Hydrodynamic and quasiballistic transport over large length scales in GaAs/AlGaAs

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We study hydrodynamic and ballistic transport regimes through nonlocal resistance measurements and high-resolution kinetic simulations in a mesoscopic structure on a high-mobility two-dimensional electron system in a GaAs/AlGaAs heterostructure. We evidence the existence of collective transport phenomena in both regimes and demonstrate that a large-scale geometry on a high-mobility system manifests a sensitivity to even weak electron-electron interactions. The combined experiments and simulations allow extraction of the momentum-conserving electron-electron scattering length, shown to be longer than predicted by microscopic theories.

Electron transport in metals is often governed by momentum dissipation from electrons to the lattice e.g. via impurity or phonon scattering. Such semiclassical diffusive Ohmic transport occurs when the momentum-relaxing (MR) electron mean-free-path \( \ell_{mr} \) (obtained from electron mobility) is the shortest length scale in the system. However, in ultraclean two-dimensional electron systems (2DESs), a departure from Ohmic transport occurs due to a long \( \ell_{mr} \), giving rise to either ballistic or hydrodynamic transport [1]. In the ballistic regime scattering mainly arises at the device boundaries, specularly or diffusively, and is delineated by the device scale \( W \sim 10 \mu m \). Yet inelastic electron-electron (e-e) interactions transfer momentum predominantly among the electrons instead of to the lattice, conserving momentum within the electron system. When such momentum-conserving (MC) scattering - characterized by MC scattering mean-free-path \( \ell_{mc} \) - dominates, electrons can move collectively like a fluid and exhibit several effects associated with fluid dynamics [1, 2, 3]. The observation of this hydrodynamic regime in electronic systems has attracted significant interest [4, 24].

The hydrodynamic regime shows a nonlocal current-voltage relation in devices, which can result in a negative nonlocal resistance (\( R_{nl} \)) [5, 10]. Such sign reversal has been exploited in recent experiments to detect the onset of the hydrodynamic regime [17, 21]. However, the ballistic regime also shows a nonlocal current-voltage relation, and can likewise produce negative \( R_{nl} \) [10, 11]. In fact, the ballistic regime also supports striking current vortices and collective motion of particles usually associated with fluid depictions [25]. In this work, we reveal notable current vortices in both hydrodynamic and ballistic regimes, uniquely supported by evidence from elaborate measurements of \( R_{nl} \). The presence of vortices in both hydrodynamic and ballistic regimes can be traced to electron momentum conservation in both regimes [10, 25]. This ballistic-hydrodynamic degeneracy is usually resolved by invoking the temperature \( T \) dependence of \( \ell_{mc} \); at sufficiently high \( T \), \( \ell_{mc} \sim 1/T^2 \) is expected to be reduced sufficiently such that the negative \( R_{nl} \) cannot be attributed to ballistic transport. This reasoning, however, is based on theoretically predicted values of \( \ell_{mc} \) which can vary depending on the theory invoked [26–31].

We present strides in experimental device design, measurement schemes, and concomitant results, as well as in one-to-one modeling of the experimental device, pivotal in understanding and demarcating the transport regimes of weak MC scattering and dominant MC scattering. We demonstrate measurements of \( R_{nl} \) in a large-scale \( \sim 30 \times 24 \mu m \) ultraclean \( \ell_{mr} \approx 65 \mu m \) at 4.2 K device, which by its scale offers exceptional sensitivity to MC scattering, and hosts 10 point contacts (PCs) to probe voltages at various distances \( \Delta x \) between the current injection point and voltage probes (Fig. 1(a)). The measurements at various \( \Delta x \) are critical to check against the predictions of ballistic or hydrodynamic models. The exceptionally long \( \ell_{mr} \), due to optimized GaAs/AlGaAs MBE growth, favors the appearance of non-Ohmic transport regimes. We interpret the experimental results using realistic high-resolution simulations of quasiparticle transport at the kinetic level, involving the actual experimental geometry in the precise contact configuration, and taking into account both MR and MC scattering. The simulations with \( \ell_{mr} \) and \( \ell_{mc} \) as inputs, determine that the device transitions from a quasiballistic regime at \( T = 4.2 \) K to a hydrodynamic regime at \( T \approx 10...15 \) K (Fig. 1(b)). The term quasiballistic conveys that the...
device geometry is so sensitive to e-e interactions that simple models of ballistic transport (such as the billiard model which omits bulk scattering) do not conform to the nonlocal measurements at even $T = 4.2$ K.

Mesoscopic geometries were patterned on a GaAs/AlGaAs heterostructure containing a 2DES with mobility $\mu$ exceeding 670 m$^2$/Vs at 4.2 K. The areal electron density is $N_S \approx 3.4 \times 10^{15}$ m$^{-2}$, corresponding to a Fermi energy $E_F \approx 11.2$ meV and $\ell_{mr} = 64.5$ $\mu$m at 4.2 K [33]. To measure $R_{nl}$, we fabricated an in-line mesoscopic geometry (Fig. 1(a)) containing 10 PCs ($a - j$) separated by $L_{\alpha\beta}$ ranging from 1.3 $\mu$m to 20.5 $\mu$m along barriers on both sides of a multiterminal Hall mesa, with sides separated by $W \approx 24$ $\mu$m. Each PC can act as a current injector $\alpha$ or voltage detector $\beta$ such that the electrons can be injected from any PC and a nonlocal voltage can be measured at any other PC. Calling $V_{nl}$ the nonlocal voltage measured at $\beta$ vs a faraway counterprobe if current $I$ is injected at $\alpha$ and drained at another faraway counterprobe (Fig. 1(a)), the 4-probe nonlocal resistance is expressed as $R_{nl} = V_{nl}/I$ and $R_{nl}$ takes the sign of $V_{nl}$. Each PC has conducting width $\sim 0.6$ $\mu$m. Measurements were performed over $4.2$ K $\leq T \leq 40$ K, using low-frequency ($\sim 44$ Hz) AC lock-in techniques without DC offsets, under small $I \sim 200$ nA to avoid electron heating. The barriers and boundaries were defined using wet etching, which results in predominantly specular boundary scattering [33, 39].

To maximize the hydrodynamic effects, we exploit the flexibility provided by the geometry, allowing testing of different configurations for current injector and drain, and for many $\Delta x$, in the same device. We use two current configurations: G1 where after injection at $\alpha$, $I$ is drained at the side of the device, and G2 where $I$ is drained at a PC at the opposite side of the device (Fig. 1(a)). The sensitivity to MC scattering turns out much higher in G1 (vicinity geometry [18]) than in G2 [33].

Transport in the device is modeled via the Boltzmann equation,

$$\frac{1}{v_F} \frac{\partial f}{\partial t} + \left( \frac{p}{m v_F} \right) \frac{\partial f}{\partial x} = -\frac{f - f_0^{mr}}{\ell_{mr}} - \frac{f - f_0^{mc}}{\ell_{mc}}$$  \hspace{1cm} (1)$$

where $f(x, p, t)$ is the electron distribution in the spatial coordinates $x \equiv (x, y)$, momentum coordinates $p \equiv (p_x, p_y)$, and time $t$. While long-range electric fields are not explicitly present in Eq. (1), they are included at linear order as the gradient of the electrochemical potential $\mu$. The left side (with $v_F$ the Fermi velocity and $m$ the effective mass) describes free advection, and the right side thermalization due to MR and MC scattering in a relaxation time approximation with $f_0^{mr}$ and $f_0^{mc}$ the local stationary and drifting Fermi-Dirac distributions. The model inputs are $\ell_{mc}$ (a free parameter) and $\ell_{mr}$ (fixed by $\mu$). We consider dynamics at the Fermi surface without thermal smearing so that $p = m v_F \zeta$, and solve for transport in the zero-frequency limit ($\partial/\partial t \rightarrow 0$); $v_F$ then factors out, leaving the circular Fermi contour as the only relevant detail. We solve Eq. (1) in the precise experimental geometry using BOLT [40], a high-resolution solver for kinetic theories. The overall prefactor of the numerical solutions is set by calibrating against the measurements in G1(ii) for each $T$ [33].

The experimental $R_{nl}$ vs $T$ for G1 and G2 are depicted in Fig. 2(a) and (b) respectively, for the specific $L_{\alpha\beta}$ used for measurements (Fig. 1(a) clarifies $L_{ac}$ as example). Two inferences appear: the negative $R_{nl}$ attests to a departure from Ohmic transport, and a striking contrast exists in $T$ dependences of G1 and G2. For G1, $R_{nl}$ shows a non-monotonic dependence on $T$, initially decreasing as $T$ increases, crossing over to negative values in a particular range of $T$ for given $L_{\alpha\beta}$, then increasing to positive values. For G2, $R_{nl}$ increases from
negative values at low \( T \) to positive values at higher \( T \). The difference in \( T \) dependence between G1 and G2 indicates that the current injector-drain configuration significantly affects transport. Figure 2(a) can be directly compared with similar results in graphene [17] and other GaAs/AlGaAs experiments [18].

In Fig. 2(a) for \( T \lesssim 10 \text{ K} \), G1 shows negative \( R_{nl} \) for small \( \Delta x \lesssim 2.6 \mu m \) (\( L_{dh}, L_{ij} \)), crossing over to positive \( R_{nl} \) for \( \Delta x \gtrsim 5 \mu m \) (\( L_{bd}, L_{ji}, \ldots \)). As \( T \) is increased, \( R_{nl} \) becomes negative for all \( \Delta x \) before finally crossing over to positive values. The observations at \( L_{ij} \) are a consequence of the interplay between \( \ell_{mc}(T) \), \( \ell_{mr}(T) \) and geometry, and present a means of bracketing values for \( \ell_{mr}(T) \) using the model. In Fig. 3(a-b) we find the values of \( \ell_{mc} \) (given \( \ell_{mr} \)) for which the modeled \( R_{nl} \) match the experiments. We first focus on \( T \lesssim 10 \text{ K} \), characterized by the distinct crossover vs \( \Delta x \) from negative \( R_{nl} \) at small \( \Delta x \) to positive further away. In Fig. 3(a) we start at \( T = 4.2 \text{ K} \) with the limiting billiard model with \( \ell_{mc}, \ell_{mr} \rightarrow \infty \). This limiting model common in ballistic transport, results in a positive \( R_{nl} \) for all the PCs and does not capture the crossover vs \( \Delta x \) (inset Fig. 3(a)); yet positive \( R_{nl} \) is not universal in the billiard model and can be heavily influenced by geometry [33]. Considering finite MR scattering with the experimental \( \ell_{mc} = 64.5 \mu m \) at 4.2 K and zero MC scattering with \( \ell_{mc} \rightarrow \infty \), the modeled \( R_{nl} \) are significantly lowered compared to the billiard model but still do not show negative \( R_{nl} \) at small \( \Delta x \) (Fig. 3(a)). Only finite MC scattering, in a range \( \ell_{mc} \approx 60 - 300 \mu m \) (\( \gg W = 24 \mu m \)), yields a crossover from negative to positive \( R_{nl} \) with increasing \( \Delta x \) (Fig. 3(a)).

The range of \( \ell_{mc} \) consistent with the data and model in Fig. 3(a) exceeds values from a commonly used theoretical expression for the quantum lifetime [26] which yields \( \ell_{mc} \approx 15 \mu m \) at 4.2 K, at which \( R_{nl} < 0 \) throughout the device. Longer \( \ell_{mc} \) could result from dielectric screening [32]. Still, even for \( T \lesssim 10 \text{ K} \) (long \( \ell_{mc} \)) the fully ballistic billiard model does not apply in the large-scale device, and finite \( \ell_{mr} \) is required. For \( T \lesssim 10 \text{ K} \) we thus speak of a quasiballistic regime in G1, where the effect of finite \( \ell_{mc} \) due to e-e interactions, although perturbative, is not negligible. Tellingly, the negative \( R_{nl} \) for \( \Delta x \lesssim 2.6 \mu m \) in Fig. 3(a) cannot be reproduced in Fig. 3(a) unless for finite \( \ell_{mc} \). And the presence of numerous current vortices of various sizes in the quasiballistic regime (Fig. 3(b) at 4.2 K), defies both fully ballistic and hydrodynamic perceptions (evidently, the dominance of MC scattering is not required for the formation of vortices).

The simulations reveal that nonlocal resistances in the G2 configuration are degenerate, supported by Fig. 2(b): \( R_{nl} < 0 \) occurs in the fully ballistic limit, the quasiballistic and the hydrodynamic regimes. Ref. [33] also shows that G2 is insensitive to MC scattering. Both properties disallow us from using G2 to determine \( \ell_{mc} \) and we therefore do not consider G2 further.

As \( T \) crosses \( \sim 10 \text{ K} \) in G1 (Fig. 2(a)), \( R_{nl} \) becomes negative for all \( L_{ij} \), with its magnitude decreasing with increasing \( \ell_{mr} \). Modeling using \( \ell_{mc} \approx 1.5-5 \mu m \) accommodates all the experimental data at \( T = 13 \text{ K} \) (Fig. 3(b)). Since \( \ell_{mc} \ll W \) is now the shortest length scale, and \( \ell_{mr} = 30.5 \mu m \gtrsim W \) is sufficiently long, the system is in the hydrodynamic regime. The range of extracted \( \ell_{mc} \) in the hydrodynamic regime does not disallow the value \( \approx 2.5 \mu m \) predicted by theory [26]. This suggests that an opportunity to test microscopic theories of \( \ell_{mc} \) lies in using a device scale in the quasiballistic regime where \( \ell_{mc} \sim W \). As expected of a fluid, Fig. 4(b) (13 K) exhibits current vortices, but with a distinct pattern compared to quasiballistic vortices: in the hydrodynamic regime only a single vortex (dashed box in Fig. 4(b)) inhabits the main chamber, obtainable from just the Navier-Stokes fluid momentum conserva-

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**FIG. 2.** Experimental nonlocal resistance \( R_{nl} \) vs \( T \) for each \( L_{ij} \) for (a) G1 and (b) G2. The dotted lines indicate \( R_{nl} = 0 \), with negative (positive) regions of \( R_{nl} \) shaded in blue (red).
FIG. 3. Modeled $R_{nl}$ for G1, plotted vs location $x$ along the barrier into which the injection PC (blue vertical bar) is placed, shown for variable $\ell_{mc}$ in G1 at (a) $T = 4.2$ K where $\ell_{mr} = 64.5$ $\mu$m and (b) $T = 13$ K where $\ell_{mr} = 30.5$ $\mu$m. The grey vertical bars represent locations of detector PCs. Inset in (a) shows $R_{nl}$ for $(\ell_{mc}, \ell_{mr} \to \infty, \infty)$. Experimental $R_{nl}$ (black dots) for G1(ii) are chosen for reference calibration [33] given the clear crossover in $R_{nl}$ in G1(ii). Schematics of subconfigurations are also depicted.

In conclusion, non-Ohmic transport, either predominantly ballistic or hydrodynamic, is realized over a wide temperature range in a large-scale GaAs/AlGaAs geometry. The appearance of both predominantly ballistic or hydrodynamic regimes at such a large scale, despite opposite required limits of the strength of MC scattering, is striking. Equally remarkable are their shared characteristics of negative nonlocal resistances and current vortices. The nonlocal resistance in both regimes can be tuned by device and contact geometry, used here to disentangle the regimes and to obtain a measure of the MC scattering length. While the importance of geometry in the ballistic regime is well-known, we additionally find that even weak electron-electron interactions can qualitatively affect measurements in an ultraclean device, whose large dimensions render it highly sensitive to MC scattering.

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Supplemental Material to “Hydrodynamic and quasiballistic transport over large length scales in GaAs/AlGaAs”

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DEVICE FABRICATION AND MATERIALS PROPERTIES

The devices were fabricated from GaAs/AlGaAs MBE-grown material (Fig. 1(a) main text). The GaAs quantum well hosting the two-dimensional electron system (2DES) is located 190 nm below the surface, has a width of 26 nm, and is top- and bottom-doped by Si δ-layers 80 nm removed from the quantum well and embedded in Al0.32Ga0.68As barriers. Optimization of heterostructure design is described in Ref. [1]. The mesoscopic geometries were patterned by electron beam lithography and wet etching of the barriers, using PMMA as the etching mask. The geometries feature mesoscopic apertures (point contacts, PCs) separated by various distances $L_{\alpha\beta}$ on both sides of a Hall mesa. Different $\alpha$ (current injector) and $\beta$ (voltage detector) are chosen such that a wide range of distances $L_{\alpha\beta}$ exists, from 1.3 $\mu$m to 20.5 $\mu$m. The PC resistance $R_{pc}$ varies between 450 $\Omega$ to 750 $\Omega$ at $T = 4.2$ K, depending on the PC.

The electron transport properties of the unpatterned 2DES were characterized by the van der Pauw method. We use the value of 2D resistivity $R_{\Box}$ from this method, and areal electron density $N_S$ from Hall measurements on the fabricated device to obtain electron mobility $\mu$. At temperature $T = 4.2$ K, we obtain $N_S = 3.4 \times 10^{15}$ m$^{-2}$ and $R_{\Box} = 2.74$ $\Omega/\Box$, yielding $\mu = 670$ m$^2$/s (confirming the quality of the material), Drude (mobility) mean-free-path $\ell_{mr} = 64.5$ $\mu$m and Fermi energy $E_F = 11.2$ meV (equivalent to $\sim 130$ K). Here, $\ell_{mr} = v_F T_{mr}$, with $v_F$ the Fermi velocity and $T_{mr}$ the Drude momentum relaxation time defined from $\mu = e \tau_{mr}/m$, where $m$ denotes the electron effective mass (0.067 $m_e$ with $m_e$ the free-electron mass) and $e$ the electron charge. The momentum-relaxing (MR) electron mean-free-path $\ell_{mr}$ describes momentum dissipation from electrons to the lattice e.g. via impurity or phonon scattering. In contrast the momentum-conserving (MC) mean-free-path $\ell_{mc}$ describes transfer of momentum internal to the 2DES between electrons via electron-electron scattering, conserving momentum within the 2DES. Since MC scattering merely causes a redistribution of momentum internally to the electron fluid, and since $\mu$ quantifies the loss of total momentum of the electron fluid to the lattice, $\mu$ cannot provide a measure of $\ell_{mc}$.

Non-parabolicity of the band structure is accounted for in calculating the transport properties [2, 3]. $N_S$ (Fig. S1(a)) and $R_{\Box}$ (Fig. S1(b)) are observed to increase with increasing $T$, while $\mu \sim 1/T$ (Fig. S1(c)), demonstrating that as expected $\mu$ is limited by scattering with acoustic phonons. However we observe a slight change in slope of $\mu$ vs $T$ for $T \approx 13$ K (inset in Fig. S1(c)), attributed to incipient scattering by LO phonons for $T > 13$ K. In a high-$\mu$ 2DES like the present, LO phonon scattering can start to be observed even at $T \approx 13$ K since the lack of residual scattering does not mask their effect, while in a 2DESs of lower $\mu$ the contribution to scattering by optical phonons is only apparent at higher $T$. As a result, the dependence on $T$ of $\mu$ is affected slightly. Figure S1(c) also shows that $\ell_{mr} \sim 1/T$, again demonstrating the expected dominance of acoustic phonons in limiting $\ell_{mr}$. The fact that $\ell_{mr}$ and $\mu$ follow closely analogous dependences on $T$, highlights that $\mu$ can provide a measure of $\ell_{mr}$ but not of $\ell_{mc}$. Figure S1(d) depicts $1/\mu$ vs $T$, indicating that a relation $1/\mu(T) = 1/\mu_0 + \alpha T$ is closely followed, where $\mu_0$ denotes $\mu$ limited by impurity scattering and $\alpha T$ describes the linear dependence on $T$ due to (predominantly) acoustic phonons where $\alpha$ is a proportionality constant [4]. The exponent of $T$ changes from 1 to 1.2 at $T > 13$ K, but since the deviation due to optical phonons is small, the dependence due to acoustic phonons is a good approximation.

The Fermi wavelength $\lambda_F = 43$ nm. Since the conducting PC width $w \approx 0.6$ $\mu$m, it is expected that $w/(\lambda_F/2) \approx 28$ spin-degenerate transverse modes contribute to transport. This number of conducting spin-degenerate modes would yield $R_{pc} \approx (\hbar/2e^2)/28 = 461$ $\Omega$, in good agreement with the measured $R_{pc}$ above. The large number of modes implies that transport through the PCs is classical.
CALIBRATION BETWEEN EXPERIMENTS AND MODELING

The solution of the kinetic model (Eq. 1 main text) is the non-equilibrium distribution function \( f(x,p) \). To obtain voltages, we would further need to solve the Poisson equation sourced by the 2D density \( N_s + \delta N_s(x) \sim \int f(x,p)d^2p \), while taking into account the full 3D electrostatic environment. We forgo this complexity, and instead directly calibrate the spatial density variation, \( \delta N_s(x) \) obtained from the model, to the measured nonlocal voltages \( V_{nl}(x) \) in one configuration (chosen to be G1(ii)), but can be any of the four configurations (Fig. 1(a) main text)). The measurements yield the electrochemical potential at given \( x \). Since the density of states is constant over energy in a 2DES, the change in electrochemical potential at \( x \) is directly proportional to the density variation at \( x \). This results in the local capacitance approximation, where \( V_{nl}(x) \propto \delta N_s(x) \), with a constant prefactor determined by the electrostatic environment. We thus have nonlocal resistance \( R_{nl}(x) = A\delta N_s(x) \), where \( A \) denotes the calibration prefactor.

Since \( |\delta N_s(x)| << N_s \) and hence \( |eV_{nl}(x)| << E_F \), the measurements are performed in a linear regime. This also holds for locations near PCs since \( |eR_{pc}I| << E_F \) indicating that close to the current injection PCs, the electrons have energies very close to \( E_F \).

To perform the calibration, we need to find \((A,\ell_{mc})\) such that \( R_{nl} \) obtained from simulations in G1(ii) match experimental values. Given freedom to set \( A \), we find that a match occurs only for a specific value of \( \ell_{mc} \), since the solution should agree with the measured \( R_{nl} \) at all PCs. The allowed range can be efficiently bisected by first matching against the sign of \( R_{nl}(\text{cfr G1(ii)}) \) in Fig. 3 main text). The calibration therefore results in a pair of unique \((A,\ell_{mc})\). After \( A \) has been set, the range of \( \ell_{mc} \) required for the modeling to match experiments in the other configurations represents an error in the method. Finally, we note that we had to perform the calibration procedure separately for each \( T \), i.e. \( A \equiv A(T) \). The reason for this dependence on \( T \) is left for future work.

The low-frequency (\( \sim 44 \) Hz) AC lock-in measurement, performed without DC offset, tracks the signs of \( \delta N_s(x) \) and \( R_{nl}(x) \) such that for \( \delta N_s(x) > 0 \) (overdensity of...
electrons at \( x \), we measure \( R_{nl}(x) > 0 \) and for \( \delta N_s(x) < 0 \) (underdensity of electrons at \( x \)), we measure \( R_{nl}(x) < 0 \).

EFFECT OF DEVICE GEOMETRY AND CONTACT CONFIGURATION ON THE BALLISTIC REGIME

![Figure S2](image-url)

In the experimental geometry, a surprising behavior is encountered in the interplay between MR and MC scattering in the presence of stronger MC scattering \((\ell_{mc} \lesssim W)\) at higher \( T \): as MC scattering increases with increasing \( T \), the susceptibility to the effects of MR scattering increases. As depicted in Fig. S4, even as \( \ell_{mc} \) is the shortest length scale, \( \ell_{mr} \) has a dominant effect on \( R_{nl} \) for \( \Delta x \gtrsim 3 \mu m \) \((\ell_{mr} = 14.3 \mu m \text{, corresponding to } T = 28 K)\). This behavior can be understood by considering the change in current vortex pattern as we fix \( \ell_{mr} \) and decrease \( \ell_{mc} \). Figure S5 reveals that for fixed model. As depicted in Fig. S2(b), for the simple rectangular geometry in Fig. S2(a), \( R_{nl} < 0 \) for distances \( \Delta x < 10 \mu m \) between current injection and voltage detection PCs, beyond which \( R_{nl} \) varies between positive and negative values. On the other hand, \( R_{nl} \) in the experimental geometry in Fig. S2(a-b) remains positive throughout \( \Delta x \). The reason for \( R_{nl} > 0 \) becomes clear on examining the carrier density distribution from the modeling: the slanting edges reflect and focus carriers back into the main chamber and increase the density relative to the grounded side ports, giving rise to \( R_{nl} > 0 \) throughout \( \Delta x \).

In our experiment, \( R_{nl} > 0 \) for the ballistic regime is advantageous because it allows us to experimentally differentiate the ballistic from the hydrodynamic regime for which \( R_{nl} < 0 \). The ballistic response also depends on the contact configuration, even within the same overall device geometry. On changing the contact configuration to G2, \( R_{nl} < 0 \) prevails at the location of almost all the PCs, becoming undifferentiated from the hydrodynamic regime (Fig.2(b) main text). Therefore, interpreting the experimental data requires taking into account the precise device geometry as well as the contact configuration.

EFFECT OF CONTACT CONFIGURATION ON SENSITIVITY TO \( \ell_{mc} \)

For a fixed device geometry the contact configuration can strongly affect the sensitivity of the device to MC scattering (Fig. S3). We find that the G2 configuration is markedly insensitive to changes in \( \ell_{mc} \), as illustrated in Fig. S3(b) by the modeled current streamline and voltage contour plots as \( \ell_{mc} \) is varied. Furthermore, \( R_{nl} < 0 \) in G2 for both ballistic and hydrodynamic regimes. G2 is therefore ineffective to probe hydrodynamic transport. In contrast, G1 shows distinctly higher sensitivity to \( \ell_{mc} \), as depicted in Fig. S3(a). For the given device geometry, G1 allows a differentiation between ballistic and hydrodynamic transport.

EFFECT OF NON-PERTURBATIVE MR SCATTERING AT HIGHER \( T \)

In the experimental geometry, a surprising behavior is encountered in the interplay between MR and MC scattering (\( \ell_{mc} \lesssim W \)) at higher \( T \): as MC scattering increases with increasing \( T \), the susceptibility to the effects of MR scattering increases. As depicted in Fig. S4, even as \( \ell_{mc} \) is the shortest length scale, \( \ell_{mr} \) has a dominant effect on \( R_{nl} \) for \( \Delta x \gtrsim 3 \mu m \) \((\ell_{mr} = 14.3 \mu m \text{, corresponding to } T = 28 K)\). This behavior can be understood by considering the change in current vortex pattern as we fix \( \ell_{mr} \) and decrease \( \ell_{mc} \). Figure S5 reveals that for fixed...
Fig. S3. Current streamline and voltage contour plots obtained from simulations for (a) G1(i) and (b) G2(i) as $\ell_{mc}$ is changed from 15 $\mu$m to 150 $\mu$m at $T = 4.2$ K where $\ell_{mr} = 64.5$ $\mu$m. G1 (i) shows higher sensitivity to $\ell_{mc}$, with a single large current vortex at $\ell_{mc} = 15$ $\mu$m breaking down to multiple smaller vortices at $\ell_{mc} = 150$ $\mu$m. In G2 (i), the vortex pattern does not significantly change under even the 10× change in $\ell_{mc}$, indicating the insensitivity of the G2 configuration to $\ell_{mc}$.

Fig. S4. Modeled $R_{nl}$ for G1 with $\ell_{mr} = 14.3$ $\mu$m (corresponding to $T = 28$ K) for $\ell_{mc} = 1 - 2$ $\mu$m and $\ell_{mc} \to \infty$, plotted vs location $x$ along the barrier into which the injection PC (blue vertical bar) is placed, along with the experimental data points (black dots). The grey vertical bars represent locations of detector PCs. At low $\ell_{mc}$ (higher $T$), $\ell_{mr}$ exerts a dominant effect on $R_{nl}$ for $\Delta x \gtrsim 3$ $\mu$m, where the curves for $\ell_{mc} = 1 - 2$ $\mu$m and $\ell_{mc} \to \infty$ lie within $\sim 1$ Ohm. Determination of $\ell_{mc}$ from such measurements is difficult, unless measurements are chosen close to injector PC(G1(ii) and G1(iii)) where $R_{nl} \lesssim 0$.

Fig. S5. Modeled current streamline and voltage contour plots for G1(iii) at fixed $\ell_{mr} = 64.5$ $\mu$m for $\ell_{mc}$ decreasing panel to panel, top to bottom. The sequence shows the disappearance of the vortex (dotted red box) with decreasing $\ell_{mc}$ for a finite $\ell_{mr}$. 
\[ \ell_{mr} = 64.5 \, \mu\text{m} \] the large device-scale current vortex increasingly localizes as \( \ell_{mc} \) decreases, before finally disappearing when \( \ell_{mc} \) decreases below a threshold. No current vortex or negative \( R_{nl} \) appear in the geometry when MC scattering dominates, and the current streamlines then resemble those of a diffusive Ohmic regime. Conversely, the hydrodynamic regime is more robust against MR scattering for sufficiently long \( \ell_{mc} \) [5].


