Entanglement and Spacetime

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+ a conjecture about what the RT area is counting in the bulk: 1704.07763 + work in progress

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Plan

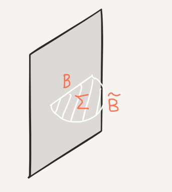
1. (15 min) Review of RT formula and applications.
2. (25 min) Conjecture: RT area is the analog of an edge term in the EE of an emergent gauge theory.
3. ($\leq$ 10 min) "Future directions" – fun facts about the $c = 1$ matrix model.

Entanglement and Spacetime

In recent years, people have suggested that "spacetime emerges from quantum entanglement."

All of the mathematically precise work in this direction comes from AdS/CFT and the Ryu-Takayanagi formula,

$$S_{EE}(B) = \frac{A}{4G_N} + S_{EE,bulk}(\Sigma) + \dots$$

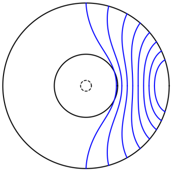


Some nice applications:

1. Linearized Einstein eq's from EE 1st law around the AdS vacuum [van Raamsdonk et al.]

$$
\begin{array}{cccl}
\delta S_{EE} & = & \delta\langle -\log\rho\rangle & \text{True } \forall\rho\text{; perturb } \mathrm{Tr}\rho\log\rho. \\
\updownarrow & & \updownarrow & \text{Specialize to ball regions in CFT's} \\
\delta S_{EE} & = & \int_B F(\langle T_{00}(r)\rangle) & \text{conformal map to Rindler wedge} \\
\updownarrow \text{ RT} & & \updownarrow \text{ GKPW} & \text{map to the bulk} \\
\int_{\tilde{B}} F_0(\delta g_{ab}) & = & \int_B F_1(\delta g_{ab}) & = \text{linearized EFE's.}
\end{array}
$$

Can we generalize to getting the linearized Einstein eq.'s around other asymptotically AdS spacetimes?



"Entanglement shadow" in generic horizonless asymptotically-AdS geometries.

A possible resolution: can more general form of entanglement geometrize in the bulk?

Algebraic EE: For $|\psi\rangle \in \mathcal{H}$ and subalgebra $\mathcal{A}_0 \in \mathcal{A}$, $\exists$

$$\rho(= \sum_{\mathcal{O}_i \in \mathcal{A}_0} p_i \mathcal{O}_i) \in \mathcal{A}_0$$

s.t.

$$\mathrm{Tr}_{\mathcal{H}}(\rho\mathcal{O}) = \langle\psi|\mathcal{O}|\psi\rangle \; \forall \; \mathcal{O} \in \mathcal{A}_0 \,.$$

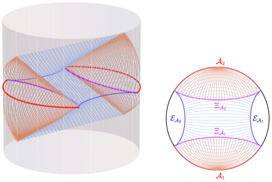
Then

$$S_{EE}(\mathcal{A}_0) = -\mathrm{Tr}\,\rho \log \rho \,.$$

Algebraic EE's can be dual to more general surfaces in the bulk. [Balasubramanian et al., JL].

2. Entanglement wedge reconstruction [Dong, Harlow, Wall ...]

In effective field theory on AdS, consider a local bulk operator at a point in the bulk. How much of the boundary CFT do we need to have access to to reconstruct it?



Any local bulk operator in the entanglement wedge $\mathcal{E}_A$ can be reconstructed as a CFT operator supported on region $\mathcal{A}$!

Moreover, "RT $=$ entanglement wedge reconstruction".
In fact, Harlow has proved a related theorem for all quantum systems...

Harlow's assumptions	AdS/CFT interpretation
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$	CFT (UV) Hilbert space
Subspace $\mathcal{H}_{IR} \subseteq \mathcal{H}$	"code subspace" of EFT on AdS
Subalgebra $\mathcal{A}_{IR}$ whose action on $\mathcal{H}_{IR}$ keeps us in $\mathcal{H}_{IR}$	gauge-inv. bulk operators.

Then, the following were proved to be mathematically equivalent:

1. $\exists$ subalgebra $\mathcal{A}_{IR,A} \in \mathcal{A}_{IR}$ s.t. $\forall\ \lvert\tilde{\psi}\rangle \in \mathcal{H}_{IR}$, $\forall\ \tilde{\mathcal{O}} \in \mathcal{A}_{IR,A}$, $\exists\ \mathcal{O}_A \in \mathcal{H}_A$ s.t. $\mathcal{O}_A\lvert\tilde{\psi}\rangle = \tilde{\mathcal{O}}\lvert\psi\rangle$.	Entanglement wedge reconstruction. ($\mathcal{A}_{IR,A}$ = bulk g-inv. operators supported on $\mathcal{E}_A$.)
2. $\exists$ an operator $\mathcal{L}_A$ in $\mathcal{A}_{IR,A} \cap \mathcal{A}_{IR,\bar{A}}$ s.t. $\forall \rho \in \mathcal{H}_{IR}$, $S_{EE}(\rho_A) =$ $\mathrm{Tr}(\rho\mathcal{L}_A) + S_{alg}(\rho, \mathcal{A}_{IR,A})$.	RT formula + 1/N correction. ($\mathcal{L}_A$ = RT area.)

Summary of the introduction

To summarize so far,

- The main argument for "entanglement = spacetime" is the Ryu-Takayanagi formula in AdS/CFT.
- Nice consequences include:
  - Steps towards understanding the CFT origin of Einstein's equations.
  - An understanding of subregion duality for bulk operator reconstruction.

In this talk, I want to discuss an idea for what the RT area might be counting from the bulk point of view.

Idea

The idea is to compare EE in emergent gauge theory to EE in AdS/CFT.

	Emergent gauge theory	**AdS/CFT**
UV	Factorizable Hilbert space	CFT
IR	Gauge theory	Effective field theory in AdS
EE	$S_{EE}^{UV}(A) = S_{alg.,ginv}(\mathrm{A})$ + boundary term	$S_{EE}^{CFT}(A) = \frac{A}{4G_N} + S_{alg.ginv}(\mathcal{E}_A)$

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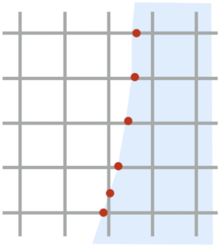
I'll now explain the gauge theory side of the table (before coming back to AdS/CFT).

A proposal for EE in gauge theories

I'll first review a completely formal proposal how to define EE in a gauge theory, then argue that it gives the UV answer when the gauge theory is emergent.

In a gauge theory, the Hilbert space doesn't factorize, so we need to get the reduced density matrix in a different way than the usual partial trace.

"Extended Hilbert space" definition: [Buividovich-Polikarpov; Donnelly]

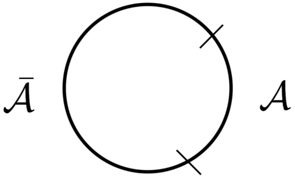


$$
\begin{aligned}
\mathcal{H} \in \mathcal{H}_{ext.} &= \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}} \\
\rho_A &= \mathrm{Tr}_{\bar{A}} \rho \in \mathcal{H}_{ext} \\
S_{EE} &= -\mathrm{Tr} \rho_A \log \rho_A \,.
\end{aligned}
$$

This definition introduces some boundary terms as advertised. Let me show this through examples.

Ex. 1: EE in abelian gauge theory on $\mathbf{S}^1$

Consider $U(1)$ gauge theory on a spatial $\mathbf{S}^1$.



- Operator algebra: $\oint \mathbf{A}$, $E(x)$ (constant by Gauss law)
- Hilbert space basis: electric field eigenstates $\{|n\rangle\}$, $n \in \mathbb{Z}$
- $\mathcal{H}_{ext.} = \{|n_1\rangle \otimes |n_2\rangle\}$, $n_{1,2} \in \mathbb{Z}$

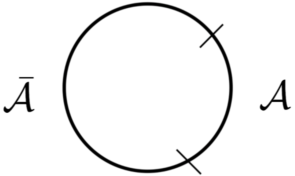
For $|\psi\rangle = \sum_n \psi_n |n\rangle \in \mathcal{H}$,

$$\rho_A \in \mathcal{H}_{ext.} = \sum_n p_n |n\rangle\langle n|, \qquad p_n = |\psi_n|^2$$

$$S_{EE} = -\sum_n p_n \log p_n \qquad \text{“Shannon edge mode”} .$$

Ex. 2: EE in nonabelian gauge theory on $\mathbf{S}^1$

Now consider Yang-Mills with gauge group G on the $\mathbf{S}^1$.



- Operator algebra: Wilson loops $\mathrm{Tr}_R \exp(i \oint \mathbf{A})$, Casimirs $E^a E^a, \ldots$
- Hilbert space basis labeled by reps of G: $\{|R\rangle\}$.
- $\mathcal{H}_{ext.} = \oplus \{|R, i, j\rangle \otimes |R, j, i\rangle\}$, $i, j \in 1, \ldots, \dim R$.

For $|\psi\rangle = \sum_R \psi_R |R\rangle \in \mathcal{H}$,

$$\rho_A \in \mathcal{H}_{ext.} = \sum_R p_R (\dim R)^{-2} \sum_{i,j} |R, i, j\rangle\langle R, j, i| \,, \qquad p_R = |\psi_R|^2$$

$$S_{EE} = -\sum_R p_R \log p_R + 2 \sum_R p_R \log \dim R \qquad \text{“log dim R edge mode”}.$$

Comments

- In $d > 2$, if we apply this definition across every boundary link of a lattice,

$$S_{EE} = \text{Shannon edge term} + \log \dim R \text{ edge term} + \text{interior EE}. \quad (*)$$

- Earlier, we saw an algebraic definition of EE: it's the von Neumann entropy of the unique element of the subalgebra that reproduces the expectation values of all the operators in the subalgebra (up to normalization).
  One can show that

$$S_{EE}^{\mathcal{H}_{ext.}}(\rho_A) = S_{alg,ginv}(A) + \log \dim R \text{ edge}.$$

- If we replace $\mathcal{H}_{ext.} \to \mathcal{H}_{UV}$ in an emergent gauge theory,

  $$S_{EE} = \text{Shannon edge term} + \text{log dim R edge term} + \text{interior EE.} \quad (*)$$

  holds (up to a state-independent constant).

  An example where this is obvious is if we take the UV Hilbert space to be that of lattice gauge theory without imposing the Gauss law at the vertices, but have a Hamiltonian term $\Delta H = U \sum_i \mathcal{G}_i$ that imposes the Gauss law dynamically.

  More generally, Wilson loops factorize by definition...

This explains the formula for EE in an emergent gauge theory, that I showed you at the beginning of the talk...

Interpretation

- From a "totally IR" point of view, $\exists$ a center operator $\mathcal{L}_A$ s.t.

$$\langle R|\mathcal{L}_A|R\rangle = \log\dim R\,.$$

  But $\mathcal{L}_A$ is a complicated, group-dependent function of the Casimirs (e.g. $\log\sqrt{4E^aE^a+1}$ for $G = SU(2)$). The completely obscures the canonical counting interpretation!

To summarize:

> In a UV-finite theory with emergent extended objects (Wilson loops), the UV-exact EE of a region can be written in a "more IR" way, as an EE assigned to the extended objects contained within each region, plus a boundary term counting UV DOF's made visible when the extended objects are cut by the entangling surface.

Analogy to AdS/CFT

If one thinks of AdS/CFT as an emergent gauge theory, with the bulk emerging from the CFT, the area term looks a lot like the "$\log\dim R$" boundary term in the more IR way of writing the EE.

Ent. wedge reconstruction $\leftrightarrow$ **RT + FLM**

Region A in CFT $\sim$ $\mathcal{E}_A$ in bulk EFT

$$S_{EE}^{CFT}(A) = S_{alg,ginv}(\mathcal{E}_A) + \langle\text{center operator}\rangle \sim \frac{A}{4G_N}$$

$\Downarrow$

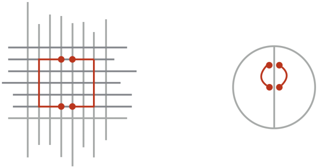
$$S_{EE}^{CFT}(A) = S_{alg,ginv}(\mathcal{E}_A) + \log\dim R$$

Assuming that the gauge theory formula can be used on the LHS, "$A/4G_N$" is a $\log\dim R$ term.

It's interesting to combine this with the interpretation of the $\log \dim R$ term as canonically counting UV DOF's correlated by an emergent gauge constraint, at the entangling surface.

A string cartoon

In particular, let's compare a Wilson loop in an emergent gauge theory to a closed string in the bulk.



$$\begin{aligned}
\frac{A}{4G_N} &= \text{log(boundary states)} \\
&= \#\text{ ways to "glue" two open strings} \\
\frac{1}{4G_N} \sim \mathcal{O}(N^2) &= (\#\text{CP factors})^2?
\end{aligned}$$

- Evidence [Lewkowycz, Maldacena]: Add a single string to the bulk by putting a $q\bar{q}$ pair in the CFT. $S_{EE}(q,\bar{q}) \sim \log N + \ldots$

Summary so far

To summarize, there were two separate conjectures in this part of the talk.

1. Conjecture 1: In AdS/CFT, the RT area term "$A/4G_N$" is the analog of the $\log \dim R$ edge term in the EE of an emergent gauge theory.

2. Conjecture 2: In string theory, closed strings can "factorize" into open ones in a UV part of the Hilbert space corresponding to BH microstates, and the BH entropy counts the Chan-Paton factors. (see Susskind-Uglum).

How can we study this?

We need an example of nonperturbative string theory (so i.e. holography), where

1. There are black holes and “RT-like” entanglement.
2. We have a notion of bulk locality.
3. We understand the “boundary to bulk algorithm”.

(The rest of this talk will be expository, ending on a computation in progress.)

c=1 matrix model

Consider the QM of a matrix with a $U(N)$-invariant Hamiltonian

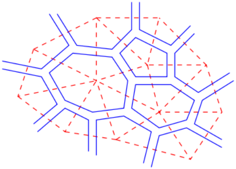
$$H = \mathrm{Tr}\left\{ \frac{1}{2} m^2 \dot{\mathbf{M}}^2 + \frac{1}{2}\omega^2 \mathbf{M}^2 + \frac{1}{3!}\lambda \mathbf{M}^3 \right\} .$$

Because the Hamiltonian is $U(N)$ invariant, the Hilbert space splits into sectors labeled by the reps of $U(N)$. This is exactly analogous to what happens in ordinary QM in a spherically symmetric potential.

Just like there, we can write a separate time-independent Schrödinger equation in each sector, and in the singlet sector we get the Schrödinger equation of N free fermions in the potential $V(\mathbf{M})$.

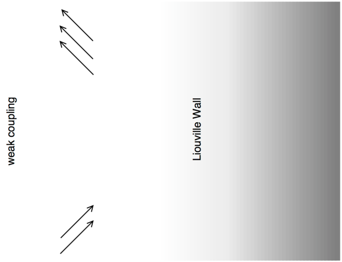
To go from the matrix model to string theory,

1. Observe that large N (double-line) diagrams of matrix QM are in 1:1 correspondence with discretized smooth surfaces.



2. To make the matrix/string diagrams quantitatively agree, we identify vertices/propagators. This relates the matrix QM action to the worldsheet CFT action (Liouville + $c = 1$ matter).
3. Finally, we have to take a continuum limit for the discretized string diagrams. In this limit, the non-singlet sectors of the matrix QM are actually gapped to infinity. So it's commonly stated that gauged matrix QM is dual to the perturbative string theory.

The bulk low energy effective field theory picture is a single massless scalar field in a 1+1d spacetime with a linear dilaton (g_s small at one end of space) and an exponentially growing scalar condensate.



Black holes?

The singlet sector of the matrix QM has no black holes [Karczmarek-Maldacena-Strominger].

However, a natural conjecture is that the strict double-scaling limit is the analog of $N = \infty$ in AdS/CFT, and the non-singlet states that we gapped out are BH microstates...

Argument [Kazakov-Kostov-Kutasov]:

- Put the matrix model on periodic Euclidean time, so

$$Z = \int_{\mathbf{M}(0)=\mathbf{M}(2\pi R)} \mathcal{D}(\mathbf{M}) \exp\left(- \int_0^{2\pi R} d\tau \left[\frac{1}{2}\dot{\mathbf{M}}^2 + V(\mathbf{M}) \right] \right) = \mathrm{Tr} e^{-\beta H} .$$

  The non-singlets in rep R schematically contribute as

$$d_R e^{\beta E_R} .$$

  Both E_R and d_R diverge in the double-scaling limit s.t. the non-singlet contribution is negligible below a critical β_c, but actually becomes dominant above it.

- In the Feynman diagram ($\simeq$ string worldsheet) expansion, the propagator includes windings around the $\mathbf{S}^1$ ($\simeq$ worldsheet vortices). Hence it's natural to guess that this phase transition at β_c is a "worldsheet vortex condensation."

- What does this mean for the bulk spacetime? Suppose that we deform the continuum worldsheet CFT by adding vortex operators to the action. Above β_c, the deformation is relevant and triggers a flow to a new fixed point. In fact, the flow is

  $$c = 1 \text{ Liouville} + \text{matter} \ \rightarrow \ \text{Sine-Liouville CFT}$$

  which is the worldsheet CFT for strings in a 2d asymptotically linear dilaton Euclidean black hole.

This logic is like a worldsheet version of the Hawking-Page transition in AdS/CFT. Hence one conjectures that the non-singlet sector of the $c = 1$ matrix model contains black holes.

Bulk locality

Hartnoll and Mazenc recently studied bulk locality in the (singlet sector of the) $c = 1$ matrix model, which is isomorphic to the QM of N free fermions. They found which fermion degrees of freedom reproduce the EE across a spatial interval in the bulk.

Summary of their result:

1. EE of a singlet matrix QM subalgebra $\xrightarrow{\text{isomorphism}}$
2. EE of N free fermions across a range of values $A = [\lambda_1, \lambda_2] \xrightarrow{\text{definition}}$
3. $-\text{Tr}(MlogM + (1 - M)log(1 - M)$ for $M_{ij} = \langle P_A\psi_i, P_A\psi_j \rangle$, the $n \times N$ matrix of wavefunction overlaps in $A \xrightarrow{\text{calculation}}$
4. EE of a scalar field across a bulk interval in $1 + 1d$ (with the UV cutoff provided by the QM dual of the bulk g_s).

Future directions

- What happens if instead of the EE for the singlet matrix subalgebra corresponding to free fermions in an interval, we compute the von Neumann entropy of the reduced density matrix in the much larger Hilbert space of matrix QM incl. the non-singlet sectors? Do we find "RT-like" $O(N^2)$ entanglement? (In progress...)
- Can we say anything at the worldsheet level?
- Besides $c = 1$ matrix model, is there any other system that we can use to better understand the edge modes of string theory?

Thank you!