Effective AdS_3/CFT_2: Life is simpler without black holes

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Introduction

- The first example of AdS/CFT: [Maldacena,……..]

Type IIB string in $AdS_5 \times S^5 \iff \mathcal{N} = 4,\ SYM$

Obtained by taking the decoupling limit of a stalk of D3 branes

- Another well-studied example: [ABJM 2008]

M-theory on $AdS_4 \times S^7/\Gamma \iff \mathcal{N} = 6,\ d = 3$ ABJM theory

Obtained by taking the decoupling limit of a stalk of M2 branes

- But what about examples of AdS_3/CFT_2 ?

Known only in the tensionless limit: [Eberhardt,Gaberdiel,Gopakumar]

Type IIB string theory in $AdS_3 \times S^3 \times T^4$ supported by 1 unit of NS-NS H-flux (tensionless limit)

$$\iff \mathcal{N} = (4,4),\ d = 2,\ \ (T^4)^N/S_N$$

Question:

What is the status of the AdS_3/CFT_2 holographic duality at a generic point in the moduli space?

Answer:

We DO NOT know the answer in full generality!

However, progress has been made recently in understanding this duality when type IIB string theory $AdS_3 \times \mathcal{N}$ is supported by pure NS-NS 2-form B-flux. In some cases, we understand the full/exact holographic duality, and in some other cases, we understand the duality in an effective sense.

Black holes are the culprits!

Results/Proposal

(I) Exact duality

Full type IIB string theory in $AdS_3 \times \mathcal{N}$ with pure NS-NS flux and $k = \frac{R_{ads}^2}{\alpha'} < 1$

$$\Longleftrightarrow$$

$$N = (2{,}2), \quad \frac{(\mathbb{R}_\phi \times \mathcal{N})^p}{S_p} + \Sigma_{[2]}\,, \quad c = 6kp$$

[Balthazar,Giveon,Kutasov,Martinec]

- $\Sigma_{[2]}$ is an exactly marginal operator in the $\mathbb{Z}_2$ twisted sector
- $\Sigma_{[2]}$ is normalizable
- Spacetime central charge: $c = 6kp$, p is the F1 string number; $g_{ws}^2 \sim 1/p$
- No normalizable ground state, no BTZ black holes in the spectrum
- AdS_3 is sub-stringy, no SUGRA description
- All $k < 1$ models are non-critical; e.g. $AdS_3 \times (S^1 \times LG_n)\big/\mathbb{Z}_n \equiv AdS_3 \times S^3_\flat$

(II) Effective duality

Perturbative type IIB string theory in $AdS_3 \times \mathcal{N}$ with pure NS-NS flux and $k = \frac{R_{ads}^2}{\alpha'} \geq 1$

$$\Longleftrightarrow$$

$$N = (2,2), \quad \frac{(\mathbb{R}_\phi \times \mathcal{N})^p}{S_p} + \Sigma_{[2]} \;, \; c = 6kp \text{ in the } p \to \infty \text{ limit}$$

[Eberhardt]

- $\Sigma_{[2]}$ is an exactly marginal operator in the $\mathbb{Z}_2$ twisted sector
- $\Sigma_{[2]}$ is non-normalizable
- Spacetime central charge: $c = 6kp$, p is the F1 string number; $g_s^2 \sim 1/p$
- Valid for states that remain finite when the spacetime central charge goes to infinity
- No black hole states
- SUGRA description for large k
- Examples: $AdS_3 \times T^3, k = 1$, $AdS_3 \times S^3 \times T^4, k \geq 2$, $AdS_3 \times S^3 \times S^3 \times S^1, k \geq 1$, etc.

All the exact string backgrounds discussed in the previous slides can be obtained as the decoupling limit of some NS5-F1 bound state systems!

Plan

1. Review of perturbative string theory in AdS_3

2. An intuitive realization of the proposal

3. Worldsheet analysis and checks

4. Conclusion

1. Review of perturbative string theory in AdS_3

Perturbative superstrings in $AdS_3 \times \mathcal{N}$ with NS-NS B-flux

- Fermionic strings in AdS_3: bosonic $SL(2,R)$ WZW model at level $k+2$ and 3 fermions ψ^a that give $SL(2,R)$ affine Lie algebra at level -2
- Total $SL(2,R)$ level is $(k+2)+(-2)=k$ and $R_{ads}=\sqrt{k\alpha'}=\sqrt{k}l_s$
- The worldsheet sigma model is invariant under $SL(2,R)_L \times SL(2,R)_R$ affine symmetry at level k (k, in general, is not an integer)
- $\mathcal{N}$ is a unitary compact $N=1$ SCFT on the worldsheet whose central charge is determined from the worldsheet criticality

$$\left(3+\frac{6}{k}\right)+\frac{3}{2}+c_{\mathcal{N}}=15$$

- If $\mathcal{N}$ has spacetime $N=(2,2)$ superconformal symmetry, then type II GSO projection is imposed by demanding the mutual locality of the vertex operators with the spacetime supercharges [Argurio,Giveon,Shomer, 2000]

$k < 1$ vs $k > 1$

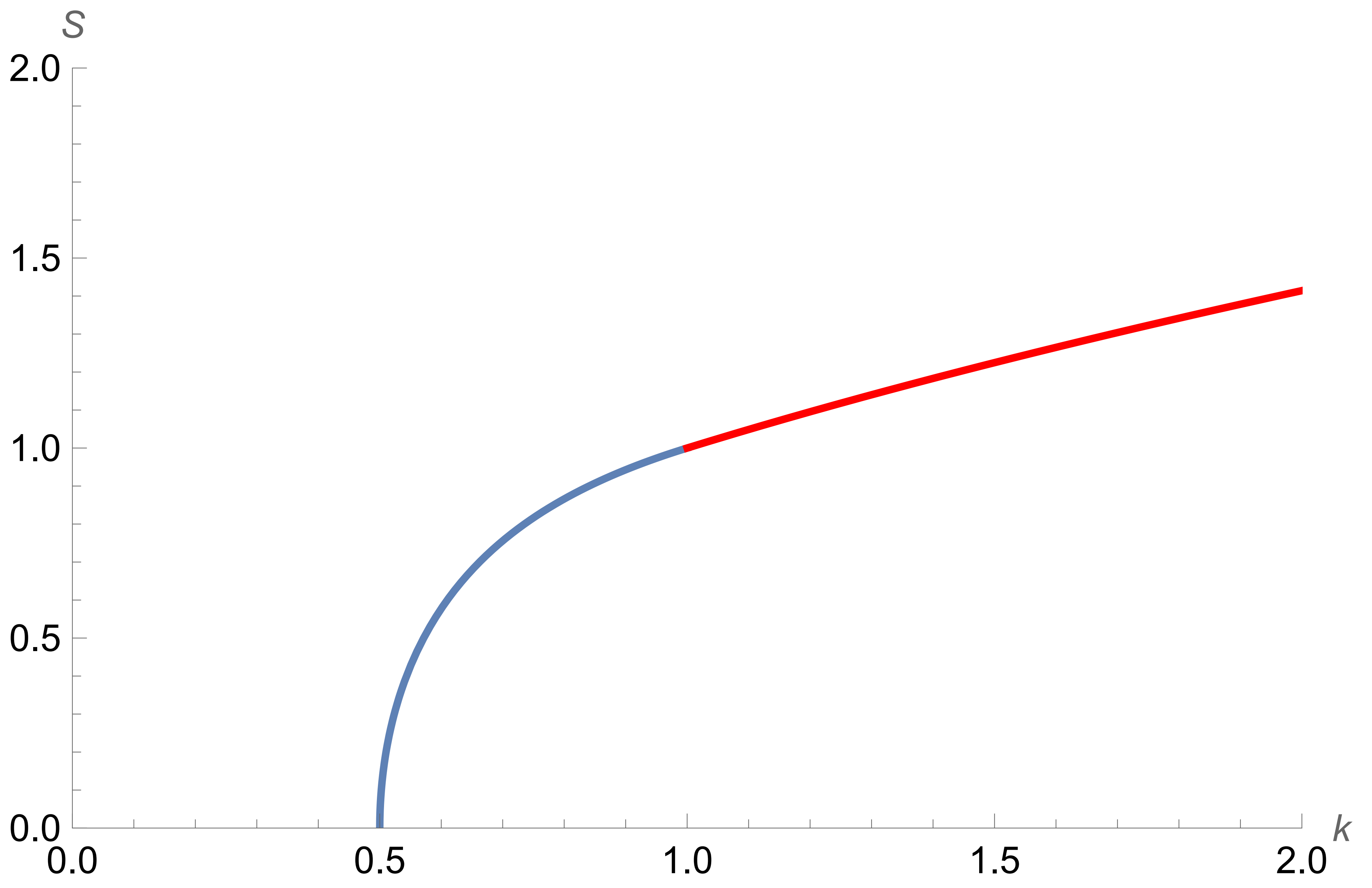
$k < 1$ models are significantly different from those with $k > 1$

- For $k > 1$, the spacetime CFT has normalizable $SL(2,C)$ invariant ground state and the high energy spectrum is dominated by BTZ black holes

- For $k < 1$, neither the $SL(2,C)$ invariant ground state nor the BTZ black holes are in the spectrum. Perturbative long strings dominate the high-energy spectrum

- $k = 1$ is the correspondence point. The black hole entropy and the perturbative string entropy match (including numerical factors)

String-Black hole transition at $k = 1$!



[Giveon,Kutasov,Rabinovici,Sever]

Perturbative string spectrum

Bosonic $SL(2,R)_{k+2}$ current algebra: $$j^a(z)j^b(0) \sim \frac{k+2}{2z^2}\eta^{ab} + i\epsilon^{ab}_c\frac{j^c(0)}{z}$$

$\Phi^{(0)}_{j,m,\bar{m}}$: primary operators under bosonic $SL(2,R)_{k+2}$ with worldsheet conformal dimension

$$h[\Phi^{(0)}_{j,m,\bar{m}}] = -\frac{j(j-1)}{k}$$

$\Phi^{(0)}_{j,m,\bar{m}}$ satisfies the following OPEs:

$$j^3(z)\Phi^{(0)}_{j;m,\bar{m}}(0) \sim \frac{m}{z}\Phi^{(0)}_{j;m,\bar{m}}(0)$$

$$j^\pm(z)\Phi^{(0)}_{j;m,\bar{m}}(0) \sim \frac{m \mp (j-1)}{z}\Phi^{(0)}_{j;m\pm1,\bar{m}}(0)$$

Some of the operators $\Phi^{(0)}_{j,m,\bar{m}}$ correspond to normalizable and delta function normalizable states.

(i) Normalizable states (short string states/bound states) belong to unitary discrete representations $D_j^{\pm}$ of $SL(2,R)$. Unitarity and normalizability range for j

$$\frac{1}{2} < j < \frac{1}{2}(k+1)$$

with $m - j \in N_{\geq 0}$ for D_j^{+}, and $-j - \bar{m} \in N_{\geq 0}$ for D_j^{-} (D_j^{-} are in states and D_j^{+} are out states).

(ii) Delta-function normalizable states (long string states/scattering states) belong to the principal continuous series C_j of $SL(2,R)$

$$j \in \frac{1}{2} + is,\ s \in R,$$

$s < 0$ are in states, $s > 0$ are out states.

The AdS_3 vacuum corresponds to $j = 1$ which is non-normalizable for $k < 1$, according to the unitarity bound on j.

Spectrally flowed states

Bosonic $SL(2,R)$ CFT also admits spectrally flowed representations.

$\Phi^{(w)}_{j,m,\bar{m}}$: spectrally flowed vertex operators with worldsheet dimension

$$h[\Phi^{(w)}_{j,m,\bar{m}}] = -\frac{j(j-1)}{k} - mw - \frac{k+2}{4}w^2$$

$\Phi^{(w)}_{j,m,\bar{m}}$ satisfies the following OPEs:

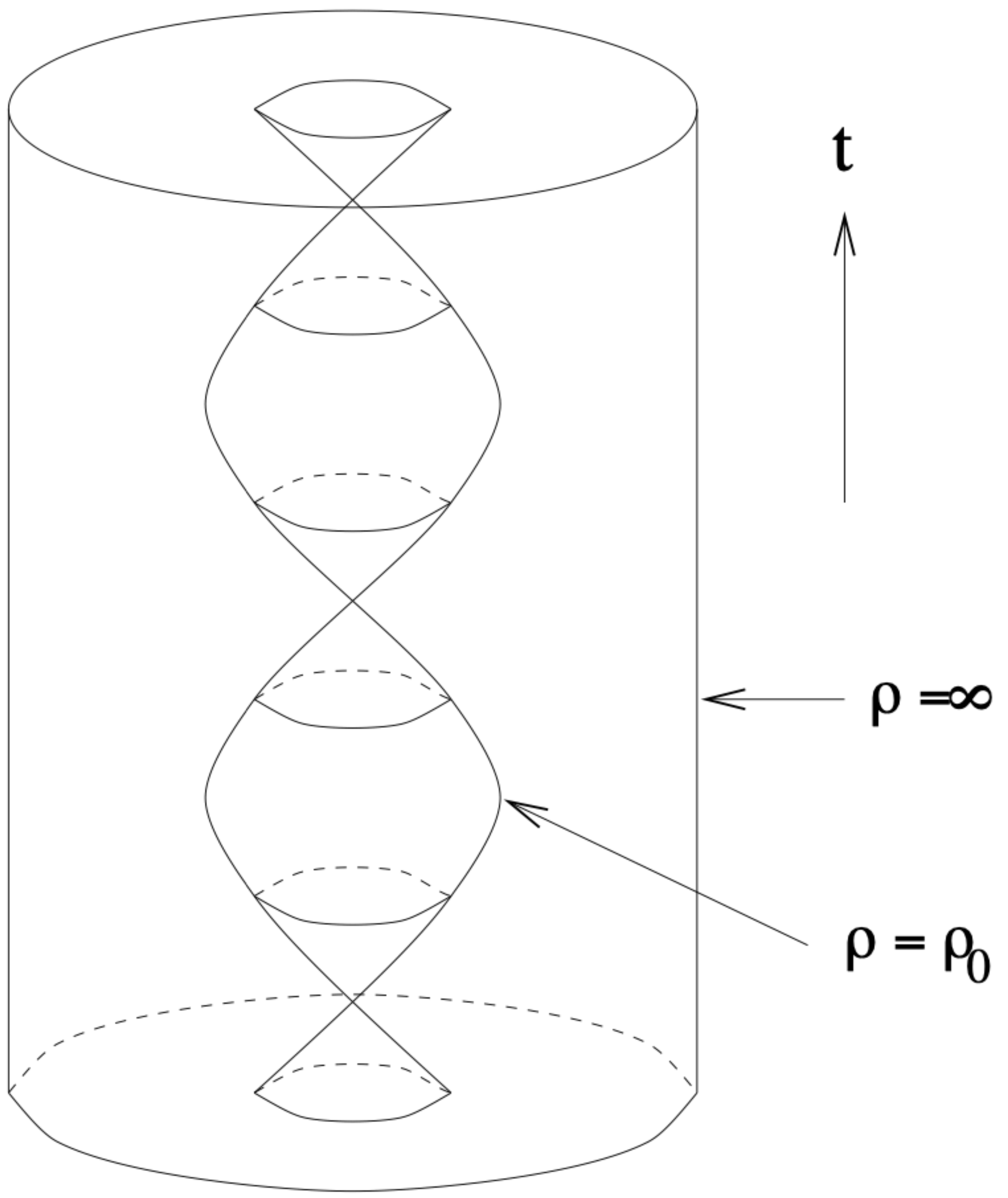
$$j^3(z)\Phi^{(w)}_{j;m,\bar{m}}(0) \sim \frac{m+\frac{k+2}{2}w}{z}\Phi^{(w)}_{j;m,\bar{m}}(0)$$

$$j^{\pm}(z)\Phi^{(w)}_{j;m,\bar{m}}(0) \sim \frac{m \mp (j-1)}{z^{\pm w+1}}\Phi^{(w)}_{j;m\pm 1,\bar{m}}(0)$$

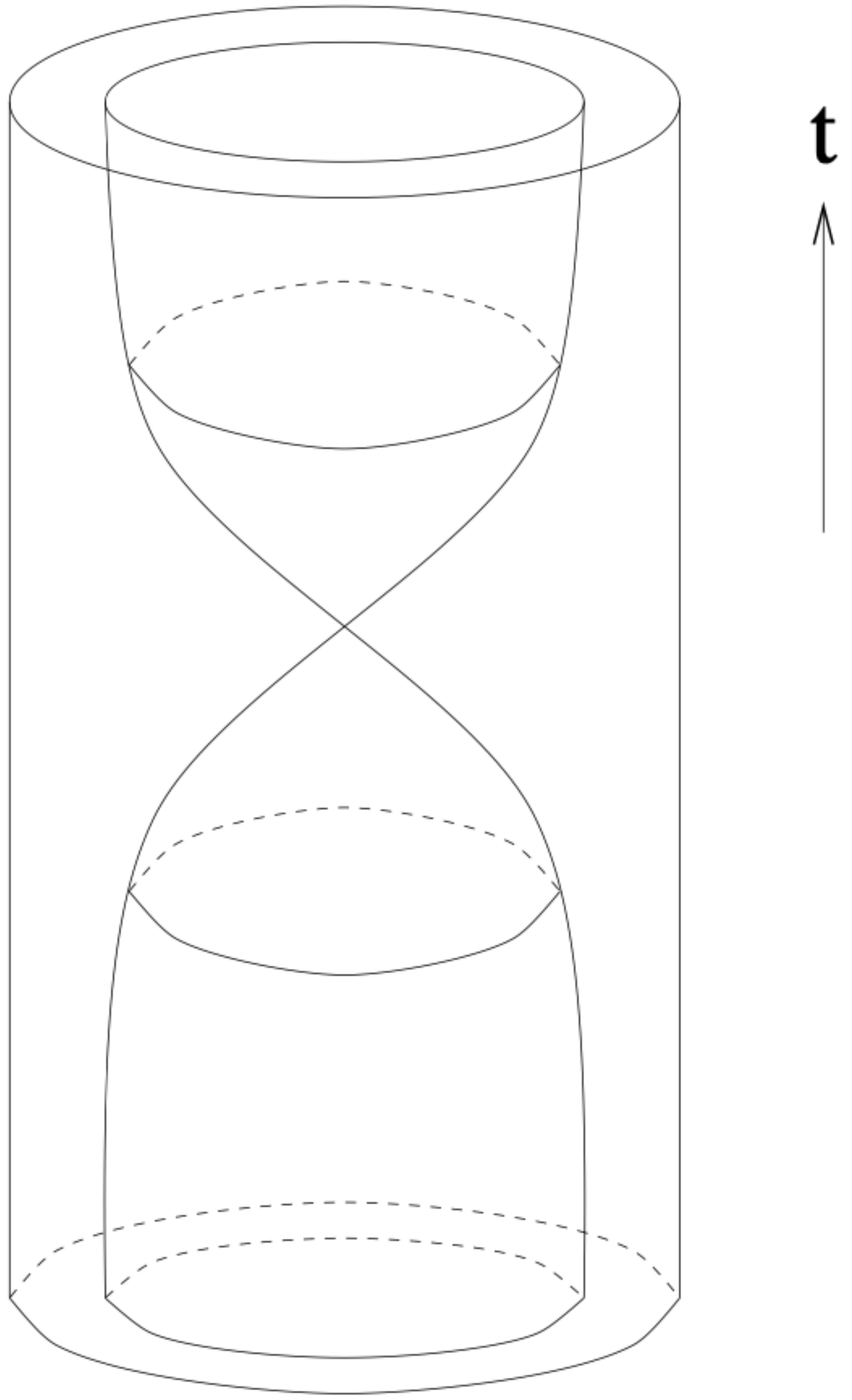
In superstrings: $\Phi^{(w)}_{j,m,\bar{m}} \to e^{-iw(H_{sl}+\bar{H}_{sl})}\Phi^{(w)}_{j,m,\bar{m}}$ which is equivalent to $k \to k-2$

However, the winding w is not a conserved quantum number!

Short strings vs Long strings



Short strings/bound states



Long strings/scattering states

[Maldacena,Ooguri]

Holographic dictionary

Via the holographic duality string theory in AdS_3 is dual to a conformal field theory living on the boundary

Normalizable/delta-function normalizable states in string theory in AdS_3

$\Longleftrightarrow$

States in the Hilbert space of the boundary CFT_2

Non-normalizable worldsheet operators

$\Longleftrightarrow$

Local operators of the boundary CFT

$$h_{st} = -m - \frac{kw}{2}, \quad \bar{h}_{st} = -\bar{m} - \frac{kw}{2}$$

with $\quad m - \bar{m} \in \mathbb{Z}$

[Giveon-Kutasov-Seiberg]
[Kutasov-Seiberg]
[Maldacena-Ooguri]
[Eberhardt-Dei]

Spacetime OPE coefficient $\Longleftrightarrow$ Worldsheet OPE coefficients

2. An intuitive realization of the proposal

Long Strings in $AdS_3 \times \mathcal{N}$

- The theory on a single long string was analyzed by Seiberg&Witten in 1999. For string theory on $AdS_3 \times \mathcal{N}$ with pure NS-NS flux, the theory on a single long string is described by a sigma model

$$\mathcal{M}_{6k}^{(\ell)} = \mathbb{R}_\phi \times \mathcal{N}\ .$$

- ϕ can be identified as the radial direction of AdS_3.
- The theory on $\mathbb{R}_\phi$ has a linear dilaton with slope:

$$Q_\ell = (1-k)\sqrt{\frac{2}{k}}\ .$$

- Effective coupling of the long strings (not to be confused with the worldsheet coupling):

$$g_\ell(\phi) \sim \exp\left(-\frac{1}{2}Q_\ell\ \phi\right).$$

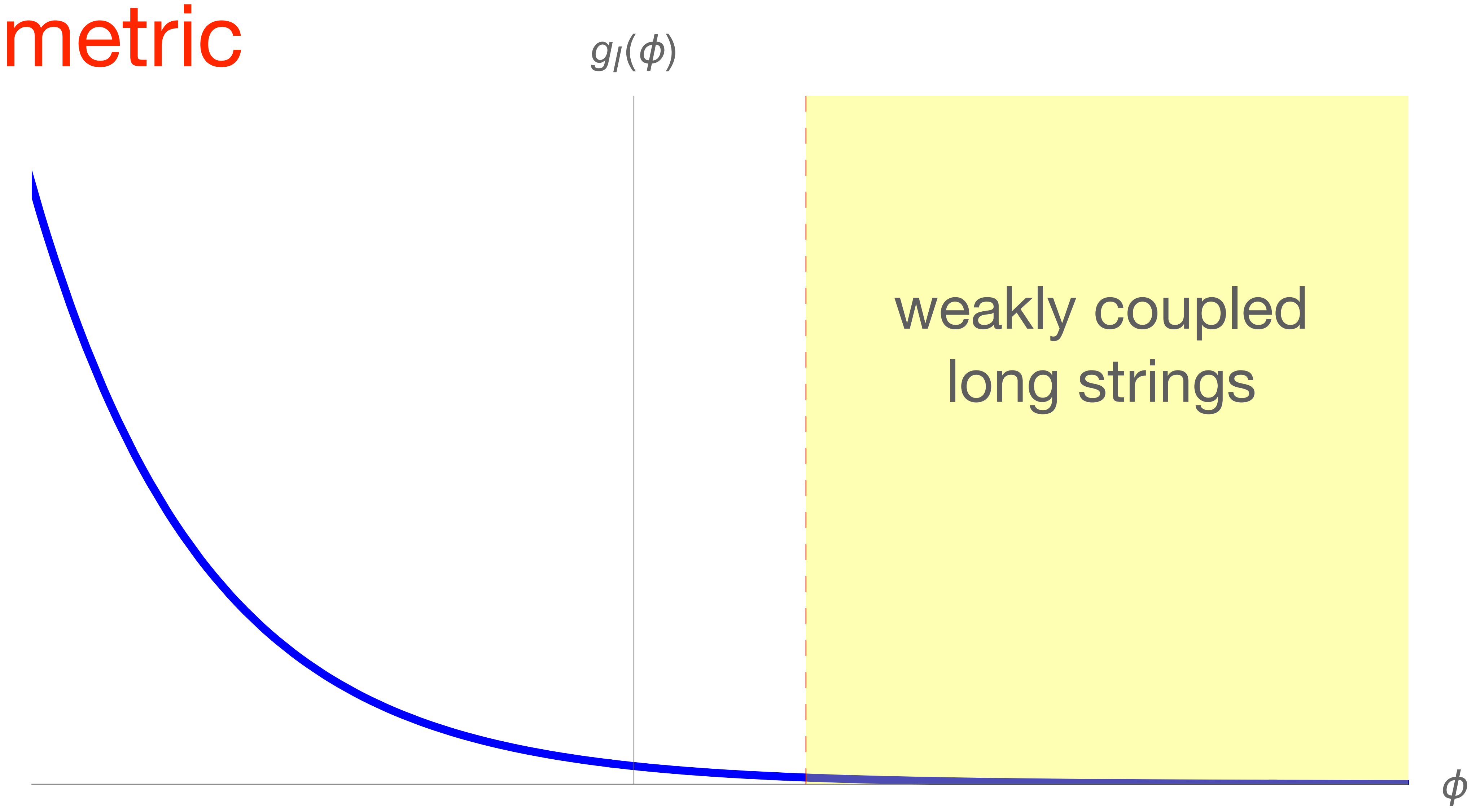
- The dynamics of the long strings heavily depend on the if $k < 1,\ k > 1$.

$k < 1$

- For $k < 1$ the Seiberg-Witten long strings are free near the boundary ($\phi \to \infty$).
- Near the boundary, the SW long strings form a symmetric product

$$\frac{(\mathcal{M}_{6k}^{(\ell)})^p}{S_p}, \quad c = 6kp.$$

- Winding $w \leftrightarrow \mathbb{Z}_{|w|}$ twisted sector
- The SW long string symmetric product description breaks down at finite ϕ.



- This is expected because the discrete states live at finite ϕ and **DO NOT** form a symmetric product.

$$\text{Full boundary CFT} = \frac{(\mathcal{M}_{6k}^{(\ell)})^p}{S_p} + \text{twisted marginal deformation } \left(\Sigma_{[2]}\right)$$

Properties of the deforming operator

1. Should be an exactly marginal operator in the $\mathbb{Z}_2$ twisted sector
2. The radial profile (or the ϕ profile) is such that it goes to zero near the boundary and shields the strong coupling region (like Liouville theory)
3. Must preserve supersymmetry
4. Must generate the full spectrum of discrete states
5. Normalizable

Remember: there are no black holes in the spectrum
(perturbative string states account for the full spectrum)

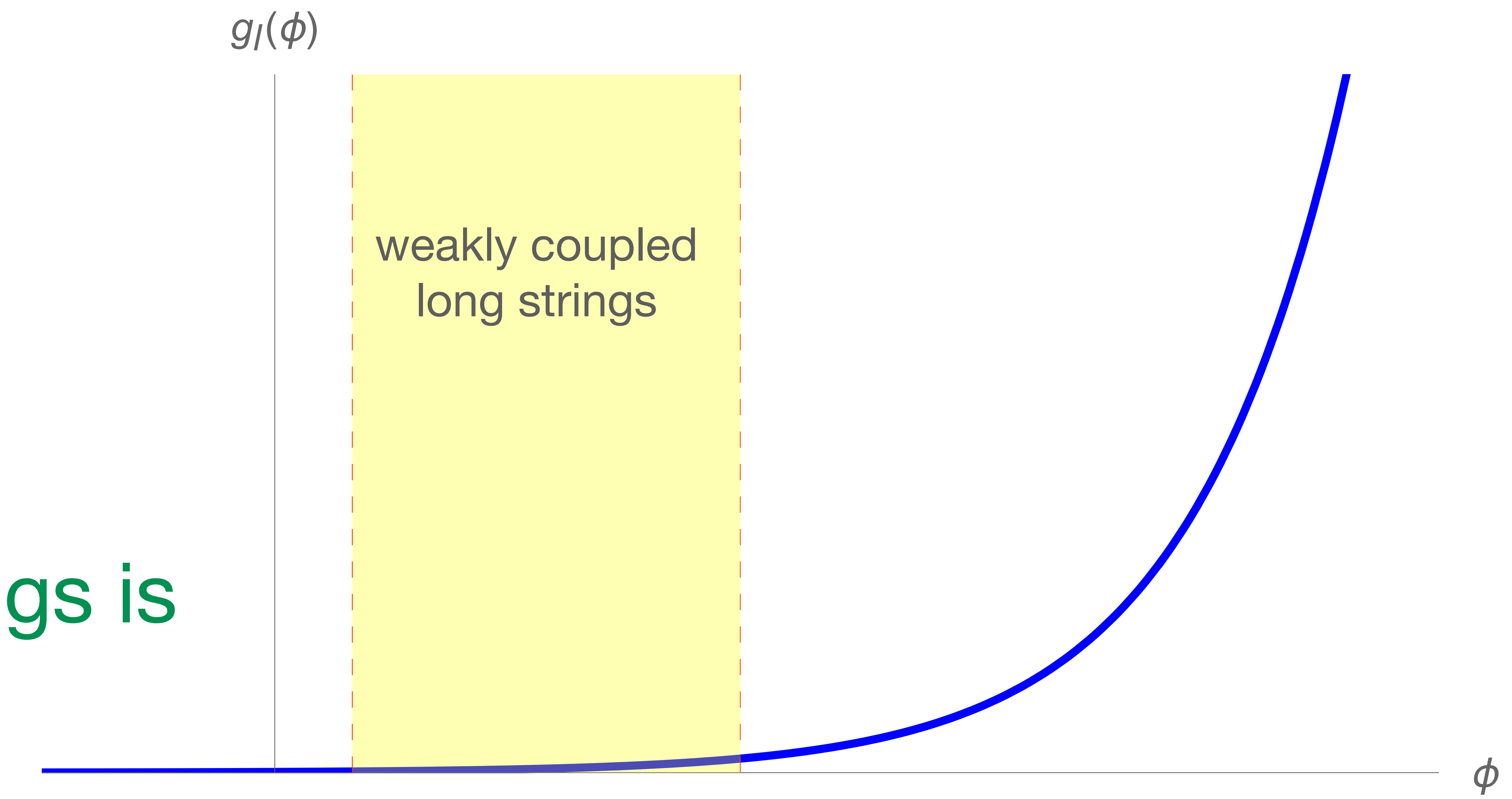
$\Longleftrightarrow$

EXACT description of the boundary CFT

Identify $\Sigma_{[2]}$ from the worldsheet analysis

$k > 1$

- For $k > 1$ the SW, long strings are strongly coupled near the boundary ($\phi \to \infty$).
- **Strong coupling of the long strings accounts for the black hole states** [Giveon,Kutasov,Rabinovici,Sever]



- **Worldsheet perturbation theory will not see the strong coupling region** $(g_{ws}^2 \sim k/p)$
- SW symmetric product description of the long strings is still valid in perturbation theory
- SW symmetric product description breaks down at finite ϕ

As an effective AdS_3/CFT_2 duality, one can still propose

$$\frac{(\mathcal{M}_{6k}^{(\ell)})^p}{S_p} + \Sigma_{[2]}, \quad \text{large } p$$

as the boundary theory for $E \ll E_{BTZ}$

Properties of the deforming operator

1. Should be an exactly marginal operator in the $\mathbb{Z}_2$ twisted sector
2. The radial profile (or the ϕ profile) is such that it goes to zero near the boundary and **shields the weak coupling region** (**UNLIKE** Liouville theory)
3. Must generate the full spectrum of discrete states
4. Must preserve supersymmetry
5. Non-normalizable

The full theory has BTZ black holes in the spectrum, but the effective AdS_3/CFT_2 duality will not see the non-perturbative states!

$k = 1$

The $k = 1$ case is a bit tricky.

The boundary CFT is still

$$\frac{(\mathscr{M}_{6k}^{(\ell)})^p}{S_p} + \Sigma_{[2]}.$$

But it's unclear whether the proposed duality is exact or effective.

This requires more analysis.

3. Identify $\Sigma_{[2]}$ from the worldsheet analysis

Worldsheet analysis $(k < 1)$

Superstrings in $AdS_3 \times \mathcal{N}$ with pure NS-NS flux.

Consider the operator $e^{\beta\phi}, \quad \beta = -\frac{Q_\ell}{2} + ip, \ \ p \in \mathbb{R}$ in the SW long string theory $\mathcal{M}_{6k}^{(\ell)} = \mathbb{R}_\phi \times \mathcal{N}$.

It has spacetime dimensions: $h_{st} = \bar{h}_{st} = -\frac{1}{2}\beta(\beta + Q_\ell) = \frac{p^2}{2} + \frac{Q_\ell^2}{8}$.

The corresponding worldsheet operator in the $(-1, -1)$ picture:

$$e^{\beta\phi} \quad \longleftrightarrow \quad e^{-\varphi-\bar{\varphi}}\, e^{i(H_{sl}+\bar{H}_{sl})}\, \Phi^{(-1)}_{j;m,\bar{m}}$$

with $\quad j = \frac{1}{2} + is, \ \ p = s\sqrt{\frac{2}{k}}$.

Taking derivative: $\partial_x e^{\beta\phi} \longleftrightarrow e^{-\varphi-\bar{\varphi}} \left[J_0^- \,,\, e^{i(H_{sl}+\bar{H}_{sl})}\, \Phi^{(-1)}_{j;m,\bar{m}} \right]$

In the limit $\beta \to 0$ (removing LSZ poles on the worldsheet side)

$$\partial_x \phi \longleftrightarrow e^{-\varphi-\bar{\varphi}} \left[\frac{1}{\sqrt{2}(1-k)} (\partial\varphi + i\partial H_{sl})\, e^{i(H_{sl}+\bar{H}_{sl})}\, \Phi^{(-1)}_{1-\frac{k}{2};\frac{k}{2}-1,\frac{k}{2}} - \psi^3_{sl}\, e^{i\bar{H}_{sl}}\, \Phi^{(-1)}_{1-\frac{k}{2};\frac{k}{2},\frac{k}{2}} \right]$$

In the SW theory, the operator $\partial_x \phi$ is holomorphic: $\partial_x \partial_{\bar{x}} \phi = 0$.
(Because it doesn't know anything about the physics at finite ϕ)

BUT

In the full spacetime theory $\partial_x \phi$ is not holomorphic.
In fact, in the full theory $\partial_x \partial_{\bar{x}} \phi =$ (information about the deforming operator $\Sigma_{[2]}$)

To probe the deformation of the SW symmetric product, one needs to calculate $\partial_x \partial_{\bar{x}} \phi$ in the bulk (full worldsheet) description.

This gives (up to BRST exact terms)

$$\partial_x \partial_{\bar{x}} \phi \quad \longleftrightarrow \quad e^{-\varphi-\bar{\varphi}}\, e^{i(H_{sl}+\bar{H}_{sl})}\, (\partial\varphi + i\partial H_{sl})(\bar{\partial}\bar{\varphi} + i\bar{\partial}\bar{H}_{sl})\, \Phi^{(-1)}_{1-\frac{k}{2};\frac{k}{2}-1,\frac{k}{2}-1} .$$

This is an operator with $w = -1$ that must correspond to an operator in the untwisted sector of the symmetric product.

The spacetime dual of the RHS is proportional to $\partial_x \phi \partial_{\bar{x}} \phi \ e^{-Q_\ell \phi}$.

The Lagrangian of the deformed SW theory: $\mathscr{L}_{block} = (1 + e^{-Q_\ell \phi})\ \partial_x \phi \partial_{\bar{x}} \phi + \mathscr{L}_{\mathscr{N}}$.

Just modifies the kinetic term on $\mathbb{R}_\phi$ which can be removed by field redefinition.

Thus, the symmetric product structure is preserved.

We want an operator in the $\mathbb{Z}_2$ twisted sector!

FZZ duality: relates winding w operators with winding $w-1$

$$\Phi^{(w)}_{j;-j,-j} \equiv \Phi^{(w-1)}_{\frac{k}{2}+1-j;\frac{k}{2}+1-j,\frac{k}{2}+1-j}.$$

So the relevant normalizable operator with winding $w = -2$ is

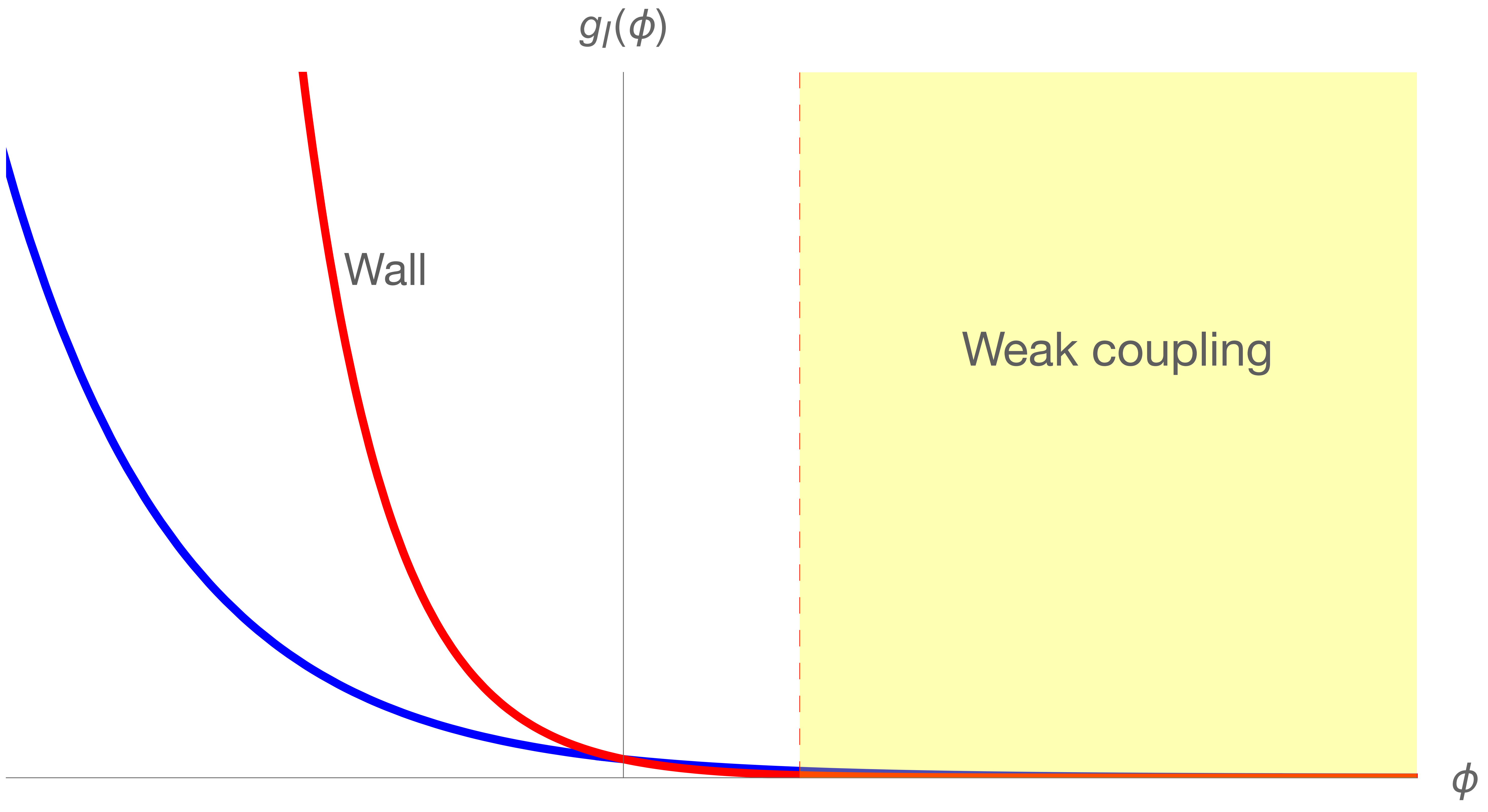
$$e^{-\varphi-\bar{\varphi}}\, e^{i(H_{sl}+\bar{H}_{sl})}\,(\partial\varphi + i\partial H_{sl})(\bar{\partial}\bar{\varphi} + i\bar{\partial}\bar{H}_{sl})\,\Phi^{(-2)}_{k;k,k}\ .$$

This corresponds to a $\mathbb{Z}_2$ twisted marginal operator in the spacetime.

Its radial profile goes like $e^{-\frac{1}{2\sqrt{k}}\phi_S}$ where

$$\phi_S = \frac{1}{\sqrt{2}}(\phi_1 + \phi_2).$$

In the $\mathbb{Z}_2$ covering space, the radial profile is $e^{-\sqrt{\frac{k}{2}}\phi}$.



- The wall shields the strong coupling region.
- The wall nicely explains the discrete states

Next step: dress it up with appropriate twist operators

$N = 2$ superconformal background $AdS_3 \times S^1 \times \mathscr{M}$

The deformation preserves supersymmetry: it must be the top component of a superfield. The bottom component should be a chiral (anti-chiral) primary of dimension $\left(\frac{1}{2}, \frac{1}{2}\right)$ of the form

$$\Sigma^{\pm}\bar{\Sigma}^{\pm}, \quad \Sigma^{\pm} = \exp\left[-\frac{1}{2\sqrt{k}}(\phi_S \mp iY_S)\right](\sigma_{\phi_A}\sigma_{Y_A}\sigma^{\pm}_{\psi_A})\Sigma^{\pm}_{\mathscr{M}} \; .$$

The $\mathbb{Z}_2$-twisted marginal operator is given by

$$\Sigma_{[2]} \sim G^{\alpha}_{-\frac{1}{2}}\bar{G}^{\bar{\alpha}}_{-\frac{1}{2}}\Sigma^{\alpha}\bar{\Sigma}^{\bar{\alpha}}.$$

Example: $AdS_3 \times (S^1_Y \times LG_n)/\mathbb{Z}_n$ a.k.a $AdS_3 \times S^3_\flat$ [Balthazar,Giveon,Kutasov,Martinec]

$k \geq 1$

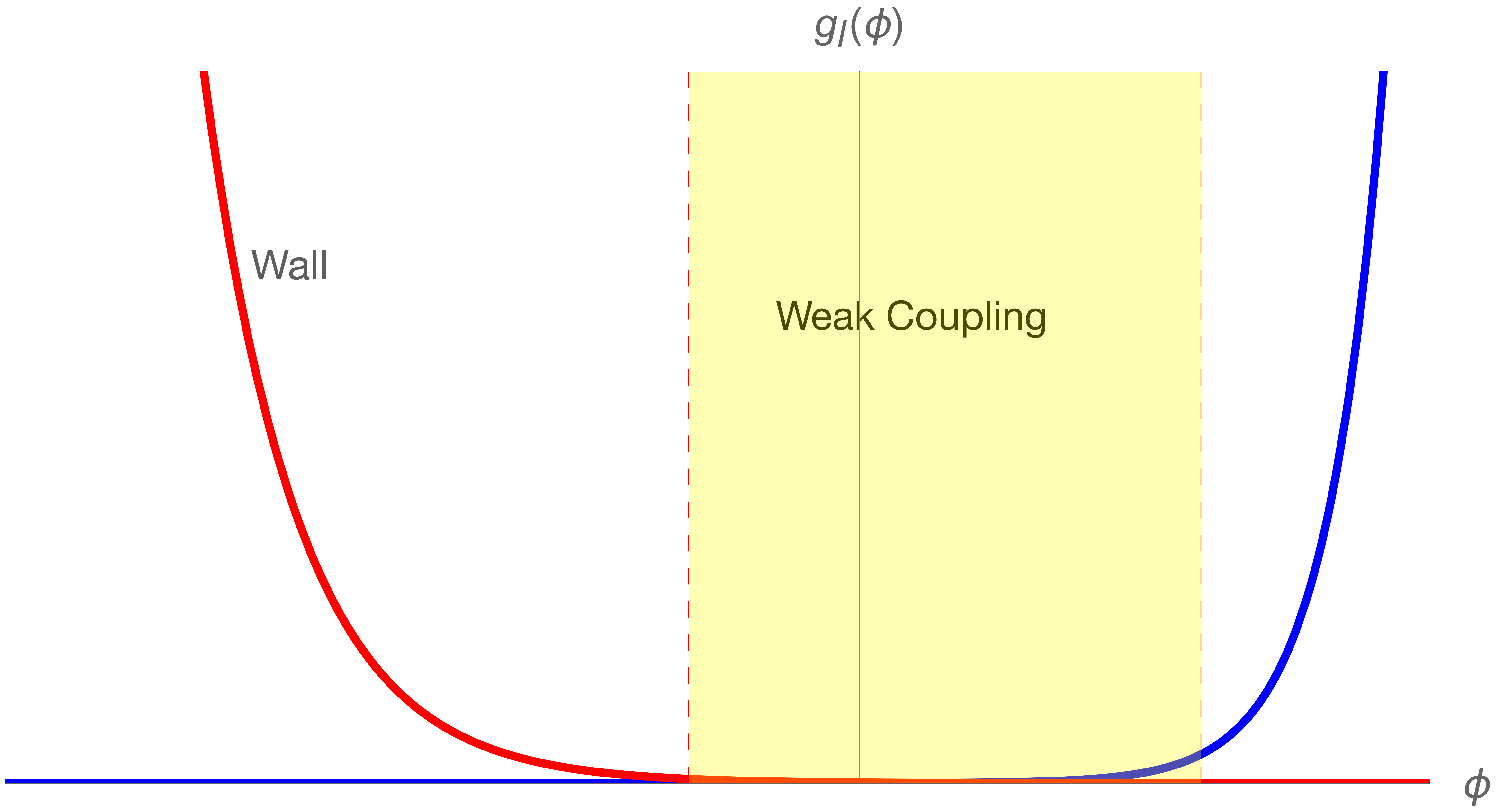
- Everything discussed for $k < 1$ also holds for $k \geq 1$

- The worldsheet operator is non-normalizable; non-normalizable deformation

- The deformation exactly reproduces the discrete states in the spectrum
  [Eberhardt] [Hikida,Schomerus]

- Tells us nothing about the BTZ black hole states; hence effective

- Not clear if black holes exist in the $k = 1$ case; requires more analysis



- The wall shields the weak coupling region
- The wall reproduces the perturbative discrete states
- Perturbative string theory will not see the strong coupling region
- Can’t say anything about the black hole states

Examples

- $AdS_3 \times S^3 \times T^4$ for $k \geq 2$, k is a positive integer

$$N = (4{,}4), \quad \left(\mathbb{R}_\phi \times SU(2)_k \times T^4\right)^p / S_p + \Sigma_{[2]},$$

$$\Sigma = \sigma_{\phi_A} \sigma_{WZW} \sigma_{\mathbb{T}^4} s_\alpha e^{-\frac{1}{2\sqrt{k}}\phi_S} \,.$$

- $AdS_3 \times S^3 \times S^3 \times S^1, \quad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2},$ $k_{1,2}$ positive integers ≥ 2 ,

$$N = (4{,}4), \quad \left(\mathbb{R}_\phi \times SU(2)_{k_1} \times SU(2)_{k_2} \times S^1\right)^p / S_p + \Sigma_{[2]},$$

$$\Sigma = \sigma_{\phi_A} \sigma_{WZW_1} \sigma_{WZW_2} \sigma_{\mathbb{S}^1} s_\alpha e^{-\frac{1}{2\sqrt{k}}\phi_S} \,.$$

Checks

- 1-1 scattering of long strings: Poles of the reflection coefficient give the full spectrum of discrete states [Hikida,Schomerus]
- Matching of the correlation function in perturbation theory [Eberhardt]

Conclusion

- String theory in $AdS_3 \times \mathcal{N}$ with NS-NS H-flux $\Longleftrightarrow$ symmetric product of SW long string CFT deformed by a twist two marginal operator.
- For $R_{ads}\big/l_s < 1$ the duality is an exact one
- For $R_{ads}\big/l_s \geq 1$ the duality is an effective one (holds for $E \ll E_{BTZ}$)
- The black holes or any non-perturbative state are not present in either of the two cases
- We do not fully understand the case $R_{ads}/l_s = 1$. Further analysis is required.
- There are twist-2, marginal RR moduli. Requires investigation
- How to bring the black holes into the spectrum? Any comments/suggestions are welcomed!

Thank you!