Ph 3455/MSE 3255

The Hall Effect in a Metal and a p-type Semiconductor

Required background reading

Tipler, Chapter 10, pages 478-479 on the Hall Effect

Prelab Questions

1. A sample of copper of thickness $18 \times 10^{-6}$ m is placed in a 0.25 T magnetic field. A current of 10 amps is flowing through the sample perpendicular to the magnetic field. A Hall voltage of $-7 \mu V$ is measured under these conditions. Assuming these numbers, what is the measured Hall coefficient for copper? From the Hall coefficient, what is the density of charge carriers in copper, and how many charge carriers are provided, on the average, by each atom?

2. For the doped sample of p-type germanium that you will use in the lab, the density of holes is about $10^{15}$ cm$^{-3}$. Assuming that the holes are the majority charge carrier in the material, what Hall voltage do you expect to measure when a sample of thickness $10^{-3}$ m is placed in a 0.25 T magnetic field with a current of 10 mA flowing through it perpendicular to the magnetic field?

3. Why is the measurement of the Hall voltage more difficult experimentally in the metal than in the doped semiconductor? What physical property of the materials causes this difference in difficulties?

Background

Read the pages 478-479 in your textbook (Tipler) on the Hall effect before reading this material.

As discussed in your textbook, the Hall effect makes use of the $qv \times B$ Lorentz force acting on the charge carriers that contribute to the flow of electrical current in a material. This is a commonly used measurement technique in solid state physics used to determine the sign and number density of charge carriers in a given material. As shown in equation 10-61 of your textbook, the Hall voltage can be written as:

$$V_H = \frac{R_H BI}{t}$$

where $B$ is the magnetic field applied to the sample, $I$ is the current flowing perpendicular to the magnetic field, and $t$ is the thickness of the sample. $R_H$ is the Hall coefficient:

$$R_H = \frac{1}{nq}$$

where $n$ is the density of charge carriers and $q$ is their sign ($-e$ for electrons, $+e$ for holes).
In this lab, you will measure the Hall coefficient in a metal (copper) and in a p-type semiconductor. In the metal, the charge carriers are conduction electrons, so we expect a negative value of the Hall coefficient. For the semiconductor, you will be using a doped semiconductor (p-type germanium) where the majority charge carriers are holes. So we expect to observe a positive value of the Hall coefficient. In both cases, you will be able to determine the charge carrier density.

You will also look at the temperature dependence of the Hall coefficient. You will see dramatically different behavior for the metal and the semiconductor. Here, we briefly review conductivity in metals versus semiconductors to understand the observations that you will make.

The theory of conduction in metals, semiconductors and insulators relies on the band theory of solids described in section 10-6 of Tipler. The presence of the lattice of atoms in the solid leads to “bands” of allowed and forbidden energy levels. The electrons in a solid fill these energy levels up to the highest filled energy level called the Fermi level. The electrical behavior of a material is critically dependent on how the highest energy band is filled.

In a metal, the highest energy band is only partially full. Thus, it is easy to raise electrons to a higher energy state with an electric field, and a metal is a good conductor. Also, the number of available electrons for conduction is just the number of electrons in the highest energy band. This number is independent of temperature, so one expects that the conduction electron carrier density (and therefore the Hall coefficient) will be independent of temperature.

In contrast, in a semiconductor the highest energy band is completely full; it is referred to as the valence band. There is an “energy gap” (typically ~ 0.3 – 2.5 eV) between the full valence band and the (almost) empty conduction band. At absolute zero temperature, the conduction band in a semiconductor would be empty. Above that, some of the electrons in the valence band get thermally excited to the conduction band. They leave behind “holes” in the valence band. The current in an “intrinsic” undoped semiconductor consists of the flow of the electrons in the conduction band and the holes in the valence band. As you found out (or will find out) in the conductivity lab, the density of intrinsic conduction electrons and holes varies rapidly with temperature as $n = e^{E_g/k_B T}$ where $E_g$ is the bandgap energy which is about 0.67 eV for germanium at room temperature, $k_B = 8.617 \times 10^{-5}$ eV/K is Boltzmann’s constant, and $T$ is the absolute temperature in Kelvin. Doped semiconductors are created through the controlled addition of impurities to intrinsic semiconductors. They result in an excess of one type of charge carrier over that present in the intrinsic semiconductor. They come in two types – n-type semiconductors have an excess of negative conduction electrons, while p-type semiconductors have an excess of positive holes. You will use p-type germanium. You will find that at room temperature it has a Hall coefficient that is consistent with a positive charge carrier. At room temperature, the density of the donor holes is larger than that of the intrinsic electrons and holes, but as the temperature increases the density of intrinsic holes and electrons becomes greater than that of the donor holes. The electrons have a higher mobility than the holes in germanium, so the intrinsic electrons have more contribution to the conductivity than the intrinsic holes. Thus, at a certain point you will see the Hall coefficient change sign, indicating the change from the conductivity being dominated by the donated holes to being dominated by the intrinsic conduction electrons that are thermally excited across the bandgap.
Procedure

There are two setups for this part – Hall effect in copper and Hall effect in a p-type semiconductor. You will start with one and then swap to the other.

Hall effect in copper

1. First pick up the Hall effect board that is not installed and identify the connections. It is useful to refer to the schematic circuit diagram below. Identify the following connectors; if you don’t understand then consult with your TA.

   - The thin \((18 \times 10^{-6} \text{ m})\) copper sample in the middle of the board
   - The connectors where the Hall current flows (one end is labeled max 20 A)
   - The connectors where the Hall voltage is read out (labeled as \(U_H\))
   - The connectors labeled “8V”; These connect to a maze of wires (see back of board) that heat up when current is passed through it
   - The connectors labeled 40 \(\mu\)V/K; these connect to a temperature probe mounted next to the copper

![Schematic circuit diagram of the Hall effect setup in copper.]](image-url)
After locating the connectors, look at the actual board that is connected up. You should find it mounted such that the copper sample is nicely in between the poles of the electromagnet. You should locate the following:

- The Hall voltage is connected up to the “Measuring amplifier”; since the Hall voltage is very small (~ µV) we need to amplify it to measure it. Your “Measuring amplifier” should have the following settings:
  - Switch set to “Low Drift”
  - Amplification set to $10^5$
  - Time constant set to 0.3 second
- The Hall current connectors should be connected up to the 30 V, 20 A power supply. To start, make sure this is off (switch in back) and put the blue cable in the blue connector and the red cable in the red connector.
- The “8V” connectors for heating the board should be connected to the right side of the top power supply. We will use this later in the lab.
- The temperature probe connectors should be connected to the PHYWE box that has the lower scaled ruled in degrees Celsius
- Look at the electromagnet; notice how its cables are wired and which supply it is attached to. Turn this supply off and hook up the red cable in the red connector and the blue cable in the blue connector to start with.
- Find the Hall probe; this device actually uses the Hall effect to measure the magnetic field. It is hooked to the Teslameter; make sure it is set to a scale of 2000 and “direct field”.

2. First you will zero the Hall probe and determine its sign. Remove the Hall probe from inside the magnet poles. If the scale does not read 0 mT, then adjust the coarse zero knob (inside the dashed square) or the fine zero knob (by the “0” ) until it reads 0 mT. Then take out one of the permanent magnets and use it to determine what direction the magnetic field points when the output is positive (recall that field lines are positive coming out of the North pole of the magnet). Reinstall the Hall probe in the apparatus and recall the direction that corresponds to positive magnetic field.

3. Make a drawing of the copper sample and indicate on it the direction that the following point when the devices read positive (as you have them hooked up):
   - Hall current
   - Hall voltage
   - Magnetic field direction

   It is important that you make a clear drawing of this for yourself. Based on the above information, predict what the sign of your measured Hall voltage should be for positive and negative charge carriers.

4. Many times during this lab you will have to adjust the current through the electromagnet or the Hall current. Often we will ask you to change the sign of these, as well. Whenever you do this please heed the following rules. If we ask you to reverse the polarity, you should turn the current and voltage knob down to zero and turn off the power supply switch in the back. Then you can reverse the leads on the front of the
supply. Then turn the power supply back on in the back. Then turn the voltage knob all the way up (leaving the current knob at zero). When you make adjustments to the current, you should only vary the current knob and leave the voltage knob alone. This is called running the power supply in “current-controlled” mode. The power supply adjusts itself to whatever voltage is necessary to provide the current you dial up (up to the maximum voltage available from the supply.)

5. First you need to “compensate” the Hall measuring circuit. In zero magnetic field, we expect to see zero Hall voltage. In practice it is impossible to align the contacts exactly one above the other. If the contact are misaligned laterally relative to the current direction, then you will observe a non-zero voltage (when current flows) even with no magnetic field. This problem is solved by having two taps at the top (instead of one) and inserting a compensating potentiometer (variable resistor). The first step is to adjust that potentiometer. Turn the supply for the magnetic field down to zero (the field between the poles will still read around 10 mT or so, but that is small enough for our purposes). Set the Hall current to 0V. Look at the voltage readout of the measuring amplifier (on the digital multimeter). Adjust the zero knob on the measuring amplifier so that the output reads in the 1 – 2 volt range. With the Hall current at 0 A, make a reading. Now turn the Hall current up to 10 A. If the reading is different by more than 0.03 V, then you need to adjust the potentiometer with the small screwdriver; your TA can show you how. Go back and forth between 0 and 10 A a few times until the output doesn’t change by more than 0.03 V between the two.

6. **Hall voltage versus magnetic field at fixed Hall current:** Set the Hall current at a fixed value of 10 A. You will now vary the magnetic field and observe how the Hall voltage varies. Since the output of the Hall voltage amplifier circuit drifts, you will have to make each measurement in a “difference mode”. So for each measurement, you need to take a measurement with the B field “off” (< 10 mT) and on. The difference of those two voltages is the Hall voltage. Take measurements at roughly 250 mT, 200 mT, 150 mT, 100 mT, 50 mT and then reverse the polarity of the magnetic field and take data at -250 mT, -200 mT, -150 mT, -100 mT, -50 mT. Determine your Hall voltages by taking the difference between the “on” and “off” values. Show your TA the results before moving on to see if they make sense. Also, based on your sign conventions from part 3, convince yourself (from your measurements) what the sign of the charge carriers is; consult with your TA and see if he/she agrees with your interpretation of the data.

7. **Hall voltage versus Hall current at fixed magnetic field:** You will now set the magnetic field to a fixed value and vary the Hall current to see how the Hall voltage varies. Before doing this, check the “compensation” again by putting the magnetic field to “zero” (< 10 mT) and making sure that the Hall voltage output is the same for 0 A and 10 A of Hall current. Now fix the magnetic field at 250 mT. You want to take data at a variety of Hall currents (10 A, 8 A, 6 A, 4 A, 2 A, -10 A, -8 A, -6 A, -4 A, -2 A) at the fixed value of 250 mT. For each measurement you need to measure the Hall voltage output with the magnetic field at 250 mT and “zero” (< 10 mT) and then take the difference to get the actual Hall voltage. Determine your Hall voltages by taking the
difference between the “on” and “off” values. Show your TA the results before moving on to see if they make sense.

8. **Temperature dependence of Hall voltage:** For a fixed magnetic field and a fixed Hall current, the Hall voltage should be constant provided the Hall coefficient remains constant. The Hall coefficient only depends on the density of charge carriers. In this activity, we will vary the temperature at fixed magnetic field and fixed Hall current and see how the Hall voltage behaves. If the Hall voltage does not change as we vary the temperature, then that means that the charge carrier density is independent of temperature for a metal. For this part, you will need to remove the Hall probe since we don’t want to damage it as you heat the sample. Before removing it, determine what the field value is when you set the electromagnet current to 2 A. For the rest of this step, when we say “turn the field on” you should set it to 2 A; that will provide a constant repeatable magnetic field. Set the Hall current to a fixed value of 10 A. Now measure the Hall voltage output with magnetic field “on” and “off” and determine the Hall voltage from the difference. This is the Hall voltage at room temperature (read the actual temperature from the temperature scale on the temperature probe box). Now you will vary the temperature of the sample to three different values. On the top power supply, hook the rotateable connector up to the 2 volt spigot. Wait 2-3 minutes; the temperature of your sample should stabilize around 44°C. When it appears to be roughly stable, take measurements with the magnetic field “on” and “off”. You will notice that the Hall voltage output will have changed a lot; that is because there is another effect called the thermoelectric effect that has a comparable voltage to the Hall voltage. But you can isolate the Hall effect from it by taking the difference measurements with magnetic field “on” and “off”. You will probably find that you need to go back and forth between “on” and “off” a few times and do some averaging. This is because the voltage is very sensitive to temperature drifts. Then go on and take measurements at the 4 volt spigot (for a temperature of about 87°C) and the 6 volt spigot (for a temperature of about 136°C). So in the end, you should have values of the Hall voltage at four different temperatures at fixed magnetic field and Hall current. **Please don’t leave the heater hooked up to any particular spigot longer than about 4 minutes; when you are done leave it disconnected like it originally was.**

**Hall effect in a p-type semiconductor**

1. First pick up the Hall effect board that is not installed and identify the connections. It is useful to refer to the schematic circuit diagram below. Identify the following connectors; if you don’t understand then consult with your TA.

   - The thin p-Ge sample in the middle of the board
   - The connectors where the Hall current flows (one end is labeled +12….30V)
   - The connectors where the Hall voltage is read out
• The connectors labeled “6V”; These connect to a maze of wires (see back of board) that heat up when current is passed through it
• The connectors labeled 40 $\mu$V/K; these connect to a temperature probe mounted next to the copper

After locating the connectors, look at the actual board that is connected up. You should find it mounted such that the copper sample is nicely in between the poles of the electromagnet. You should locate the following:

• The Hall voltage is connected up to a digital multimeter in voltage mode and also directly to the Labview interface board for computerized data taking.
• The Hall current connectors should be connected up to connectors that go to the two white boxes. The two white boxes have circuitry that functions as a constant current supply. The current can be varied from 0 – 40 mA by adjusting the knob on top. The current is read out by a digital multimeter (in current mode) in series and it also passes through a 10 ohm resistor on the data acquisition board. The data acquisition boards reads the voltage across that 10 ohm resistor to monitor the current. Vary the current knob and make sure the current read on the DMM goes from 0 to 40 mA.
• The “6V” connectors for heating the board should be lying loose at the moment; we will use them later in the lab.
• The temperature probe connectors should be connected to a digital multimeter and also to the data acquisition board.
• Look at the electromagnet; notice how its cables are wired and which supply it is attached to. Turn this supply off and hook up the red cable in the red connector and the blue cable in the blue connector to start with.
• Find the Hall probe; this device actually uses the Hall effect to measure the magnetic field. It is hooked to the Teslameter; make sure it is set to a scale of 2000 and “direct field”.

2. First you will zero the Hall probe and determine its sign. Remove the Hall probe from inside the magnet poles. If the scale does not read 0 mT, then adjust the coarse zero knob (inside the dashed square) or the fine zero knob (by the “0” ) until it reads 0 mT. Then take out one of the permanent magnets and use it to determine what direction the magnetic field points when the output is positive (recall that field lines are positive coming out of the North pole of the magnet). Reinstall the Hall probe in the apparatus and recall the direction that corresponds to positive magnetic field.

3. Make a drawing of the p-Ge sample and indicate on it the direction that the following point when the devices read positive (as you have them hooked up):
   - Hall current
   - Hall voltage
   - Magnetic field direction
It is important that you make a clear drawing of this for yourself. Based on the above information, predict what the sign of your measured Hall voltage should be for positive and negative charge carriers.

4. Many times during this lab you will have to adjust the current through the electromagnet. Often we will ask you to change the sign of it, as well. Whenever you do this please heed the following rules. If we ask you to reverse the polarity, you should turn the current and voltage knob down to zero and turn off the power supply switch in the back. Then you can reverse the leads on the front of the supply. Then turn the power supply back on in the back. Then turn the voltage knob all the way up (leaving the current knob at zero). When you make adjustments to the current, you should only vary the current knob and leave the voltage knob alone. This is called running the power supply in “current-controlled” mode. The power supply adjusts itself to whatever voltage
is necessary to provide the current you dial up (up to the maximum voltage available from the supply.)

5. First you will bring up the computer data acquisition program (written in LabView) and get familiar with it. Click to Class Notes -> ph3455_stuff -> hall_effect.vi. In addition to the main window, there will be a “Controls” window that comes up; you can close the “controls” window. To start data-taking, click on the white right arrow in the upper left hand corner. Here is an explanation of the various quantities in boxes:

- Resistor – ohms: This should be set to 10.00; otherwise don’t mess with it. It is the value of the resistor that the Hall current flows through.
- Voltage: This is the voltage across the 10 ohm resistor that the Hall current flows through; so this voltage monitors the Hall current.
- Current: This is the voltage above divided by 10 ohms; so this is just the Hall current. It should agree with the Hall current read on the DMM.
- Hall Effect Voltage: This is the Hall voltage; it should agree with the voltage read on the DMM.
- Ambient temperature: ignore this
- Hall Temperature (volts): This provides a voltage that is proportional to the temperature of the sample. At room temperature is ~ 0 mV. Above that the calibration is 40 µV/K.
- Hall Temperature – K: Ignore this; it is not working properly.

When you want to record a data point, simply press the “Acquire Data” button and it will enter the current values as a row in the table. When you are done with a set of data, you can print the window to the printer in the room. Print enough copies for all partners. To clear the data, press the “STOP” button and then press the white right arrow again if you want to restart. If you want to take more data than the 16 entries in the table, then you have to print out the data, clear it, and take more.

6. Hall voltage versus magnetic field at fixed Hall current: Set the Hall current at a fixed value of 30 mA. You will now vary the magnetic field and observe how the Hall voltage varies. Take measurements at roughly 250 mT, 200 mT, 150 mT, 100 mT, 50 mT and then reverse the polarity of the magnetic field and take data at -250 mT, -200 mT, -150 mT, -100 mT, -50 mT. Show your TA the results before moving on to see if they make sense. Also, based on your sign conventions from part 3, convince yourself (from your measurements) what the sign of the charge carriers is; consult with your TA and see if he/she agrees with your interpretation of the data. Make sure to print out a copy of the data for each partner.

7. Hall voltage versus Hall current at fixed magnetic field: You will now set the magnetic field to a fixed value and vary the Hall current to see how the Hall voltage varies. Now fix the magnetic field at 250 mT. You want to take data at a variety of Hall currents (40 mA, 35 mA, 30 mA, 25 mA, 20 mA, 15 mA, 10 mA, 5 mA, -40 mA, -35 mA, -30 mA, -25 mA, -20 mA, -15 mA, -10 mA, -5 mA) at the fixed value of 250 mT. Ask your TA how to reverse the Hall current direction when you need to do that to get the negative values. Show your TA the results before moving on to see if they make sense. Make sure to print out a copy of the data for each partner.
8. **Temperature dependence of Hall voltage:** For a fixed magnetic field and a fixed Hall current, the Hall voltage should be constant provided the Hall coefficient remains constant. The Hall coefficient only depends on the density of charge carriers. In this activity, we will vary the temperature at fixed magnetic field and fixed Hall current and see how the Hall voltage behaves. If the Hall voltage does change as we vary the temperature, then that means that the charge carrier density changes as a function of temperature. Set the magnetic field to a fixed value of 250 mT. Then, remove the Hall probe, since we don’t want to damage it as the sample is heated. Set the Hall current to a fixed value of 40 mA. Now you will vary the temperature of the sample. Make sure the data taking program is ready to go; record a point at room temperature. Then hook the cables that connect to the “6 V” connectors to the 6 VAC output on your power supply (ask your TA to be sure you know where it is). This will start the sample heating up. As you see the “Hall Temperature –V” change record new values (it will flicker between values, so you need to wait till it is stable before recording a point). **When you are done taking data, unhook the 6VAC cables immediately; they shouldn’t remain hooked up longer than a total of about 5 minutes.** You should observe the Hall voltage change with temperature. Show your data to your TA. Make sure to print out a copy of the data for each partner.

**Report**

Your report should address all of the following points:

1. **Introduction**

2. First, analyze the data for the copper sample. Include a drawing like you made in procedure step 3 that shows the copper sample and the direction of the Hall current, Hall voltage, and magnetic field when the measuring devices read positive.

3. For both the “Hall voltage versus magnetic field at fixed Hall current” and the “Hall voltage versus Hall current at fixed magnetic field” do the following:

   - Include a table with your raw data values (and the Hall voltages extracted from the “on-off” differences. Note that your raw Hall voltages need to be multiplied by a factor of $10^{-5}$ since the amplification factor of our amplifier was $10^5$.)
   - Make a graph of Hall voltage versus the changing quantity (current or magnetic field)
   - Perform a least squares fit to your sets of data to determine the best fit values of the slope and intercept. Do not use a canned fitting routine; follow the procedure in section V of the error analysis handout. Use the formulae there to determine the best fit value of the slope and intercept (and their errors). Put you best fit lines on your plots. (Note: the measured data points should be plotted as individual data points and they should NOT be connected by a jagged line to guide the eye.
The best fit line should be drawn as a continuous line which, of course, will not go through every data point.

- From the measured slopes (and their errors), you should be able to determine a value of the Hall coefficient (as defined in the background material in the introduction) and its error. You should have two independent Hall coefficient measurements. (Note that the thickness of the copper sample was $18 \times 10^{-6}$ m.)

- From the sign of the measured Hall coefficients (and the conventions in your drawings in step 2) make an argument about the sign of the charge carriers in copper.

- Use your two measured Hall coefficients to determine two values of the charge carrier density in copper. Compare to the accepted value in Table 10-3 of Tipler. (You will probably find that you are off by about a factor of 2; that is what I found. I am not quite sure why; it could be that our amplifier is a factor of 2 off from being exactly $10^5$.)

4. Make a table of your raw data (and extracted Hall voltages) for the temperature dependence of the Hall voltage. Plot the Hall voltage versus temperature. Do you see any evidence of a temperature dependence of the Hall voltage for copper? What do you expect and why?

5. Next, analyze the data for the p-type germanium sample. Include a drawing like you made in procedure step 3 that shows the p-type germanium sample and the direction of the Hall current, Hall voltage, and magnetic field when the measuring devices read positive.

6. For both the “Hall voltage versus magnetic field at fixed Hall current” and the “Hall voltage versus Hall current at fixed magnetic field” do the following:

   - Include a table with your raw data values.
   - Make a graph of Hall voltage versus the changing quantity (current or magnetic field)
   - Perform a least squares fit to your sets of data to determine the best fit values of the slope and intercept. Do not use a canned fitting routine; follow the procedure in section V of the error analysis handout. Use the formulae there to determine the best fit value of the slope and intercept (and their errors). Put your best fit lines on your plots. (Note: the measured data points should be plotted as individual data points and they should NOT be connected by a jagged line to guide the eye. The best fit line should be drawn as a continuous line which, of course, will not go through every data point.

   - From the measured slopes (and their errors), you should be able to determine a value of the Hall coefficient (as defined in the background material in the introduction) and its error. You should have two independent Hall coefficient measurements. (Note that the thickness of the p-type germanium sample was 1 mm = $10^{-3}$ m.)
• From the sign of the measured Hall coefficients (and the conventions in your drawings in step 5) make an argument about the sign of the charge carriers in p-type germanium at room temperature.

• Use your two measured Hall coefficients to determine two values of the charge carrier density in your sample of p-type germanium at room temperature.

7. Make a table of your raw data for the temperature dependence of the Hall voltage in p-type germanium. Plot the Hall voltage versus temperature. (Note that the calibration of the thermocouple was 40 $\mu$V/K and you can assume that the thermocouple reads 0 mV at room temperature). Explain what is going on in the curve (see the background material in the introduction). From the part of the data where the Hall voltage changes sign pick out the maximum value of the Hall voltage in that region. Use it to determine the Hall coefficient at that temperature. Compare the number you obtain to the number you obtain from the following (empirical) expression formula for the intrinsic charge carrier density in germanium:

$$n(T) = \left(5.7 \times 10^{20} \text{ m}^{-3}\right) e^{\frac{5110}{T(K)}}$$

where the temperature is in Kelvin.

8. Conclusion