Objective

1. To demonstrate that the motion of the torsion pendulum satisfies the simple harmonic form in equation (3)

2. To show that the period (or angular frequency) of the simple harmonic motion of the torsion pendulum is independent of the amplitude of the motion

3. To make measurements to demonstrate the validity of equation (6), which relates the angular frequency of motion to the torsion constant and the moment of inertia of the torsion pendulum

Useful background reading
Young and Freedman, section 13.1, 13.2, 13.4 (the “angular SHM” section, specifically)

Introduction

A torsion pendulum is shown schematically to the right. A disk with moment of inertia $I_0$ is fastened near the center of a long straight wire stretched between two fixed mounts. If the disk is rotated through an angle $\theta$ and released, the twist in the wire rotates the disk back toward equilibrium. It overshoots and oscillates back and forth like a pendulum, hence the name. Torsion pendulums are used for the timing element in some clocks. The most common variety is a decorative polished brass mechanism under a glass dome. You may have seen one. The balance wheel in an old fashioned mechanical watch is a kind of torsion pendulum, though the restoring force is provided by a flat coiled spring rather than a long twisted wire. Torsion pendulums can be made very accurate and have been used in numerous precision experiments in
physics. In this session you will use one to study simple harmonic motion (SHM) because the restoring torque is very nearly proportional to the angular displacement over quite large angles of twist, so that the motion is close to being ideal SHM.

When the torsion pendulum disk is twisted away from equilibrium by an angle \( \theta \) the twisted wire exerts a restoring torque proportional to that angle

\[
\tau = -\kappa \theta
\]  

(1)

If the wire is thick and make of stiff material, \( \kappa \) is large. If the wire is long, \( \kappa \) is small. The rotational analog of Newton’s Second Law is \( \tau = I_0 \alpha \), where \( \alpha = d^2 \theta / dt^2 \) is the angular acceleration. So we obtain an equation in the simple harmonic form:

\[
I_0 \frac{d^2 \theta}{dt^2} = -\kappa \theta
\]  

(2)

The solution to this equation is:

\[
\theta(t) = A \cos(\omega t + \phi)
\]  

(3)

so the disk oscillates with time according to this equation. Here, \( A \) is the amplitude of the oscillation and \( \omega \) is the angular frequency. The period of the motion (the time for one cycle) is \( T = 2\pi / \omega \). The angular velocity and acceleration of the torsion pendulum are given by:

\[
\frac{d\theta}{dt} = -A \omega \sin(\omega t + \phi)
\]  

(4)

\[
\frac{d^2 \theta}{dt^2} = -A \omega^2 \cos(\omega t + \phi)
\]  

(5)

so the maximum (absolute) value of the angular velocity is \( A \omega \) and the maximum (absolute) value of the angular acceleration is \( A \omega^2 \).

The angular frequency of oscillation is determined by the physical parameters of the torsion pendulum:

\[
\omega = \sqrt{\frac{\kappa}{I_0}}
\]  

(6)

So the time for a single oscillation of the torsion pendulum is the period:

\[
T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I_0}{\kappa}}
\]  

(7)
1. The function shown has the form given in equation (3) of the introduction to this lab. From the graph, estimate the value of the amplitude $A$ and the angular frequency $\omega$.

2. The angular position of a torsion pendulum as a function of time is given by $\theta(t) = A \cos(\omega t + \phi)$.

   a) What do you expect the maximum (absolute) value of the angular velocity $d\theta/dt$ to be? (show your work) (write your answer in Prediction 1-3 for reference during the lab)

   b) What do you expect the maximum value of the angular acceleration $d^2\theta/dt^2$ to be? (show your work) (write your answer in Prediction 1-5 for reference during the lab)
3. A metal ring has a mass M, an inner radius a, and an outer radius b. The ring is of uniform thickness and the metal is of uniform density throughout. Write down the moment of inertia of this ring about an axis perpendicular to it running through its center. You can look the formula for this moment of inertia up in the textbook if you want; you don’t have to derive it. (write your answer by Question 2-1 for reference during the lab)

4. A “reference ring” with moment of inertia $I_C$ about the axis shown rests on a solid disk torsion pendulum with a moment of inertia $I_0$ as shown to the right.

a) What is the period $T_0$ for motion with the disk alone? Assume that the torsion constant of the wire is $\kappa$. (see equation 7 in the introduction)

b) What is the period $T_1$ for motion with the combined system (ie. when the reference ring is placed on top of the disk and is rotating with it)? Assume that the torsion constant of the wire is $\kappa$. (see equation 7 in the introduction)

c) Use the expressions from a) and b) to derive an expression for $I_0$ in terms of $T_0$, $T_1$, and $I_C$. (hint: you will find it most convenient to solve if you take the ratio of the equations in a) and b) ) (write your answer by Question 2-2 for reference during the lab.)
**Equipment**

You will use the following equipment:

- Torsion pendulum apparatus
- Rotary motion sensor (yellow plug in digital 1, black plug in digital 2)
- Force sensor (hooked to analog A)
- Vernier caliper
- Electronic balance

Reminder: Whenever you write a number down in the spaces below you should indicate what its UNITS are. As always, your computed results will not come out correctly if your units are incorrect.

**Activity 1: Investigating Simple Harmonic Motion**

The apparatus for this investigation is shown below. The aluminum disk at the top has the bulk of the moment of inertia and the oscillating torsion pendulum is attached to the shaft of a rotary motion sensor (RMS) to record the angular motion.

The apparatus has three different wires that you can use. The wires have the same length, but varying thicknesses. We will refer to these wires as the thin, medium and thick wires. First install the thin wire by attaching it at both the upper and lower wire clamp. The easiest thing is to get the upper wire clamp in place first; then have one person hold that in place while the other person adjusts the lower wire clamp. Ask your instructor if you don’t understand what to do. Bring up the `torsion_pendulum_1.ds` file from the Ph2305 class folder. If the round black ring is sitting on top of the torsion pendulum platform, remove it. The aluminum disk should be in place on the top of the torsion pendulum, secured in place by a bolt.
Part 1: Parameters of simple harmonic motion

We will first study the characteristic quantities (amplitude and period) that describe simple harmonic motion. Initially, leave the platform at rest. Start data-taking and then rotate the torsion pendulum platform (aluminum disk) and release it to start it oscillating. You should see three graphs (angular position, velocity, and acceleration). When you have a reasonable looking set of graphs, print them out. Don’t erase the data set from the screen; you need to do some analysis on it.

**Question 1-1:** Indicate on your printout how you determine the period and amplitude from the angular position data.

**Question 1-2:** Use the “Smart Tool” in DataStudio to precisely determine the amplitude and the period from your angular position data. How is the angular frequency related to the period? From the period, determine the angular frequency.

Amplitude = ______________________

Period = __________________________

Angular frequency = $\omega = __________________________$

**Prediction 1-3:** From some of the quantities measured above, predict what you expect the maximum value of your angular velocity ($d\theta/dt$) to be. (Recall the answer to prelab question 2a.)

**Question 1-4:** Use the “Smart Tool” to determine what the maximum value of the angular velocity was for your data; how does it compare to your prediction?
**Prediction 1-5:** Predict what you expect the maximum value of your angular acceleration \((\frac{d^2\theta}{dt^2})\) to be. (Recall the answer to pre-lab question 2b).

**Question 1-6:** Use the “Smart Tool” to determine what the maximum value of the angular acceleration was for your data; how does it compare to your prediction?

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**Part 2: Dependence of Period on Amplitude**

For this activity, you will study how the period of simple harmonic motion depends on the amplitude of the motion. By amplitude, we mean the maximum angular displacement from equilibrium that the torsion pendulum experiences during its motion.

**Prediction 1-7:** Consider two runs with the torsion pendulum. Assume that one run has larger amplitude than the other. Does the run with the larger amplitude have a period that is greater, the same, or less than the other run? Explain your answer.

To check your prediction, take three runs with different initial amplitudes (use the torsion_pendulum_1.ds DataStudio file). DO NOT print out the results for each run, but use your “Smart Tool” to obtain the amplitude and period for each run and record them below:

Run 1:   Amplitude = _____________________     Period = _______________________
Run 2:   Amplitude = _____________________     Period = _______________________
Run 3:   Amplitude = _____________________     Period = _______________________
**Question 1-8:** From your data, what do you conclude about how the period depends on the amplitude in simple harmonic motion? If your observations differ from your prediction in Prediction 1-7, then consult with your TA to make sure you understand the explanation for your observations.

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**Activity 2: Verification of the Angular Frequency Equation**

In this investigation, you will make measurements of the torsion constant and moment of inertia to compute the expected angular frequency (see equation 6 in the introduction). You will then see how that prediction compares to your measured angular frequency.

**Part 1: Determination of the Platform’s Moment of Inertia**

You first need to determine the moment of inertia of the rotating platform (aluminum disk) (and the rotating part of the rotary motion sensor that is attached to it). One way to determine the moment of inertia of these parts would be to disconnect them, weigh them, and measure the relevant radii. We will use another technique. It is the same technique that you worked out in pre-class assignment question 4. We will use a ring with a known moment of inertia to help us indirectly determine the moment of inertia of the free platform.

First, make the measurements necessary to determine the moment of inertia of the black ring that you should find on your lab tabletop. We will call this the “reference ring”.

- Mass of black ring = ______________________________
- Inner radius of black ring = a = __________________________________________
- Outer radius of black ring = b = __________________________________________

**Question 2-1:** From the above data, determine the moment of inertia of the reference ring. (recall the answer to prelab question 3).

Leave the thin wire installed in the torsion pendulum apparatus. Measure the period (before mounting the reference ring) using the torsion_pendulum_1.ds DataStudio file. We will call this period $T_0$, the period with the rotating platform alone.

$T_0$ = ______________________________
Now install the reference ring on top of the rotating platform. Measure the period of the combined system, which we will call $T_1$.

$$T_1 = \text{________________________________________}$$

**Question 2-2:** From the data you have taken, determine the moment of inertia of the rotating platform alone. Use the expression you derived for pre-lab question 4.

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**Part 2: Determination of the Torsion Constant**

In this activity, you will measure the torsion constant and the oscillation period for three different diameters of wire. We will refer to these three as thin, medium, and thick. The thin wire should still be installed in your apparatus from the previous activity. Remove the black reference ring that was used in the previous activity.

To measure the torsion constant, you will apply a torque to the system using a string wrapped around the **largest pulley** on the rotary motion sensor. You will determine the magnitude of the applied torque using the force sensor. By applying varying torques, you can plot the angular displacement of the disk versus the applied force; the slope of this line is related to the torsion constant $\kappa$ (see equation 1 in the introduction). The setup is shown in the figure below.
**Question 2-3:** What you will actually plot is the angular displacement versus applied force. The data studio file will do a linear fit and report the slope. From the measured slope, how will you determine the torsion constant $\kappa$? To answer this, write a simple expression that shows how the torsion constant is related to the slope; your expression will involve the radius of the pulley where the string is attached ($R$). (You should use equation 1 in the introduction as your starting point; you can ignore the minus sign. Then you need to recall how to relate the torque $\tau$ to the applied force $F$ and the radius of the pulley ($R$). From the resulting expression, you should be able to see what the slope will be when you plot angular displacement ($\theta$) versus applied force ($F$).)

Remove the aluminum platform temporarily and use the vernier caliper to measure the diameter of the **largest pulley**, which you will use for the torsion constant measurement. Record the radius (not diameter!) of the pulley below. (For using the vernier caliper, you only need to do a “coarse” reading. On the scale attached to the bottom of the sliding piece, look at where the vertical line above the leftmost “0” is. Then just read off the scale like you would a ruler where that vertical line is located.) Replace the aluminum platform.

$R =$ ______________________________________

Open the DataStudio file **torsion_pendulum_2.ds** for this part. It has a top plot showing angular displacement versus time that you can use to determine the period of the oscillation in each case. It has a bottom plot that you will use for your force probe measurements to measure the torsion constant in each case.

Set up the string so that you can attach the force sensor to it as shown in the figure on page 9. The string should be attached to the **largest pulley** so that it provides a force that is tangential to the pulley. Wind several turns of string onto the **largest** pulley. Wind it in such a way that you will pull the string from the right hand side of the pulley as you face the apparatus. Then hook the force probe onto the loop on the string, start data-taking and gently pull on the force probe in a smooth motion. The lower screen should plot the angular displacement versus applied force and perform a linear fit for you. (Note: you have five seconds to do this after you press START before the data-taking times out.)
For each of the three wires, you should measure the oscillation period and the torsion constant. (Make sure you leave the rotating aluminum platform in place the whole time; it is not necessary to remove it as shown in the figure). If you are not sure how to attach the new wires, ask your TA.

**Thin wire:**

Oscillation period = $T_1 =$

Slope of angular displacement versus applied force =

**Medium wire:**

Oscillation period = $T_2 =$

Slope of angular displacement versus applied force =

**Thick wire:**

Oscillation period = $T_3 =$

Slope of angular displacement versus applied force =

**Question 2-4:** From your measured slopes, determine the torsion constant for each of the three wires; write down the formula that you use to compute these values.

**Thin wire:** $\kappa_1 =$

**Medium wire:** $\kappa_2 =$

**Thick wire:** $\kappa_3 =$
**Prediction 2-5:** From your measured torsion constants and moment of inertia, write down the predicted angular frequency and period for each of the three wires. Write down the formulae that you use for your computations.

**Thin wire:** \[ \omega_1 = \ldots \quad T_1 = \ldots \]

**Medium wire:** \[ \omega_2 = \ldots \quad T_2 = \ldots \]

**Thick wire:** \[ \omega_3 = \ldots \quad T_3 = \ldots \]

**Question 2-6:** Summarize your predicted and measured periods for each of the three wires below. Determine the percentage difference = \((100)^* |(predicted - measured)|/predicted.\)

**Thin wire:** \(T_{\text{pred}} = \ldots \quad T_{\text{meas}} = \ldots \quad \% \text{ diff} = \ldots\)

**Medium wire:** \(T_{\text{pred}} = \ldots \quad T_{\text{meas}} = \ldots \quad \% \text{ diff} = \ldots\)

**Thick wire:** \(T_{\text{pred}} = \ldots \quad T_{\text{meas}} = \ldots \quad \% \text{ diff} = \ldots\)

**Question 2-7:** What is your conclusion about the validity of equation 6 in the introduction, the relation between angular frequency and the physical parameters of the system (torsion constant and moment of inertia)?
Make sure to tidy up your work space when you are done.

Reinstall the thinnest wire on the torsion pendulum apparatus.

When you are done with the lab (or if there are only 10 minutes left in the period), skip to the next page and answer the postlab questions.
Postlab questions:

1. You are observing the motion of a torsion pendulum and you observe that its angular displacement is well-described by the equation:

\[ \theta(t) = A \cos(\omega t) \]

where \( A = 0.5 \) radians is the amplitude and \( \omega = 1.0 \, \text{s}^{-1} \) is the angular frequency. What is the maximum absolute value of the angular velocity that is achieved during the motion?

2. Does the period of a torsion pendulum depend on the amplitude of its oscillations? Explain why or why not.