

Measuring the Signal-to-Noise Ratio S/N of the CCD Image of a Star or Nebula

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Measuring the total flux of light collected from a star is of fundamental importance to astronomy. Equally important is the measurement of the light arriving at one spot in the image of a nebula. In a CCD image such measurements are made by counting the number of electrons released by the incident photons. This number of electrons is a “signal” S . Any measurement has some inherent error, or uncertainty, which is called the “noise” N . A large signal-to-noise ratio means we can be very confident of the measured value of the signal. This handout explains how to estimate S/N .

Electron Counting Statistics

To estimate S/N one must first understand a few facts about the statistical nature of the counting process inherent in CCD images. All electrons released by incident photons (from the object of interest, or from the “sky” along the line of sight) obey Poisson statistics. All electrons released by other processes (e.g., “dark current”) in the CCD chip also obey Poisson statistics. This means that if a large set of measurements is performed, the variation in the number of released electrons (rms variation) would be equal to the square-root of the typical (the mean) number of electrons released. Therefore, the uncertainty assigned to any contribution to the electron count is set equal to the square-root of that contribution to the count. This fact simplifies much of our work.

These Poisson statistics apply only to the released number of electrons, not to pixel values displayed in an image. A pixel value (said to be in units of “counts” or “ADU” for Analog to Digital Units) is equal to the number of electrons detected in that pixel, divided by the gain G of the CCD, where G is in units of electrons/ADU. (For the SBIG ST-8E, the gain is $G = 2.3 \text{ e}^-/\text{ADU}$. Strictly speaking, one should first subtract 100 from any pixel value in an SBIG image before converting to electrons, since an artificial value of 100 has been added to each pixel value for every displayed or stored SBIG image. If one works with differences in pixel values, as in our final equations, or with rms values, this artificial 100 will be automatically removed in the calculation.)

The error for the number of electrons collected in one pixel is independent of the error for any other pixel in the image. This means that if a set of n pixels each has the same uncertainty, the sum of the electrons in those n pixels will have an uncertainty larger than the uncertainty for one pixel by a factor of \sqrt{n} ; the average over n pixels will have an uncertainty equal to that of one pixel divided by \sqrt{n} . These results come from a standard “propagation of errors” calculation: the squared uncertainty for the sum (or difference) of a set of measurements is equal to the sum of the squares of the uncertainties of the individual measurements.

Estimating S/N of a Star in a CCD Image

If we image an extremely bright star so the number of electrons S produced by its photons is very large compared to the number of electrons produced by any other process (e.g., large compared to the electrons produced by photons from the “sky” along that line of sight), then due to the nature of Poisson statistics, we would have

$$\frac{S}{N} \approx \frac{S}{\sqrt{S}} \approx \sqrt{S}.$$

In terms of pixel values (in ADU or counts), where C is the sum over all the pixels containing counts due to incident photons from the star, this result could also be written as

$$\frac{S}{N} \approx \frac{GC}{\sqrt{GC}} \approx \sqrt{GC},$$

where G is the gain of the CCD. This last result is an approximate upper bound which can serve as a rough ballpark estimate of S/N when the star (or some region in a nebula) is extremely bright. Note that S/N

could be rewritten as

$$\frac{S}{N} \approx \frac{C}{\sqrt{C/G}}$$

implying that S/N can be measured at some location in the image using the pixel values to compute the signal, but then the Poisson noise deduced from these pixel values must be scaled down by \sqrt{G} in calculating the signal-to-noise ratio.

In a more typical situation, the total signal from a star is determined by the sum of the electrons in a box (or other “aperture”) centered on the star, and containing a number of pixels n , *after* subtraction of a background (“sky”) value from each pixel’s contribution. Let C_{on} be the total number of counts (ADU) in the on-star box, and $\langle x_{\text{sky}} \rangle$ be the average counts in a pixel in a off-star box containing p pixels (i.e., in the nearby “sky” or “background”) while σ_{sky} is the rms value in the off-star box. Clearly the background measurement should be made in a region of the image devoid of obvious stars, but representative of the expected background below the star’s image. The value of σ_{sky} in that background region will take account of all processes that contribute noise to the measurement of the signal except for noise due to the star’s incident photons (e.g., it will take account of noise due to dark current, noise due to “readout noise,” even noise due to numerous, but individually undetected stars in the background, and uncertainties introduced by dark-corrections or flat-field corrections). Then we have

$$\boxed{\frac{S}{N} \approx \frac{C_{\text{on}} - n\langle x_{\text{sky}} \rangle}{\sqrt{(C_{\text{on}} - n\langle x_{\text{sky}} \rangle)/G + n\sigma_{\text{sky}}^2 + n\sigma_{\text{sky}}^2/p}}}$$

for the signal-to-noise ratio of the star, as measured from the image pixel values (ADU).

The uncertainty in the star’s magnitude, as a result of this S/N , can be determined from the definition of magnitude differences in terms of flux ratios. The magnitude uncertainty is

$$\begin{aligned} \Delta m &\approx 2.5 \log\left(\frac{S+N}{S}\right) \\ &\approx (2.5)(0.43429) \ln\left(\frac{S+N}{S}\right) \\ &\approx (2.5)(0.43429) \frac{N}{S} \\ &\approx 1.0857 \frac{N}{S} \end{aligned}$$

where the final result is appropriate for $S/N \gg 1$. In effect, $\Delta m \approx N/S$ as long as S/N is at least 5 or so.

Estimating S/N of a Specific Pixel in the Image of a Nebula

The estimation of S/N for a specific location (pixel) in the image of a nebula is done by using the result for a star, given in the boxed equation above, but taking $n = 1$ and taking C_{on} to be the value of the pixel (in ADU). For that matter, a signal-to-noise ratio can be calculated for any size region by summing the pixel values in that region, and setting n to the appropriate value.

Further Information

Newberry, M.V. 1991, “Signal-to-Noise Considerations for Sky-Subtracted CCD Data,” Publications of the Astronomical Society of the Pacific, 163, 122.

(http://adsabs.harvard.edu/cgi-bin/nph-bib_query?bibcode=1991PASP..103..122N&db_key=AST&high=3c321cbf8301567)