Lab 1: Determination of e/m for the electron

Background Reading: Tipler, Llewellyn pp. 125 – 130; this covers the original method of Thomson which is somewhat different from that used in this experiment.

Apparatus: Helmholtz coil, power supplies, permanent magnet.

Prelab Questions:

1. Use Eq. (4) to calculate the magnetic field at the midpoint of the two coils used in this experiment when the current through each coil is 1 amp. The number of turns on each coil is 130 and the distance between the coils is 15 cm. If the error in the current is 0.02 amp and the error in the measurement of the distance between coils is 1.0 mm, use equations for the propagation of error from the error analysis handout to find the error in the magnetic field.

2. Why is it important to have a homogeneous B field in this measurement?

I. Introduction

This experiment measures e/m, the charge to mass ratio of the electron. This ratio was first measured by J. J. Thomson in 1897. He won a Nobel prize for his study of electrons. In the present experiment a beam of electrons is accelerated through a known potential, so the velocity of the electrons is known. A pair of Helmholtz coils produces a uniform and measurable magnetic field at right angles to the electron beam. This magnetic field deflects the electron beam in a circular path. By measuring the accelerating potential, the current to the Helmholtz coils, and the radius of the circular path of the electron beam, the ratio e/m is calculated.

II. Description of Experiment

The source of electrons is an electrically heated metal oxide surface called a cathode which is mounted inside an evacuated glass tube. A diagram of the apparatus is shown in Fig. 1. The cathode is heated by a filament which produces a visible glow when the tube is operating. Electrons are boiled off the cathode (the name for this process is thermionic emission) with a spectrum of initial velocities which range from zero to a maximum of the order of
where \( \hat{k} \) is Boltzmann's constant, \( T_C \) is the absolute temperature of the cathode, and \( m \) the electron mass. For a characteristic temperature of \( T = 2500 \text{ K} \), one has \( v_o = 4 \times 10^5 \text{ m/s} \). It will be seen later that this initial velocity is negligible compared to the velocity the electron gains after acceleration.

\[
v_o = \left( \frac{3kT_C}{m} \right)^{1/2}
\]

(1)

The electron gun containing the heated cathode is shown in Fig. 2. The electrons emitted from the cathode are accelerated by a potential applied between the cathode and the anode. The grid is held positive with respect to the cathode and negative with respect to the anode. It helps to focus the electron beam. Electrons, accelerated from the cathode to the anode through an accelerating potential \( V \), gain kinetic energy equal to their charge times the accelerating potential. Neglecting the small initial kinetic energy which the electrons have upon emission from the cathode, energy conservation can be used to find the electron's velocity after acceleration through the potential \( V \).

\[
\frac{1}{2} m v^2 = eV
\]

(2)

After passing through a small hole in the anode, the electrons are in a region with no electric field but with a uniform magnetic field \( \mathbf{B} \) oriented in a horizontal direction. The magnetic force \( \mathbf{F}_m \) acting on a particle of charge \( q \) moving with velocity \( \mathbf{v} \) in a magnetic field \( \mathbf{B} \) is given by the equation

\[
\mathbf{F}_m = q \mathbf{v} \times \mathbf{B}
\]

Since the electron beam in this
experiment is perpendicular to the magnetic field, the equation can be written in scalar form as

\[ F_m = e\nu B \]

where \( e \) is the charge of the electron.

If the region inside the tube were actually a vacuum, the electron beam would be invisible. To visualize the beam, the tube is filled with helium at a pressure of 10.2 mm Hg. The electron beam leaves a visible trail in the tube because some of the electrons collide with helium atoms which are excited and then radiate visible light.

A homogeneous magnetic field is produced in the region of the cathode ray tube by a current through two circular coils. The coils have radius \( R \) of 15 cm and are positioned about a common axis with a spacing of \( R \). Such a configuration is called a Helmholtz pair. The magnetic field produced by the coils is proportional to the current through the coils. The unique feature of the Helmholtz pair is its very uniform magnetic field throughout the central region, on and off axis. The field in the central region of the pair is spatially uniform like that in a solenoid. The Helmholtz arrangement has many practical advantages over a solenoid, the most important being ready access to equipment placed inside the uniform field region.

A mirrored scale is attached to the back of the rear Helmholtz coil. It is illuminated by lights that light automatically when the heater of the electron gun is powered. By aligning the electron beam with its image in the mirrored scale, you can measure the radius of the beam path without parallax error.
Safety Warning: The anode voltage in this experiment is high enough to cause a painful, or possibly, serious shock. Do not work with wet hands or when standing on a wet floor. It is always best when working on a high voltage circuit to keep one hand in your pocket so as to avoid getting a shock from one hand to the other and thus passing current near your heart.

III. Procedure

1. Before applying power, set the current and voltage adjustment knobs as follows:
   a. Helmholtz Coil current, counterclockwise
   b. Low Voltage:
      DC current, clockwise
      DC voltage, counterclockwise
   c. High Voltage, counterclockwise

2. In order to shade the e/m tube from room lights, place the hood over the e/m apparatus.
3. Flip the toggle switch up to the e/m MEASURE position.
4. Turn the current adjust knob for the Helmholtz coils to the OFF position.
5. Connect your power supplies and meters to the front panel of the e/m apparatus, as shown in Fig. 3.

6. There are two power supplies which need to be set. One supply, the Pasco model SF-9585 High Voltage Power Supply, provides power to the electron gun. The other supply, the Pasco Model SF-9584 Low Voltage Power Supply, provides power to the Helmholtz coils. Both of these supplies are described in attachments at the end of this handout.

   Adjust the power supplies to the following levels:
   - Electron Gun
     Heater. 6.3 VAC
     Electrodes: 150 to 300 VDC
     Helmholtz Coils: 6-9 VDC (set this using the instructions for “constant voltage mode” on page 2-8 of this handout)

   CAUTION: the voltage to the heater of the electron gun should NEVER exceed 6.3 volts. Higher voltages will burn out the filament and destroy the e/m tube.
7. Slowly turn the current adjust knob for the Helmholtz coils clockwise. *Watch the ammeter and take care that the current does not exceed 2 A.*

8. Wait several minutes for the cathode to heat up. When it does, you will see the electron beam emerge from the electron gun. It will be curved by the field from the Helmholtz coils. Check that the electron beam is parallel to the Helmholtz coils. If it is not, turn the tube until it is. Don't take it out of its socket. As you rotate the tube, the socket will turn.

9. Carefully read the current to the Helmholtz coils from your ammeter and the accelerating voltage from your voltmeter. Record your result.

10. Carefully measure the radius of the electron beam. Look through the tube at the electron beam. To avoid parallax errors, move your head to align the electron beam with the reflection of the beam that you can seen on the mirrored scale. Measure the radius of the beam as you see it on both sides of the scale, then average the results. Record your result.

11. Gently turn the tube and observe the electron beam when the velocity of the electrons is not perpendicular to the magnetic field.

12. Observe the path of the electrons under the influence of a permanent magnet. With the Helmholtz coils turned off, can you predict the direction the electron beam will bend?
IV. Data Taking

Adjust the anode voltage and the current in the coil and repeat the measurement of the electron radius so as to have measurements for eight different circular electron paths. For each measurement you will need the radius of the path, the current in the coils and the accelerating voltage. Try to take data so that you include solenoid currents that range over 1-2 amps and accelerating voltages that range over 150 – 300 volts. This completes the in-class part of the lab.

V. Analysis (see page 1-7 for details on what to include in the lab report)

\[ B = \frac{\mu_0}{2} \left( \frac{ia^2}{a^2 + b^2} \right)^{3/2} \]  

(3)

where \( b \) is the perpendicular distance from the plane of the coil and \( \mu_0 = 4\pi \times 10^{-7} \text{ tesla-meter/ampere} \).

![Helmholtz coil arrangement](image)

**Figure 4. Helmholtz coil arrangement.**

For the special case of two coils with \( N \) turns each, the magnetic field at the midpoint between the coils is given by:

\[ B = \mu_0 \left( \frac{4}{5} \right)^{3/2} \frac{Ni}{a} \]  

(4)

where the distance between the two coils is equal to the radius of the coils. The number of turns on each coil is 130 and the distance between the coils is 15 cm. Compute the magnetic field for each reading taken.
The electrons are accelerated through a potential difference equal to the voltage on the anode of the tube. By setting the potential energy of the electrons equal to its kinetic energy, write down an expression for the velocity in terms of e and m.

Now consider the forces acting on the electron due to the magnetic field and the circular motion of the electron. Use the expression for velocity found above to derive an expression for the ratio e/m in terms of measurable quantities. Compute the ratio e/m for each reading taken during the experiment. Using the equations given in the appendix for the propagation of errors, derive an expression for the error in e/m in terms of your measured quantities. Use this equation to calculate the error in each of your measured values of e/m. Then compute the average and its error. Compare the average value with the accepted value and discuss any discrepancy.

Finally calculate the electron velocity when it has reached the anode for the two cases corresponding to the highest and lowest value of V used in your experiment. Compare to the initial electron velocity at the cathode \( v_o = 4 \times 10^5 \text{ m/s} \) estimated at the beginning of this description.

VI. Questions

Your lab report should include an introduction, a data analysis section following the outline in section V, a section answering the questions below, and a conclusion.

1. Discuss the effect of the earth’s magnetic field on the result of this experiment. You can get a value for the earth’s magnetic field in Blacksburg by going to the following website: [http://www.geomag.bgs.ac.uk/gifs/igrf.html](http://www.geomag.bgs.ac.uk/gifs/igrf.html). The numbers you need to input are:
   - Blacksburg’s latitude: 37 degrees 15 minutes, longitude: -80 degrees 25 minutes, and its altitude 0.6 km.
2. In the experiment you were asked to turn the tube, thus causing the velocity of the electrons to no longer remain perpendicular to the magnetic field. What path did the electrons follow? Physically, why must this be the case?
3. Use the \( \chi^2 \) test for consistency described in section IV of the error analysis handout to determine the value of \( \chi^2 \) for your set of eight data points. Do the computation in two ways. First assume that \( Y_{\text{theory}} \) is equal to the average value of your data points. This will provide a quantitative measure of the internal consistency of your various measurements of e/m. Next assume that \( Y \) is equal to the accepted value for e/m. This will provide a quantitative measure of how well your data agrees with the accepted value. In each case, quote the reduced \( \chi^2 \) defined in Table 2 of the error analysis handout. To interpret this number, recall that for a large number of degrees of freedom (\( \nu \)) \( \chi^2 \) becomes Gaussian distributed with a mean of 1 and a standard deviation of \( \frac{2}{\nu} \).
The PASCO scientific Model SF-9584 Low Voltage AC/DC Power Supply provides regulated AC and DC voltages ranging from 0 to 24 volts. The DC output can be adjusted either for constant voltage or for constant current—up to 8 amperes for low resistance loads. A meter selection switch allows monitoring of either voltage or current output. The AC output is adjustable in 2 volt increments, with a maximum output of 5 amperes.

Both AC and DC outputs are regulated and thermally protected against current overload.

To Use the Power Supply:

**NOTE:** If at any time the power supply fails to come on, or if it shuts down during operation due to excessive current draw, press the RESET button on the front panel of the supply. Of course, it may be necessary to reduce the voltage or increase the load in order to resume operation. If the RESET button does not work, and if the ON/OFF button is not lit, check the fuse on the back panel of the power supply. If the fuse is blown, replace it only with a similarly rated fuse (see the Parts List at the end of this manual).

**AC OPERATION:**
1. Plug the power cord into a well-grounded (three-pronged), 117 VAC, 60 Hz outlet.
2. Connect the AC output terminals of the power supply to the circuit. (Connecting wires are not provided with the supply.)
3. Set the AC VOLTAGE ADJUST knob to the 2 volt position.
4. Flip the ON/OFF switch on the power supply to ON. The switch will light to show that the power supply is on.
5. Set the AC VOLTAGE ADJUST knob to the desired setting.

**DC OPERATION:**
1. Plug the power cord into a well-grounded (three-pronged), 117 VAC, 60 Hz outlet.
2. Connect the DC output terminals of the power supply to the circuit. (Connecting wires are not provided with the supply.)
3. Turn the DC VOLTAGE ADJUST knob and the DC CURRENT ADJUST knob fully counterclockwise.
4. Flip the ON/OFF switch on the power supply to ON. The switch will light to show that the power supply is on.
5. **Constant Voltage Mode:** Turn the DC CURRENT ADJUST knob fully clockwise. Then adjust the DC VOLTAGE ADJUST knob to obtain the desired voltage output. The DC voltage may be read on the meter. To determine the DC current output, press the METER SELECT button. The meter will then show the DC current output until the button is released.

**Constant Current Mode:** Turn the DC VOLTAGE ADJUST knob fully clockwise. Press the METER SELECT button as you adjust the DC CURRENT ADJUST knob to obtain the desired current output. With the button pressed, the meter will show the DC current output. (When you release the button, the meter will revert to showing DC voltage.)

**SPECIFICATIONS**

**POWER OUTPUT:**
- AC: 2 to 24 volts (RMS), selectable in 2 volt increments; current up to 6 amperes.
- DC: 0 to 24 volts up to 8 amperes in constant voltage mode; or 0 to 8 amperes up to 24 volts in constant current mode.

AC and DC outputs are available simultaneously, and both are protected from current overload by a thermally activated circuit breaker.

**rippLE:** Less than 0.1%.

**Regulation:** Less than 1% change in current or voltage output (depending on mode) for a 10% change in line or load.
The PASCO scientific Model SF-9585 High Voltage Power Supply provides regulated DC power up to 50 mA in two voltage ranges: 0 to 50 and 0 to 500 volts. The output for the two ranges are independently adjustable, though a single terminal is used as the positive terminal for the 50 volt output and as the negative terminal for the 500 volt output. The meter may be switched to read current or voltage in either range.

In addition, a separate, constant voltage filament supply provides unregulated AC power up to 3 amperes at 1, 2, 4, 5, 6, and 7 volts. Voltage selection for the filament supply is simply a matter of plugging into terminals with the desired voltage differential, as shown on the front panel of the supply.

To Use the Power Supply:

**CAUTION—HIGH VOLTAGE:** Play it safe:

- *Always work with a partner.
- *Wear rubber soled shoes.
- *Work with one hand only. Keep the other hand in your pocket or behind your back.

1. Plug the power cord into a well-grounded (three-pronged), 117 VAC, 60 Hz outlet.
2. Connect the appropriate output terminals of the power supply to the circuit. (Connecting wires are not provided with the supply.) The 50 and 500 volt ranges, as well as the filament supply, may all be used simultaneously.
3. Turn the 50 VOLT ADJUST knob and the 500 VOLT ADJUST knob fully counterclockwise.
4. Flip the ON/OFF switch on front panel to ON. The switch will light to show that the power is on.
5. Normally the meter reads the voltage between the 500 V output terminals. To read the voltage between the 50 V terminals press and hold down the upper METER SELECT pushbutton. To read the output current, in milliamperes, from the 500 V terminals, press the lower pushbutton and hold it down. Hold down both pushbuttons to read the output current of the 50 V terminals. In each case, the adjacent LEDs will light to show which variable is being measured.
6. With the meter set for the corresponding range, turn the 50 VOLT ADJUST or the 500 VOLT ADJUST knob to obtain the desired output.

**NOTE:** All output circuits are protected against current overload. If the power supply fails to come on, or if it shuts down during operation because of a current overload, check the fuses on the rear of the supply (see the fuse locations in the illustration at the top of the page). If one is blown, replace it only with a similarly rated fuse: (see the Parts List at the end of this manual for fuse specifications).

**SPECIFICATIONS**

DC Output: 0 to 50 volts and 0 to 500 volts, independently adjustable; up to 50 mA in each range.

AC Output: 1, 2, 4, 5, 6, and 7 VAC; up to 3 amperes.

RIPPLE: Less than 0.1%

REGULATION: Less than 1% variation in output for a combined variation in line and load of up to 10%.