Radio Astronomy Fundamentals I
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Spring 2012

Radio astronomy provides a very different view of the universe than optical astronomy. Radio astronomers and optical astronomers use different terminology to describe their work. Here I present some basic concepts and terms of radio astronomy.

Radio astronomers talk about **sources** of radio emission. Cas A is a strong source, for example. Other strong sources are Cyg A and Tau A. The Sun is also a strong source. The **flux density** of a source (often denoted as S_ν) is a measure of the strength of a source. The flux density is the energy arriving at Earth per second (W), per unit area perpendicular to the incoming radiation (m^2), per unit frequency interval (Hz), at the frequency of the observation (ν). So the units of flux density are W m^{-2} Hz^{-1}. Since radio flux densities are typically small, the commonly used unit for flux density is the **Jansky**, where 1 Jansky = 1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

Radio astronomers often describe sources as if they were blackbody radiators, even if they are not: they specify the emission of a source in terms of a temperature, called the **brightness temperature**. If the radio source is an actually blackbody radiator (e.g., the Moon, the Earth, you), then the brightness temperature equals the actual temperature of the source (in your case, about 300K). If the source is not a blackbody, brightness temperature is still used and it equals the temperature of a blackbody that would emit the same intensity of radiation as the source, at the observed radio frequency.

Radio telescopes are **antennas**. An antenna has an **effective collecting area** for receiving incoming radiation energy, and this area depends on the direction of the incoming radiation. In other words, point a radio telescope (e.g., a **single dish radio telescope**) at a source, and it will present the greatest effective collecting area to the incoming radiation from that source. But the collecting area will be smaller for off-axis sources, i.e., sources in directions not along the symmetry axis of the dish. The range of off-axis angles for which the antenna is reasonably sensitive is often specified by the **half-power beamwidth** (**HPBW**), Θ_{HPBW}. The **beam** can be thought of as the cone of "light" that would be produced by the telescope if it were used as a transmitter (e.g., in radar). The actual effective collecting area A_{eff} for a **point source** observed on-axis is not equal to the full geometric collecting area A_{geo} (= πr^2) of the dish. The fraction $\eta_A = A_{eff}/A_{geo}$ is called the **aperture efficiency**.

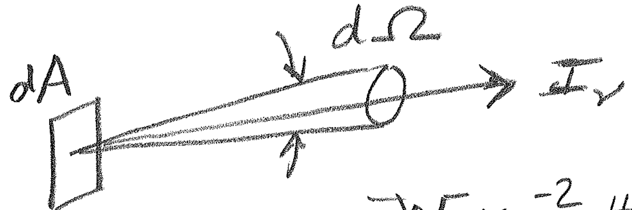
Continuing the discussion of sources in terms of temperatures, observations by radio telescopes are often quoted in terms of **antenna temperature,** T_A. A radio telescope can pick up radiation from all directions, but it is most sensitive to radiation coming within the beam of the telescope. The measured antenna temperature is the weighted average of the temperature of all sources in all directions, where the weighting is the effective collecting area of the telescope --- a function which depends on the direction of the source. So, the antenna temperature is dominated by the temperature of sources in the beam of the telescope; the antenna temperature is weakly dependent on sources in other directions.

If a radio telescope is placed in a blackbody container --- an oven at temperature T --- then the measured antenna temperature would be T. The universe is an oven of temperature T=3K, so if there were no other radio sources in the universe, the measured antenna temperature for any observation would be T_A=3K. But, now assume there is a source of radiation of brightness temperature T_b >> 3K filling the beam of the telescope. "Filling the beam" means that the solid angle of the source Ω_{sou} approximately equals the solid angle of the antenna beam Ω_{HPBW}. Then, the measured antenna temperature would be dominated by T_b, but not quite equal to T_b since the telescope is also receiving radiation of temperature 3K from all other directions. Thus the measured antenna temperature $T_A = \eta_B T_b$ where η_B is called the **beam efficiency** and would be 1 if the telescope was only sensitive to directions within the beam. In any practical situation, η_B is less than 1. If a source does not fill the beam, then $T_A \approx \eta_B\, T_b\, \Omega_{sou}/\Omega_{HPBW}$, where Ω_{sou} and Ω_{HPBW} are the solid angles of the source and the antenna beam, and $\Omega_{HPBW} \approx \pi(\Theta_{HPBW}/2)^2$.

The following pages give the physical and mathematical details of these concepts.

EM Radiation

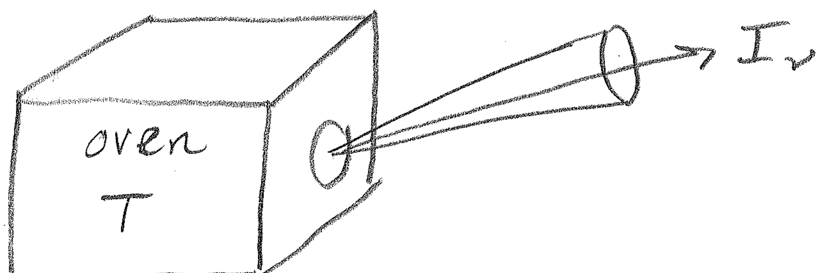
I_ν = "specific Intensity"



$W\, m^{-2}\, Hz^{-1}\, sr^{-1}$

$E\, dA^{-1}\, d\nu^{-1}\, d\Omega^{-1}\, dt^{-1}$

Blackbody Radiation



$$I_\nu\, d\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}\, d\nu$$

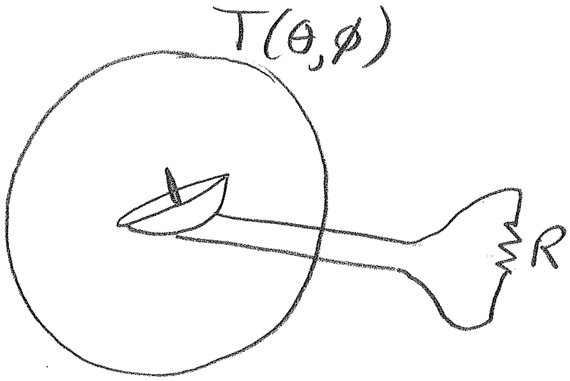
"Planck function"

For $h\nu \ll kT$ (typically true for radio)

$$I_\nu\, d\nu = \frac{2kT}{\lambda^2}\, d\nu$$

"Rayleigh-Jeans law"

Antenna placed in a Universe



For $T(\theta,\phi) = T =$ constant

Resistor R $\rightarrow$ temperature T

Johnson, Nyquist (1920's): power dissipated in R, due to Thermal noise is

$$P_R = kTB$$

where B = bandwidth of frequencies of e^- oscillations.

"Johnson noise"

Antenna must be receiving/transmitting same power, for equilibrium.

$$P_{received} = P_R = kTB$$

where T = temperature of Universe.

"Antenna Temperature" T_A

$$T_A \equiv \frac{P_R}{kB}$$

$T(\theta, \phi)$ not constant over sphere

$$P_R = \int_{4\pi} d\Omega \ \frac{1}{2} \ I_\nu(\theta, \phi) \, B \, A_{eff}(\theta, \phi)$$

where

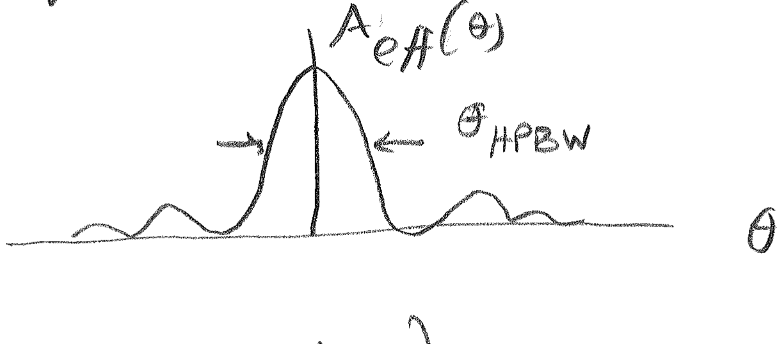
I_ν = intensity at θ, ϕ $\quad W \, m^{-2} \, Hz^{-1} \, sr^{-1}$

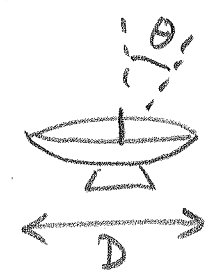
B = bandwidth of receiver/antenna

"$\frac{1}{2}$" present since only 1 polarization can be received by a real antenna

$A_{eff}(\theta, \phi)$ = effective collecting area of antenna, to incoming radiation. Depends on geometry of antenna and diffraction, etc.

For typical, single-dish antenna





$$\theta_{HPBW} \simeq 1.2 \, \frac{\lambda}{D}$$

[$\theta_{HPBW} = \frac{\lambda}{D}$ for "uniformly illuminated" circular aperture, but radio dishes typically not uniformly illuminated]

$$P_R = \int_{4\pi} d\Omega \; \frac{1}{2} I_\nu(\theta,\phi)\, B\, A_{eff}(\theta,\phi)$$

$$\uparrow \; P_R = k T_A B$$

In radio astronomy: $I_\nu = \dfrac{2kT_b}{\lambda^2}$, where

T_b = "brightness temperature" ($\equiv$ actual T of "source" if source is a blackbody)

For any source

$$S_\nu = \int d\Omega \; I_\nu(\theta,\phi) \simeq I_\nu \, \Omega_{source}$$

$$= \text{"flux density"} \quad W\,m^{-2}\,Hz^{-1}$$

$$10^{-26}\ W\,m^{-2}\,Hz^{-1} = 1 \text{ Jansky} = 1 \text{ Jy}$$

a) Source is very small on sky

$$\theta_{source} << \theta_{HPBW}, \quad \Omega_{source} << \Omega_{HPBW}$$

$\boxed{\text{" point source "}}$

$$kT_A B = \frac{1}{2} S_\nu A_{eff}(0,0) B$$

$$\frac{T_A}{S_\nu} = \frac{A_{eff}(0,0)}{2k} = \frac{\eta_A A_{geometric}}{2k}$$

$$= \text{" sensitivity " } K/Jy$$

$$\eta_A = \text{" aperture efficiency "}$$

$$T_A = \eta_A A_{geo} \frac{S_\nu}{2k}$$

Actual result is:
T_A = eta_A A_geo S_nu / 2k

b) Source covers whole sky

$$T_A = T_b$$ ...as on page 2

c) General story

$$T_b = T_b(\theta, \phi)$$

$$P_R = B \int_{4\pi} d\Omega \, \frac{1}{2} I_\nu(\theta,\phi) \, A_{eff}(\theta,\phi)$$

$$= B \int_{4\pi} d\Omega \, \frac{1}{2} \, \frac{2kT_b(\theta,\phi)}{\lambda^2} \, A_{eff}(\theta,\phi)$$

$$kT_A B = kB \int_{4\pi} d\Omega \, T_b(\theta,\phi) \, \frac{A_{eff}(\theta,\phi)}{\lambda^2}$$

[For $T_b(\theta,\phi) = \text{constant}$, then $T_A = T_b$, so it must be that $\int_{4\pi} A_{eff}(\theta,\phi) = \lambda^2$]

$$\therefore \quad T_A = \frac{\int_{4\pi} d\Omega \, T_b(\theta,\phi) \, A_{eff}(\theta,\phi)}{\int_{4\pi} A_{eff}(\theta,\phi) \, d\Omega}$$

$\therefore$ T_A = weighted average of $T_b(\theta,\phi)$ over sky; weighting by effective collecting area of antenna, which is function of (θ,ϕ).

d) "Extended source" (ie, not a point source; not covering all of sky).

Let's say $\Omega_{source} < \Omega_{HPBW}$

$$kT_A B = kB \int_{4\pi} d\Omega \, T_b(\theta,\phi) \frac{A_{eff}(\theta,\phi)}{\lambda^2}$$

assume $T_b(\theta,\phi) \simeq$ constant across source

Take $A_{eff}(\theta,\phi) \simeq A_{eff}(0,0)$ across source

$$T_A = T_b \, \Omega_{source} \frac{A_{eff}(0,0)}{\lambda^2}$$

$$= T_b \, \Omega_{source} \frac{A_{eff}(0,0)}{\int_{4\pi} A_{eff}(\theta,\phi) \, d\Omega}$$

$$= T_b \, \Omega_{source} \frac{A_{eff}(0,0) \, \Omega_{HPBW}}{\int_{4\pi} A_{eff}(\theta,\phi) \, d\Omega} \frac{1}{\Omega_{HPBW}}$$

$$\simeq T_b \, \Omega_{source} \underbrace{\left(\frac{\int_{beam} A_{eff}(\theta,\phi) \, d\Omega}{\int_{4\pi} A_{eff}(\theta,\phi) \, d\Omega} \right)}_{\eta_B = \text{"beam efficiency"}} \frac{1}{\Omega_{HPBW}}$$

$$\boxed{T_A = T_b \, \eta_B \frac{\Omega_{source}}{\Omega_{HPBW}}}$$

for extended source $\theta_{source} \lesssim \theta_{HPBW}$