Objective

To study the behavior of transverse standing waves on a string.

Required background reading
Young and Freedman, section 15.2, 15.3, 15.4, 15.7, 15.8

Introduction

A transverse wave is created on a length of string under tension when one end is displaced. The disturbance thus created propagates down the length of the string. Although the wave appears to travel, the string itself (the medium) does not, each piece of it moving sideways instead, about an equilibrium position. If the string displacement is harmonic in time, the resulting wave has a sinusoidal shape, and any point on the string is displaced with a harmonic motion. A sinusoidal wave produced on a string with one fixed end will reflect from the fixed end. Using the principle of superposition, it can be shown that the original wave combines with the reflected wave to result in a standing wave. A characteristic feature of standing waves is the existence of nodes and antinodes at points along the string. If the amplitude of a standing wave is large enough you can see the wave envelope, which will look similar to Figure 15-23 (12th edition) or Figure 15-20 (11th edition) in Young and Freedman.

A standing wave may also be produced in a string with two fixed ends if, for instance, you pluck a string of a musical instrument. As the fixed ends cannot move they are required to be nodes, and this constrains the wavelengths of the standing waves which can be produced. In other words, a standing wave can exist on a string of length \( L \) only if its wavelength \( \lambda \) satisfies

\[
L = n \frac{\lambda}{2} \quad (n = 1, 2, 3, \ldots)
\]

Different standing wave patterns (corresponding to different \( n \)) are called modes, or harmonics. These are defined in your text (page 512, 12th edition; page 576, 11th edition). Although these waves appear to stand still, they can be thought of as being...
composed of a superposition of two waves traveling at the same speed \( v \) and same wavelength \( \lambda \) in opposite directions on the string. The speed of a wave is related to its frequency \( f \) by

\[
v = \lambda f
\]  

(1)

The wave speed \( v \) depends on the string tension \( F \) and the string’s linear mass density \( \mu \), and is given by

\[
v = \sqrt{\frac{F}{\mu}}
\]  

(2)

Since \( v \) is fixed for a given string under constant tension, standing waves with different wavelengths must have different frequencies.

This lab uses a wave driver apparatus to generate standing waves on a string. The lab has two major objectives:

1. Verify that the wave speed \( v \) is fixed for a given string under a constant fixed tension \( F \).

2. Verify the relationship between \( v \) and \( F \) given in equation (2) by measuring the wave speed \( v \) for various values of the string tension \( F \).
A traveling sinusoidal wave is propagating in the +x direction on a string. You make the following observations about the wave:

- The maximum transverse displacement of the string from its equilibrium position is 5 cm.
- At a fixed instant in time, the distance between two points where the string has zero displacement from its equilibrium position is 40 cm.
- At a fixed position along the string, you observe that the string has zero displacement from its equilibrium position, and then it starts to be displaced. After 0.1 sec. elapses, you again observe that the string has zero displacement from its equilibrium position (at that position along the string).

Write down the mathematical equation that describes this traveling wave (like equation 15.4 in the textbook). Your equation should have numerical values in it that you can deduce from the information given above.
In this exercise, we are considering standing waves on a string as described in the introduction to this lab.

2. a) For a given string under constant tension, find an expression for the frequency of the \( n^{th} \) harmonic, \( f_n \), given the wave speed \( v \) and the length of the string \( L \).

b) Derive the expression, \( v = 2 \, l_n \, f_n \), where \( l_n \) is the shortest distance between nodes for the \( n^{th} \) harmonic.

c) Read the description of the apparatus at the top of page 8. For one of the measurements in this lab, you will be using a hanging mass of \( M = 150 \) grams. Assume that the linear mass density of the string is \( \mu = 4.1 \) g/m and the length of the string is \( L = 1.8 \) m. Determine the expected frequency \( f_2 \) of the second harmonic \( (n = 2) \) standing wave. Copy your numerical answer to the space near the top of page 9; this value will be useful to you when you do the lab.
Activity 1: The Mathematical Description of a Traveling Wave

In this lab you will be making measurements on standing waves. Before we have you do that, we want to have you think a little about traveling waves. These are difficult to study in our lab space, so we will have you study traveling wave concepts with some computer animations. Recall that the mathematical representation of the displacement from equilibrium $y$ of particles on a stretched string with a sinusoidal traveling wave is given by:

$$y(x,t) = A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right]$$

where $\lambda$ is the wavelength and $T$ is the period of the wave for a wave traveling in the $+x$ direction. For a fixed time, the above equation gives the shape of the wave as a function of distance $x$ along the string. If one instead concentrates on a particular string particle at distance $x$ along the string (fixed $x$), the above equation gives the time dependence of the displacement of that particle. These concepts are illustrated nicely with an animation. On your computer desktop, click on the “Physlets” shortcut and then click on “Illustration 17.1”. Select “Animation 2” and press “Play”. This animation shows the generation of a traveling wave on a string, similar to the way the wave driver will generate waves in this lab. The animation shows you the overall wave on the string, but it also shows you a graph of the motion of a particle segment of the string (the red dot).

**Question 1-1:** Describe the motion of the red dot as a function of time (in words; there is no need to use a mathematical formula to answer this question).

Now try another Physlet, exploration 17.2 “Measure the Properties of a Wave”. Traveling waves are difficult to demonstrate quantitatively in the lab, so you will use this simulation to aid your understanding of the properties that characterize a traveling wave. You can use the play and pause buttons to start and stop the progress of the traveling wave. The step buttons step the wave in time steps of 0.1 seconds (note that the time is written right on the screen in the lower left hand corner of the simulation). If you place the cursor on the graph and left click with the mouse, you can read off the $(x,y)$ coordinates in cm. (Note: in the box that says $f(x,t)$, you should erase the formula given there; it will only confuse you later.)
Question 1-2: What is the wavelength of the simulated traveling wave? Describe how you determine it and show your calculations.

Question 1-3: What is the period of the simulated traveling wave? Describe how you determine it and show your calculations.

Question 1-4: Write down the mathematical equation that describes the behavior of the simulated traveling wave; it should be a function both of $x$ and $t$. You can type in your result in the result box of the simulation and click on “check” to see if you have determined the proper functional form. (You should refer to the formula at the beginning of this activity for help.)

Activity 2: Combining Traveling Waves to make Standing Waves

In the next part of this lab, you will be studying standing waves. Standing waves are made up out of a superposition of traveling waves. Go through Physlet “Illustration 17.4, Superposition of Traveling Waves”. Read the text and try all three animations.
Question 2-1: Describe what must happen for traveling waves to combine to form standing waves.

Question 2-2: Which of the three animations is most similar to the way you will generate standing waves in this lab? Describe how it is similar.

Finally, play Physlet “Illustration 17.5, Resonant Behavior on a String”. Read the text and try the animation.

Question 2-3: Describe the timing that is necessary to achieve the resonance. How is the time interval between “shakes of the string” related to the travel time of the wave along the string? (note: a numerical answer is not needed here; just write a description in words)
Activity 3: Measuring the Normal Modes of a Vibrating String

The vibrating string apparatus is shown below. A function generator and wave driver are used to generate disturbances at specific frequencies.

The string is tied to a vibrator at one end, passes over a pulley, and is attached to a weight at the other end which creates a tension in the string. Different weights can be hung on the string to create different tensions. The string is vibrated by a wave driver connected to a function generator. When the vibrator moves up and down with a frequency $f$, a wave of frequency $f$ propagates down the string.

Make sure that the wave driver apparatus is set up as shown in the figure above. The length of string between the wave driver piston and the pulley should be about 1.8 m.

To apply an oscillating signal to the wavedriver, you will use the function generator built into the PASCO Science Workstation. Make sure the Pasco Science Workstation and the Power AMP II are turned on. On the computer, click on the 2306_lab1.ds file in the 2306 folder (which is in the Class Notes folder on the desktop). You should see the following screen come up when Data Studio starts
You can leave the amplitude at 5.00 V. The size of the frequency steps is controlled by the controls indicated by the red arrow above.

Hang 150 g of mass (total) on the end of the string (note that the weight holder weighs 50 g). Slowly adjust the frequency setting until you observe the second harmonic (note: best results are obtained when you step the frequency with a step size of 0.1 Hz). The frequency of the resonance should be near the value that you got for the prelab: WRITE PRELAB VALUE HERE ______. Harmonics are easily identified because as the frequency approaches it, the amplitude increases and the wave envelope will show the distinctive shape. Write your measured value of the second harmonic frequency in the space in the first part of question 3-1.

**Question 3-1:** Write down the frequency where you found the second harmonic. BEFORE hunting around for the next harmonics, use your measured value of the 2nd harmonic frequency to predict the values of the 3rd and 4th harmonic frequencies. (Remember that for the prelab you determined that the frequency of the $n^{th}$ harmonic is just a constant times the mode number $n$ or just $f_n = n f_1$.) Show your work.

$2^{nd}$ harmonic frequency (measured) = __________________________

$3^{rd}$ harmonic frequency (predicted) = __________________________

$4^{th}$ harmonic frequency (predicted) = __________________________

Now find the 3rd and 4th harmonics.

$3^{rd}$ harmonic frequency (measured) = __________________________

$4^{th}$ harmonic frequency (measured) = __________________________

**Question 3-2:** Do your values above agree with your prediction in question 3-1? If not, consult with your TA. If necessary, show your work for how to properly predict the values below:
Now you will make some measurements that will allow you to determine the wave speed $v$. Recall that for the prelab, you determined that $v = 2 l_n f_n$, where $f_n$ is the frequency of the $n^{\text{th}}$ harmonic and $l_n$ is the shortest distance between nodes for the $n^{\text{th}}$ harmonic. For the total 150 g hanging weight, you will now reﬁnd the three harmonics you found above. For each case, measure the shortest distance between nodes using the moveable position markers on the meter stick. (Note that the end of the string attached to the drive arm is not exactly a node, but the end at the pulley is, so it is best to measure relative to it.) Record your data below, and compute the wave speed for each case.

150 gram weight:

$f_2 = \underline{\hspace{2cm}}$  $l_2 = \underline{\hspace{2cm}}$  $v = \underline{\hspace{2cm}}$

$f_3 = \underline{\hspace{2cm}}$  $l_3 = \underline{\hspace{2cm}}$  $v = \underline{\hspace{2cm}}$

$f_4 = \underline{\hspace{2cm}}$  $l_4 = \underline{\hspace{2cm}}$  $v = \underline{\hspace{2cm}}$

**Question 3-3:** Do you expect the three different wave speed determinations to be in reasonable agreement with each other? Why or why not? How do your data agree with your expectation?
Now you should perform the same measurements you made above for total hanging masses of 200, 250, 300, 350 g. Put those values and the values from your 150 g data in the table below. You should have concluded from the above exercise that the wavespeed is constant for a fixed string tension. So for each hanging mass, you only need to make the measurement for one harmonic. Be sure to indicate in the table which harmonic \((n = 2, 3 \text{ or } 4)\) you choose for each measurement. Compute the value of the wavespeed \(v\) for each measurement. Make sure to write what units you are using at the head of each column.

<table>
<thead>
<tr>
<th>Hanging mass</th>
<th>(N)</th>
<th>(f_n)</th>
<th>(l_n)</th>
<th>(V)</th>
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<tbody>
<tr>
<td>150 g</td>
<td>2</td>
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<tr>
<td>150 g</td>
<td>3</td>
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<tr>
<td>150 g</td>
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<td>350 g</td>
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**Question 3-4:** In question 3-3, you should have concluded that the wavespeed is constant for a given string tension. For each hanging mass you used, record the tension and the computed wavespeed value (from the table on page 11) in the table below (for the 150 g case, put in the average of your three wavespeed determinations). In the space provided show a sample calculation of how you compute the tension for the 150 g hanging mass. Make sure to put the tension and wavespeed in proper units and indicate what units you are using.

<table>
<thead>
<tr>
<th>Tension: $F$</th>
<th>Wavespeed: $v$</th>
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**Question 3-5:** You have learned that the relationship between wavespeed and string tension is given by \( v = \frac{F}{\sqrt{\mu}} \). Use this relationship and your five \((F,v)\) pairs to compute five separate determinations of the linear mass density, \(\mu\). Summarize your results in the table below, and compute the average of the five values of \(\mu\).

<table>
<thead>
<tr>
<th>Tension: (F)</th>
<th>Wavespeed: (v)</th>
<th>Linear mass density: (\mu)</th>
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Average value of \(\mu\)

**Question 3-6:** The wavespeed is given by \( v = \frac{F}{\sqrt{\mu}} \), where \(F\) is the string tension and \(\mu\) is its linear mass density. In this experiment, you used the same piece of string for every measurement. Now you will measure its linear mass density directly. What two quantities do you need to measure to determine the linear mass density of the string?

Go to the instructor’s table. There is a sample piece of string there. Measure the quantities necessary to determine the linear mass density.

**Question 3-7:** Record your measurements here, and compute the linear mass density. Make sure that you use the correct (SI) units for linear mass density.

Linear mass density = \(\mu = \) __________________________
**Question 3-8:** If the relationship between wave velocity and string tension is correct, your indirect determination of the linear mass density (from the tension and wave velocity values) should agree with the direct determination that you just made. Write them both down here to see how they compare; do they appear to be in agreement?

**Question 3-9:** You will probably observe that all the linear mass density values you computed in question 3-5 do not agree with each other. In fact, you will probably notice a systematic trend in the linear mass density values as the tension $F$ varies. Can you explain why this might be the case? (To understand what is going on, attach some weights to the string again and watch what happens to it).
When there is 10 minutes left in the lab, please stop what you are doing and skip to these questions. These questions are intended to emphasize some of the concepts you have dealt with in the lab session today.

Post lab questions

1. Assume that you are using the same experimental setup that you just used. For a given amount of total hanging mass, you observe the third harmonic to be at 22.0 Hz.
   
a) Sketch what the fifth harmonic would look like.

b) What frequency would you find the fifth harmonic at (assuming you did not change the amount of hanging mass on the string)?

2. Assume that you halve the amount of hanging mass attached to the string. What would the resonant frequency of the fifth harmonic be now?