Discrete torsion in gauging non-invertible symmetries

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Based on (P-L. arXiv:2406.02676)

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Introduction

Symmetries are a fundamental concept in Physics. In recent times, it has become a subject of intense study, due to the extension to **generalized symmetries** (Gaiotto et al. '15).

This talk is concerned with

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- controlled by some appropriate **cohomology**.

Main result

Discrete torsion for **non-invertible symmetries** admits **two generalizations**, one of which is naturally **classified** by a **cohomology group** and **acts** on **gaugeable (sub)symmetries** and via **non-invertible B field actions**.

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Phases in partition functions

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$$Z_{\mathcal{T}/G}(\tau,\bar{\tau}) = \frac{1}{|G|}\sum_{g,h\in G;\,[g,h]=1} Z_{g,h}(\tau,\bar{\tau}), \tag{1}$$

where $Z_{g,h}(\tau,\bar{\tau})$ are (g,h)-twisted sector contributions.

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The partition function can be **consistently** changed by adding $U(1)$ **phases** to the twisted sectors (Vafa '86)

$$Z^{[\omega]}_{\mathcal{T}/G}(\tau, \bar{\tau}) \;=\; \frac{1}{|G|} \sum_{g,h \in G;\, [g,h]=1} \frac{\omega(g,h)}{\omega(h,g)}\, Z_{g,h}(\tau, \bar{\tau}). \tag{2}$$

Hence, there are **inequivalent** ways of **gauging** a G-symmetry, uniquely **specified** by a **cohomology class** $[\omega] \in H^2(G, U(1))$, a choice of **discrete torsion**.

B field actions

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Given a G-action on X the **target space** of a 2d σ-model

$$\rho : G \to \mathrm{Diff}(X),$$

and a B field over X, sometimes there exist lifts to an **action** on the B field, a G-**equivariant** structure on B

$$\hat{\rho} : G \to \mathrm{Aut}(X, B), \tag{3}$$

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Any two such lifts are related by **discrete torsion** (Sharpe '03).
Schematically,

$$(\hat{\rho}/\hat{\rho}') \in H^2(G, U(1)). \tag{4}$$

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Non-invertible symmetries then correspond to topological line operators $\{L_i\}_{i\in\mathcal{I}}$ that can be added and fused according a fusion law

$$L_i \otimes L_j = \sum N_{ij}^k L_k, \tag{6}$$

for $N_{ij}^k \in \mathbb{N}$ some *fusion coefficients*. This fusion law is **not necessarily group-like** (5), and in particular, **line operators may not have inverses** under fusion.

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Fusion categories are **finitely-generated**, meaning every object admits a decomposition $A = \sum_{i \in \mathcal{I}} N_A^i L_i$ into a "basis" of "simples" $\{L_i\}_{i \in \mathcal{I}}$. It is in this sense that **fusion categories are the non-invertible analogue of finite groups.**

Fusion category examples

Examples.

Examples of fusion categories include

- The ordinary group case is recovered by $\mathcal{C} = \mathrm{Vec}(G)$ with

$$L_g \otimes L_h = L_{gh}, \tag{7}$$

- Representation categories $\mathrm{Rep}(G)$, whose **simple** objects are the **irreps** of G,
- Representation categories $\mathrm{Rep}(\mathcal{H})$ of particular algebras $\mathcal{H}$.

Basics of gauging non-invertible symmetries

Given a theory $\mathcal{T}$ with symmetries $\mathcal{C}$, the **gaugeable subsymmetries** are in one-to-one **correspondence*** with special symmetric Frobenius (or **gaugeable**) **algebras** (A, μ, Δ), for $\mu : A \otimes A \to A$, $\Delta : A \to A \otimes A$ **(co)multiplication** morphisms. This again gives rise to a **gauged theory** $\mathcal{T}/(A, \mu, \Delta)$.

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On T^2, the partition function of $\mathcal{T}/(A, \mu, \Delta)$ takes the form[1]

$$Z_{\mathcal{T}/(A,\mu,\Delta)}(\tau, \bar{\tau}) = \sum_{L_1, L_2, L_3} \mu_{L_1, L_2}^{L_3} \Delta_{L_3}^{L_2, L_1} Z_{L_1, L_2}^{L_3}(\tau, \bar{\tau}), \qquad (8)$$

where $\mu_{L_1,L_2}^{L_3} \Delta_{L_3}^{L_2,L_1}$ expands the morphism $\Delta \circ \mu : A \otimes A \to A \otimes A$ in terms of simples L_1, L_2, L_3.

[1] For a fusion category where the hom-spaces of simple objects are at most one-dimensional. For more general cases see (P-L et al: arxiv:2408.16811).

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2. Discrete torsion **twists**: **differences** of choices (of algebras, of actions on B fields...), controlled by a (nonabelian) **cohomology group**, generalizing $H^2(G, U(1))$.

We will discuss the twists next, and discuss how they relate to the choices.

Discrete torsion twists

Mathematically, **discrete torsion twists** are based on the concept of **2-cocycles** $\omega \in Z^2(\mathcal{C})$ of monoidal **categories**. These are **natural isomorphisms**

$$\omega_{A,B} : A \otimes B \xrightarrow{\cong} A \otimes B, \tag{9}$$

satisfying the (normalized) 2-cocycle conditions

$$\omega_{A,1} = \omega_{1,A} = \mathrm{id}_A, \tag{10}$$

$$(\omega_{A,B\otimes C}) \circ (\mathrm{id}_A \otimes \omega_{B,C}) = \omega_{A\otimes B,C} \circ (\omega_{A,B} \otimes \mathrm{id}_C). \tag{11}$$

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By quotienting out the 2-coboundaries $B^2(\mathcal{C})$, $\delta_A : A \xrightarrow{\cong} A$

$$\omega_{A,B} = \delta_{A\otimes B} \circ (\delta_A^{-1} \otimes \delta_B^{-1}), \tag{12}$$

one gets the **lazy cohomology group** of $\mathcal{C}$ (Panaite et al. '10):

$$H^2_\ell(\mathcal{C}) := Z^2(\mathcal{C})/B^2(\mathcal{C}).$$

Discrete torsion twists

Lazy cohomology group examples

Some examples include (Guillot et al. '10):

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- $H^2_\ell(\mathrm{Rep}(A_4)) = \mathbb{Z}_2$,
- $H^2_\ell(\mathrm{Rep}(\mathcal{H})) = H^2_\ell(\mathcal{H}^*)$, the lazy cohomology group of the dual Hopf algebra $\mathcal{H}^*$.

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- pure non-invertible gauge actions on B fields,
- *any* gaugeable algebra (A, μ, Δ) in $\mathcal{C}$,
- that is independent of the used cocycle representative.

Non-invertible B field actions

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For a G-group action, the B field changes as

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for A^g the connection of a **line bundle** L^g labeled by $g \in G$.
In (Waldorf '07) these actions were extended to *non-invertible actions*. For an object $y \in \mathrm{ob}(\mathcal{C})$ with dimension $\dim(y) = n \in \mathbb{N}$ in a fusion category $\mathcal{C}$, its action on a B field is

$$B \mapsto B + \frac{1}{n}\mathrm{tr}(F_A^y), \tag{14}$$

for F_A^y the curvature of a connection A^y of a rank n-**vector bundle** E^y labeled by y.

Discrete torsion twists and B field actions

Just as G-actions are described by *group* homomorphisms $\rho : G \to \mathrm{Aut}(B)$, **non-invertible actions** are *tensor* functors

$$(F, J) : \mathcal{C} \to \mathrm{End}(B) \cong \mathrm{HVbdl}_{\nabla}(M), \qquad (15)$$

for $J_{X,Y} : F(X) \otimes F(Y) \xrightarrow{\cong} F(X \otimes Y)$ some natural isomorphisms and $\mathrm{HVbdl}_{\nabla}(M)$ are hermitian vector bundles with connection.

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Different non-invertible **actions** that *fix* the bundles can be obtained by **precomposing** by a *tensor automorphism* $(\mathrm{Id}_{\mathcal{C}}, \omega) : \mathcal{C} \to \mathcal{C}$. These automorphisms are **precisely** the **lazy 2-cocycles** $\omega \in Z^2(\mathcal{C})$.

We thus identify **discrete torsion twists** with the *nonabelian* lazy **cohomology group**

$$H^2_\ell(\mathcal{C}) := Z^2(\mathcal{C})/B^2(\mathcal{C}).$$

Twisting B field actions

Proposition 1. Twists on B field actions

Let $(F, J) : \mathcal{C} \to \mathrm{HVBdl}_\nabla(X)$ be a pure gauge action on a B field over a space X. Then given any cocycle $\omega \in Z^2(\mathcal{C})$, the tuple

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Moreover, if $[\omega] = 1 \in H^2_\ell(\mathcal{C})$, then

$$(F, J) \sim_{\text{mon. eq.}} (F, J_\omega). \tag{17}$$

In this sense, only cohomology classes are physically relevant.

Twists of algebras

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Proposition 2. Twists of gaugeable algebras

Let (A, μ, Δ) be a gaugeable algebra in $\mathcal{C}$, for $\mathcal{C}$ a symmetry of a theory $\mathcal{T}$. Given a cocycle $\omega \in Z^2(\mathcal{C})$, the tuple

$$(A, \mu_\omega, \Delta_\omega), \tag{18}$$

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In particular, the **partition function** becomes

$$Z^{[\omega]}_{\mathcal{T}/(A,\mu,\Delta)}(\tau, \bar{\tau}) = \sum_{L_1, L_2, L_3} (\mu \circ \omega_{A,A})^{L_3}_{L_1, L_2} \, (\omega^{-1}_{A,A} \circ \Delta)^{L_2, L_1}_{L_3} \, Z^{L_3}_{L_1, L_2}(\tau, \bar{\tau}). \tag{20}$$

Discrete torsion choices + twists

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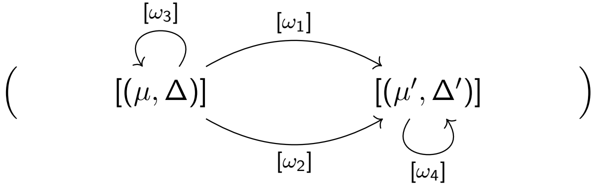
$\text{d.t.}(A) =$



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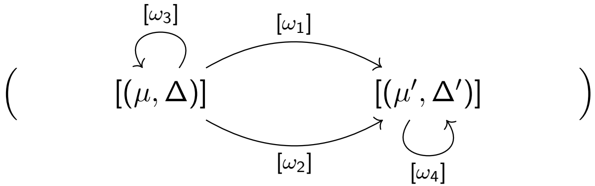

(21)

Note in particular that *nontrivial* cohomology classes can **fix** Morita classes.

- All the **consistency** conditions (**modular invariance**, **multiloop factorization**) demanded in (Vafa '86) are **automatically satisfied**.

Rep(A_4)

Application. Rep(A_4) discrete torsion.

A nontrivial example is the case $\mathcal{C} = \text{Rep}(A_4)$. For the regular object

$$R = \sum_{\rho \in \text{Irrep}(A_4)} \dim(\rho)\rho, \tag{22}$$

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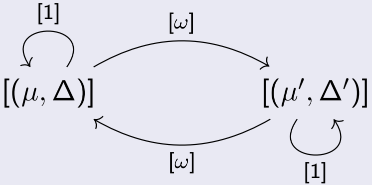
there are two Morita classes of algebra structures. One can show these are related by the unique nontrivial twist $[\omega] \in H^2(\mathrm{Rep}(A_4)) = \mathbb{Z}_2$:

$$\mathrm{d.t.}(R) = \Bigg(\qquad \Bigg) . \tag{23}$$



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Questions are welcome!