Example: $\mathcal{M} = \mathbf{P}^1$ $\quad G = SU(2)$

(Bagger, 1983)

$$\mathrm{Hom}(SU(2), U(1)) = 0$$

so equivariant lift unique

For Bagger-Witten $\mathcal{L} = \mathcal{O}(-n)$

$$(D^{(1)})^2 + (D^{(2)})^2 + (D^{(3)})^2 = \left(\frac{n}{2\pi} \right)^2$$

[Use $D^a = \phi T^a \phi$ on $\mathbf{P}^1$, plus fact that D's obey Lie algebra rel'ns to fix the value above.]

susy always broken

Math interpretation:

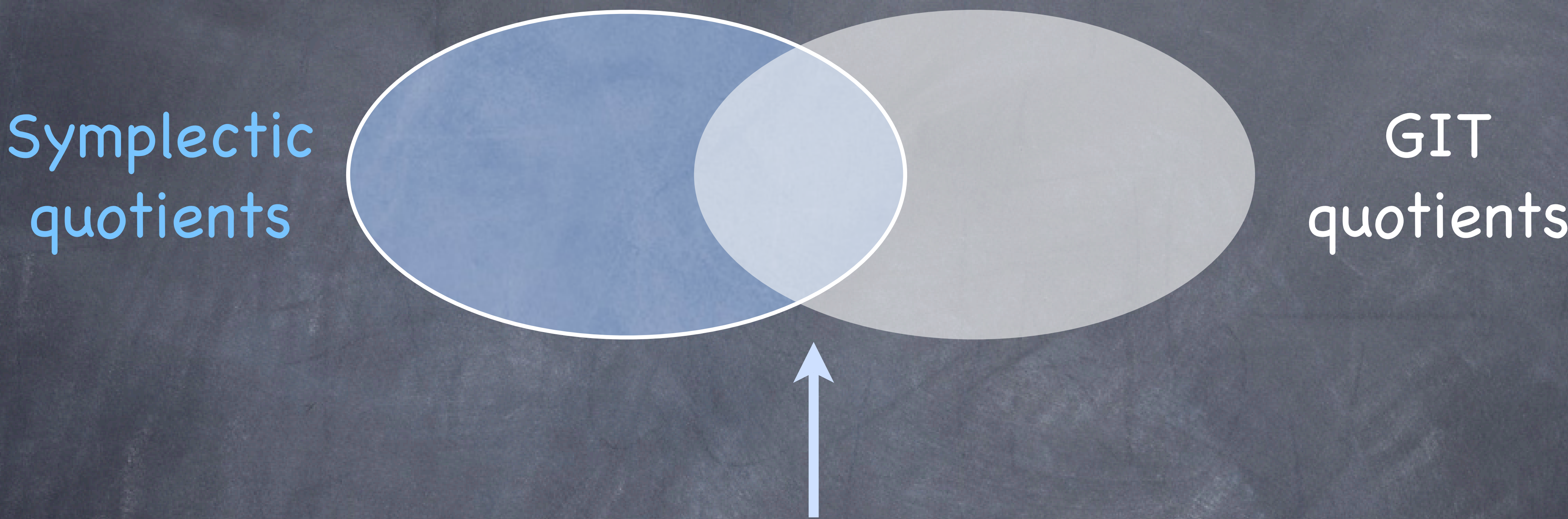
* In rigid susy, gauging ~ symplectic reduction

* Symplectic quotients do **not** have a restriction to integral Kahler classes;

this can**not** be a symplectic quotient.

* Instead, propose: **GIT quotients**.

* Symplectic/GIT sometimes used interchangeably; however, GIT quotients restrict to integral classes.



complex Kahler manifolds,

integral Kahler forms

Why should GIT be relevant ?

* 1st, to specify GIT,
need to give an ample line bundle on original space,
that determines a projective embedding.
(= Bagger-Witten line bundle)

* 2nd, must specify a group action on that line bundle;
Kahler class ultimately determined by that group action.

Same structure as here: thus, sugrav = GIT

Summary:

* reviewed Bagger-Witten

* quantization of FI parameters in sugrav

Thank you for your time!