Discrete symmetries in string theory and supergravity

Eric Sharpe
Virginia Tech

w/ J Distler, S Hellerman, T Pantev, ...,
0212218, 0502027, 0502044, 0502053, 0606034, 0608056, 0709.3855, 1004.5388, 1008.0419, 1012.5999

Also: N Seiberg, 1005.0002; Banks, Seiberg 1011.5120

My talk today will concern NLSM's describing:

spaces with trivial group actions
(which physics will, nevertheless, see)

equivalently

NLSM's with restrictions on nonperturbative sectors

in both 2d strings (new SCFT's, GW, GLSM's)
and also 4d sugrav's (gen'l of Bagger-Witten).

There is considerable literature on this going back a decade or so, to which Seiberg, Banks-Seiberg have recently contributed.

Let's begin with a prototypical example.

Consider an analogue of the susy $\mathbf{P}^N$ model in two dimensions, which physically can be described by a U(1) susy gauge theory with N+1 chiral fields, but give them charge k, instead of charge 1.

This has a trivially-acting $\mathbf{Z}_k$ everywhere, a prototype for our discussion of discrete symmetries.

(Also looks like $\mathbf{P}^N$ model w/ restriction on nonpert')

How can this be different from ordinary $\mathbf{P}^N$ model? After all, perturbatively identical.

The difference lies in nonperturbative effects.
(Perturbatively, having nonminimal charges makes no difference.)

Example: Anomalous global U(1)'s

$$\mathbf{P}^{N-1}: \quad U(1)_A \mapsto \mathbf{Z}_{2N}$$
$$\text{Here}: \quad U(1)_A \mapsto \mathbf{Z}_{2kN}$$

Example: A model correlation functions

$$\mathbf{P}^{N-1}: \quad < X^{N(d+1)-1} > = q^d$$
$$\text{Here}: \quad < X^{N(kd+1)-1} > = q^d$$

Example: quantum cohomology

$$\mathbf{P}^{N-1}: \quad \mathbf{C}[x]/(x^N - q)$$
$$\text{Here}: \quad \mathbf{C}[x]/(x^{kN} - q)$$

Different physics

General argument:

Compact worldsheet:

To specify Higgs fields completely, need to specify what bundle they couple to.

If the gauge field $\sim L$
then Φ charge Q implies

$$\Phi \in \Gamma(L^{\otimes Q})$$

Different bundles => different zero modes
=> different anomalies => different physics

Argument for noncompact worldsheet:

Utilize the fact that in 2d,
theta angle acts as electric field.

Want Higgs fields to have charge k
at same time that instanton number is integral.

Latter is correlated to periodicity of theta angle;
can fix to desired value by adding massive charge 1,
-1 fields -- for large enough sep', can excite, and that
sets periodicity.

(J Distler, R Plesser, Aspen 2004 & hepth/0502027, 0502044, 0502053;
N Seiberg, 2010)

An example in string orbifolds:

Consider $[X/D_4]$, where D_4 acts by first projecting to $\mathbf{Z}_2 \times \mathbf{Z}_2$, letting $\mathbf{Z}_2$ center act trivially:

$$1 \longrightarrow \mathbf{Z}_2 \longrightarrow D_4 \longrightarrow \mathbf{Z}_2 \times \mathbf{Z}_2 \longrightarrow 1$$

$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$
$$\mathbf{Z}_2 \times \mathbf{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$$

Let's compute the 1-loop partition function; we'll see this is **not** the same as $[X/\mathbf{Z}_2 \times \mathbf{Z}_2]$.

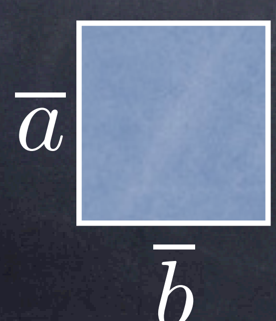
An example in string orbifolds:

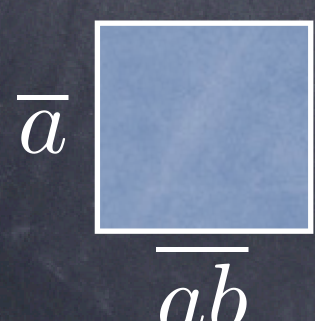
$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$

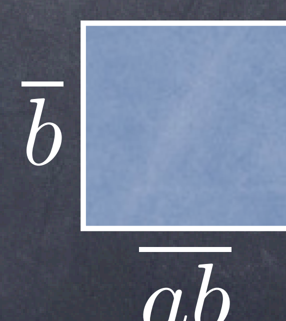
$$\mathbf{Z}_2 \times \mathbf{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$$

$$Z(D_4) = \frac{1}{|D_4|} \sum_{g,h \in D_4, gh=hg} Z_{g,h}$$

Each of the $Z_{g,h}$ twisted sectors that appears, is the same as a $\mathbf{Z}_2 \times \mathbf{Z}_2$ sector, appearing with multiplicity $|\mathbf{Z}_2|^2 = 4$ except for the







sectors.

Partition functions, cont'd

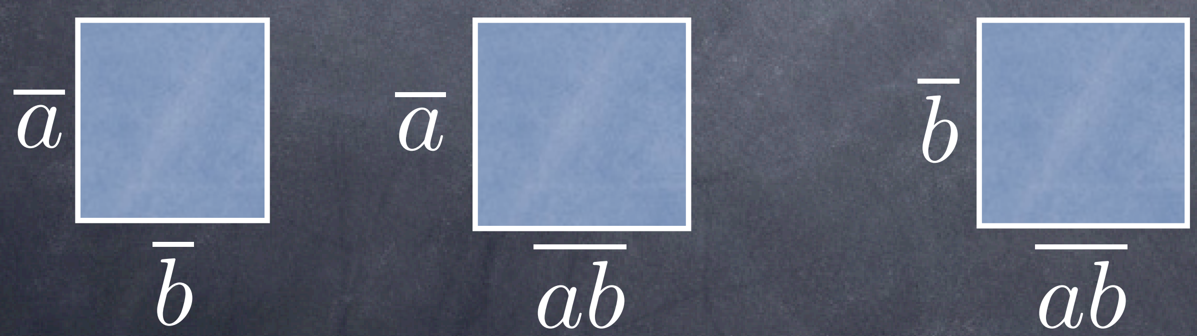
$$
\begin{aligned}
Z(D_4) &= \frac{|\mathbf{Z}_2 \times \mathbf{Z}_2|}{|D_4|} |\mathbf{Z}_2|^2 \left(Z(\mathbf{Z}_2 \times \mathbf{Z}_2) \; - \; (\text{some twisted sectors}) \right) \\
&= 2 \left(Z(\mathbf{Z}_2 \times \mathbf{Z}_2) \; - \; (\text{some twisted sectors}) \right)
\end{aligned}
$$

Note:

* not the same as $[X/\mathbf{Z}_2 \times \mathbf{Z}_2]$

(* we've restricted nonperturbative sectors)

Discrete torsion acts as a sign on the



twisted sectors

so we see that $Z([X/D_4]) \, = \, Z\left([X/\mathbf{Z}_2 \times \mathbf{Z}_2] \coprod [X/\mathbf{Z}_2 \times \mathbf{Z}_2] \right)$

with discrete torsion in one component.

Lesson:

Physics knows about even trivial group actions.

So far, only discussed 2d case.

There is a closely analogous argument in analogous four-dimensional models coupled to gravity.

Instead of theta angle,
use Reissner-Nordstrom black holes.

Idea: if all states in the theory have charge a multiple of k, then, gerbe theory is same as ordinary one.

However, if have massive minimally-charged fields, then a RN BH can Hawking radiate down to charge 1, and so can sense fields with mass > cutoff.

(J Distler, private communication)

Example: moduli spaces in string theory

Consider toroidally-compactified Spin(32)/$\mathbf{Z}_2$ heterotic string.

Low-energy theory has only adjoints,
hence all invariant under $\mathbf{Z}_2$ center of Spin(32)/$\mathbf{Z}_2$

But, there are massive states that do see the center.

-- exactly the setup just discussed.

Worldsheet realization: quantum symm' assoc to GSO

Also: such structures along subvarieties

Example in Seiberg duality:

Matt Strassler, Spin/SO duals

hepth/9507018, 9510228, 9709081, 9808073

* Spin(8) gauge theory with N_f fields in $\mathbf{8}_V$, and one massive $\mathbf{8}_S$

Seiberg dual to

* SO(N_f - 4) gauge theory with N_f vectors (from Higgsing SU(N_f-4) theory)

massive $\mathbf{8}_S$ <--> $\mathbf{Z}_2$ monopole

$$\pi_2(SU(N_f-4)/SO(N_f-4)) = \mathbf{Z}_2$$

$\mathbf{Z}_2$ center of Spin(8) acts trivially on massless matter, but nontrivially on the massive $\mathbf{8}_S$

An application: 4d N=1 sugrav

Old result of Bagger-Witten:

Metric on sugrav moduli space is quantized:
[Kahler form] = $c_1(L^2)$

Theories w/ trivial group actions can, naively, provide counterexamples....

Example: 4d analogue of susy $\mathbf{CP}^N$ model

* U(1) gauge theory
* N+1 chiral superfields charge k

(ignore the anomaly in this toy example, it plays no role)

D-terms: $$\sum_i k|\phi_i|^2 = r$$ (r integer)

which this is same as

$$\sum_i |\phi_i|^2 = r/k$$

-- looks like ordinary $\mathbf{CP}^N$ model (albeit w/ triv' $\mathbf{Z}_k$), but now with fractional Kahler class.

We'll see this is a generalization of BW, but 1st....

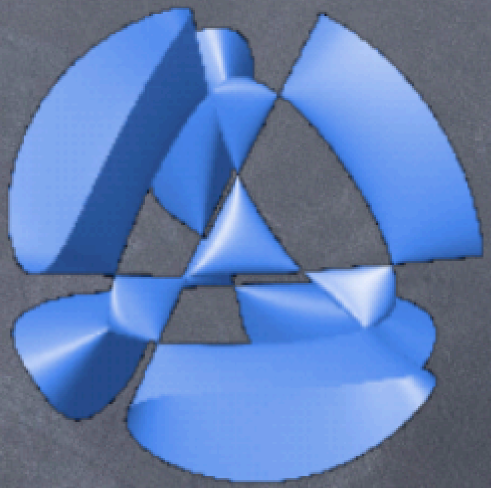
There exists a simple, unified mathematical description of these models.

Briefly: these are examples of
`stacks,'
generalizations of spaces
(hence, potential sources of new
SCFT's).

In fact, spaces w/ trivial group actions
(= NLSM's with restrictions on nonpert' sectors)
are special stacks called gerbes.

NLSM on a stack

A stack is a generalization of a space.



Idea: defined by incoming maps.

(and so nicely suited for NLSM's;
just have path integral sum over what
the def'n gives you)

Most moduli `spaces' are really stacks;
thus, to understand sugrav, need to understand stacks
as targets of 4d NLSM's.

Example: A space X as a stack

For every other space Y, associate to Y the set of continuous maps Y ---> X

Example: A quotient stack [X/G]

Maps Y ---> [X/G] are pairs

(principal G bundle (w/ connection) E on Y, G-equivariant map E --> Y)

If $Y = T^2$ & G finite,

= twisted sector maps in string orbifold

All smooth `Deligne-Mumford' stacks (over **C**)
can be described as [X/G]
for some X, some G

(G not nec' finite, not nec' effectively-acting
-- these are not all orbifolds)

Program:

A NLSM on a stack
is a G-gauged sigma model on X

Problem: such presentations not unique

If to [X/G] we associate ``G-gauged sigma model,"
then:

$[\mathbf{C}^2/\mathbf{Z}_2]$ defines a 2d CFT

=

$[X/\mathbf{C}^\times]$ defines a 2d theory w/o conformal invariance

$\left(X = \frac{\mathbf{C}^2 \times \mathbf{C}^\times}{\mathbf{Z}_2} \right)$

Try to fix with renormalization group flow

Other issues:

* Deformations of stacks $\neq$ Deformations of physical theories

* Cluster decomposition issue for gerbes
(ie, multiple gravitons in (2,2) gerbe compactifications)

Does RG flow wash out presentation-dependence, giving physics that only depends on the stack, and not on the choice of X, G?

Two dimensions: Yes

Extensive work & checks by myself, T Pantev, J Distler, S Hellerman, A Caldararu, and others in physics; extensive math literature on Gromov-Witten

Four dimensions: No

-- the stack does not determine gauge coupling

-- in low energy effective field theory, W bosons generate effects that can swamp NLSM interp'

Can associate stack to physics, but not physics to stack.

Let's consider a particularly interesting kind of stack.

Consider NLSM's in which the sum over nonperturbative sectors has been restricted; only sum over maps of degree divisible by k, say.

Equiv'ly: trivial $\mathbf{Z}_k$ action everywhere

Since stacks describe, in essence, all possible NLSM's, naturally this is a kind of stack.

Specifically, this sort of stack is known as a **gerbe**.

Decomposition conjecture

(Hellerman, Henriques, Pantev, Sharpe, etc)

For (2,2) susy worldsheet theories, we believe
SCFT(gerbe) = SCFT(disjoint union of spaces)

In special cases (`banded' gerbes),

$$\mathrm{CFT}(G - \mathrm{gerbe\ on\ } X) \;=\; \mathrm{CFT}\left(\coprod_{\hat{G}} (X, B) \right)$$

where the B field is determined by the image of

$$H^2(X, Z(G)) \stackrel{Z(G) \rightarrow U(1)}{\longrightarrow} H^2(X, U(1))$$

More gen'ly, disjoint union of **different** spaces.

Why should gerbe ~ disjoint union ?

Formally, a path integral for a NLSM with restrictions on degrees of nonperturbative sectors is of form

$$\int [D\phi] e^{iS} \underbrace{\left(\sum_{m=0}^{k-1} e^{i(m/k) \int \omega} \right)}_{\text{projection operator}}$$

$$= \sum_{m=0}^{k-1} \int [D\phi] e^{iS} e^{i(m/k) \int \omega}$$

= partition function of a disconnected union, with rotating B fields.

Example:

Consider $[X/D_4]$ where the center acts trivially.

$$1 \longrightarrow \mathbf{Z}_2 \longrightarrow D_4 \longrightarrow \mathbf{Z}_2 \times \mathbf{Z}_2 \longrightarrow 1$$

Already seen in part that

$$[X/D_4] = [X/\mathbf{Z}_2 \times \mathbf{Z}_2] \coprod [X/\mathbf{Z}_2 \times \mathbf{Z}_2]_{\rm d.t.}$$

Example:

Consider $[X/H]$ where <i> acts trivially:

$$1 \longrightarrow \langle i \rangle (= \mathbf{Z}_4) \longrightarrow H \longrightarrow \mathbf{Z}_2 \longrightarrow 1$$

Can show $[X/H] = [X/\mathbf{Z}_2] \coprod [X/\mathbf{Z}_2] \coprod X$

Gromov-Witten prediction

There is a prediction here for Gromov-Witten theory of gerbes:

GW of gerbe

should match

GW of disjoint union of spaces

Numerous checks by H-H Tseng, Y Jiang, & collab's:
0812.4477, 0905.2258, 0907.2087, 0912.3580, 1001.0435, 1004.1376, ...

GLSM's

This result can be applied to understand GLSM's.

Example: **CP**^{3}[2,2]

Superpotential: $$\sum_a p_a G_a(\phi) = \sum_{ij} \phi_i A^{ij}(p) \phi_j$$

$r \ll 0:$

* mass terms for the ϕ_i, away from locus $\{\det A = 0\}$.
* leaves just the p fields, of charge -2
* $\mathbf{Z}_2$ gerbe, hence double cover

Examples:

(A. Caldararu, J. Distler, S. Hellerman, T. Pantev, E.S., 0709.3855;
K. Hori, 1104.2853)

$\mathbf{CP}^3[2,2]$ (= T^2) ⟵ Kahler ⟶ branched double cover of $\mathbf{CP}^1$ (= T^2)

$\mathbf{CP}^7[2,2,2,2]$ ⟵ Kahler ⟶ nc res'n of branched double cover of $\mathbf{CP}^3$

Novel physical realization of geometry in GLSM's
New SCFT's (nc res'ns)
Non-birational twisted derived equivalence in gen'l
Physical realization of Kuznetsov's homological projective duality

Return to Bagger-Witten & 4d N=1 sugrav.

(S Hellerman, ES 1012.5999)

We argued that for gerby **CP**^{N}, Kahler class fract'l.

Over a gerbe, there are `fractional' line bundles.

Ex: gerbe on **CP**^{N}

$$[x_0, \cdots, x_N] \cong [\lambda^k x_0, \cdots, \lambda^k x_N]$$

Can define a line bundle L by $y \mapsto \lambda^n y$

Call it $\mathcal{O}(n/k)$

The model discussed, has Kahler form in the cohomology class of this line bundle.

Not a loophole, but a **generalization** of BW.

New heterotic CFT's

Although (2,2) models decompose into a disjoint union, (0,2) models do not seem to in general.

Prototype: $\mathcal{O}(1) \longrightarrow \mathbf{P}_{[k,k,\cdots,k]}$ "$\mathcal{O}(1/k)$"

-- understanding of some of the 2d (0,4) theories appearing in geometric Langlands program

-- genuinely new string compactifications

A lesson for the landscape:
many more string vacua may exist than previously enumerated.

Summary

* Exs of gauge theories with trivial gp actions, that physics nevertheless knows about.

* Interpretation: stacks, gerbes (generalized spaces)

* Decomposition conjecture for (2,2) worldsheets

* Applications to Gromov-Witten, GLSM's

* Generalization of Bagger-Witten (4d N=1 sugrav)

* (0,2): new SCFT's ?