In class we showed that if $f(z)$ is a holomorphic function with simple isolated zeroes located at a_1, a_2, $\cdots$, none of the a_i zero, then $f(z)$ could be written in terms of $f(0)$, $f'(0)$, and the a_i as

$$f(z) \;=\; f(0) \exp\left(z \frac{f'(0)}{f(0)} \right) \prod_n \left(1 \;-\; \frac{z}{a_n} \right) \exp(z/a_n)$$

1. Suppose $f(z)$ is an Nth order polynomial:

   $$f(z) \;=\; \alpha(x - x_1)(x - x_2)\cdots(x - x_N)$$

   for α some constant and none of the x_i vanishing. Show that the expression above correctly captures the polynomial nature of $f(z)$. In other words, plug in $f(0)$, $f'(0)$, and the locations of the zeroes, and verify that the abstract formula at top reduces to the Nth order polynomial above.

2. The function $1/\Gamma(z + 1)$ can be shown to have simple zeroes at $z = -1, -2, -3, \cdots$. Use the formula above to derive the Weierstrass representation of $1/\Gamma(z)$.