1. An example of convolution

Consider a function

$$\begin{aligned} f(x) &= \cos x + \cos 3x \\ &= \frac{1}{2}(\exp(ix) + \exp(-ix)) + \frac{1}{2}(\exp(3ix + \exp(-3ix)) \end{aligned}$$

Imagine passing this function through a low-pass filter, that multiplies the coefficients of 1, $\exp(\pm ix)$ by 1, and multiplies all other Fourier components by zero.

If we write

$$f(x) = \sum_{n=-\infty}^{\infty} (c_f)_n \exp(inx)$$

then the action of the low-pass filter can be described as

$$\sum_{n=-\infty}^{\infty} (c_f)_n (c_g)_n \exp(inx)$$

for

$$(c_g)_n = \begin{cases} 1 & n = 0, \pm 1 \\ 0 & n \neq 0, \pm 1 \end{cases}$$

a) Show that

$$\sum_{n=-\infty}^{\infty} (c_g)_n \exp(inx) = 1 + 2\cos x$$

b) For $g(x) = 1 + 2\cos x$, compute

$$(f * g)(x) \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} f(s) g(x-s) ds$$

c) Show that the convolution product above matches

$$\sum_{n=-\infty}^{\infty} (c_f)_n (c_g)_n \exp(inx)$$

(See next page for another problem)

2. The values of a 2π-periodic function are sampled at four times: 0, $\pi/2$, π, $3\pi/2$, where the function has values 0, 1, 0, 1, respectively.

Compute the discrete-time Fourier series of this function.