1. Legendre polynomials can be represented by the *Schlaefli integral*:

$$P_n(z) \;=\; \frac{2^{-n}}{2\pi i}\oint \frac{(t^2-1)^n}{(t-z)^{n+1}}dt$$

   By direct evaluation of the Schlaefli integral, show that $P_n(1) = 1$.

2. The quantum mechanical angular momentum operators $L_x \pm iL_y$ are given by

$$L_x \pm iL_y \;=\; \pm e^{i\varphi}\left(\frac{\partial}{\partial\theta} \;\pm\; i\cot\theta\frac{\partial}{\partial\varphi}\right)$$

   Show that

$$(L_x + iL_y)Y_\ell^m(\theta,\varphi) \;=\; \sqrt{(\ell-m)(\ell+m+1)}\,Y_\ell^{m+1}(\theta,\varphi)$$

3. The *Pochhammer symbol* $(a)_n$ is defined as

$$(a)_n \;=\; a(a+1)\cdots(a+n-1), \quad (a)_0 \;=\; 1$$

   Show that

$$(a)_{n+k} \;=\; (a+n)_k(a)_n$$