5. The Michelson Interferometer

Equipment: Michelson interferometer, sodium lamp, He-Ne laser, white light source, desk lamp

Prelab Questions

1. Why are the fringes observed in the Michelson interferometer circular in shape?

2. Using the He-Ne laser as a source, how many new fringes would be observed when the mirror \( M_2 \) in Figure 1 is moved a distance of 1 cm?

Introduction

Interference devices can be divided into two classes, those based on a division of wave front and those based on division of amplitude. The diffraction grating, which divides the wave front into sections laterally, is an example of the first. The Michelson interferometer and the Fabry-Perot interferometer are examples of the second. They divide the wave by partial reflection, the two resulting wave fronts maintain the original size but with reduced amplitudes.

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The arrangement of the interferometer is shown in Fig. 1. An extended source (such as a laser beam passing through a diffusing ground-glass plate) emits a wave, part of which travels to the right. The beam-splitter at \( O \) divides the wave into two, one segment traveling to the right and one up toward mirror \( M_2 \). The two waves are reflected by mirrors \( M_1 \) and \( M_2 \) and return to the beam-splitter. Part of the wave coming from \( M_2 \) passes through the beam-splitter going downward and part of the wave coming from \( M_1 \) is reflected by the beam-splitter toward the detector. Thus the two waves are united, and interference occurs.

Figure 1. The Michelson Interferometer
The mirror $M_2$ is mounted on a carriage and can be moved. This slow and accurately controlled motion is accomplished by means of a screw which is calibrated to show the exact distance the mirror has been moved. To obtain fringes the mirrors $M_1$ and $M_2$ are made exactly perpendicular to each other by means of screws on mirror $M_1$.

Circular fringes are produced with monochromatic light when the mirrors are in exact adjustment. To understand how fringes are formed, refer to Fig. 2. The real mirror $M_1$ has been replaced by its virtual image $M_1'$ formed by reflection in the beam splitter at $O$. Suppose the mirrors are positioned so that the beams reaching the screen are exactly parallel. $M_1'$ is then parallel to $M_2$. With the reflections in the real interferometer, we may now think of the extended source as being at $L$, behind the observer, and as forming two virtual images $L_1$ and $L_2$ in $M_1'$ and $M_2$. That is, the light from each partial beam striking the screen looks as if it had originated in the corresponding virtual source. These virtual sources are coherent in that the phases of corresponding points in the two are exactly the same at all instants. If $d$ is the separation between $M_1'$ and $M_2$, the virtual sources will be separated by $2d$. When $d$ is exactly an integral number of half wavelengths (i.e., the path difference $2d$ is equal to an integral number of whole wavelengths) all rays of light reflected normal to the mirrors will be in phase. Rays of light reflected at an angle, however, will generally not be in phase.

\[ \frac{P' - P''}{2d \cos \theta} \]

Figure 2. Formation of circular fringes in the Michelson interferometer

Now consider two rays approaching the eye at an angle $\theta$. The path difference between the two rays coming to the eye from corresponding points $P'$ and $P''$ is $2d \cos \theta$, as shown in the figure.

Note that one beam passes through $O$ three times whereas the other traverses it only once. The compensating plate $C$, inserted in the arm $OM_1$, ensures that the path in glass of the two rays is equal.
The angle $\theta$ is necessarily the same for the two rays when $M_1'$ is parallel to $M_2$ so that the rays are parallel. Hence when the eye is focused to receive parallel rays, the rays will reinforce each other to produce maxima when the angle $\theta$ satisfies the relation

$$2d \cos \theta = m\lambda \quad m = 1, 2, 3, ... \quad (1)$$

For a given $m$, $\lambda$, and $d$, the angle $\theta$ is constant, and the maxima will lie in the form of circles about the perpendicular from the eye to the mirrors.

Now look at Fig. 3 to see how the circular fringes look under different conditions. Starting with $M_2$ a few centimeters beyond $M_1'$, the fringe system will have the general appearance shown in (a) with the rings very closely spaced. If $M_2$ is now moved slowly toward $M_1'$ so that $d$ is decreased, Eq. (1) shows that a given ring, corresponding to a given value of the $m$, must decrease its radius because the product $2d \cos \theta$ must remain constant. The rings therefore shrink and vanish at the center, a ring disappearing each time $2d$ decreases by $\lambda$, or $d$ by $\lambda/2$. This follows from the fact that at the center $\cos \theta = 1$, so that Eq. (1) becomes

$$2d = m\lambda.$$ 

To change $m$ by unity, $d$ must change by $\lambda/2$.

![Figure 3. Appearance of the various types of fringes observed in the Michelson interferometer. Upper row, circular fringes. Lower row, fringes with mirrors out of alignment. The path difference $d$ increases outward, in both directions, from the center.](image)

As $M_1'$ approaches $M_2$ the rings become more widely spaced, as indicated in Fig. 3b, until finally we reach a critical position where the central fringe has spread out to cover the whole field of view, as shown in (c). This happens when $M_1'$ and $M_2$ are exactly coincident, for it is clear that under

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these conditions the path difference is zero for all angles of incidence. If the mirror is moved still farther, it effectively passes through $M_1'$, and new widely spaced fringes appear, growing out from the center. These will gradually become more closely spaced as the path difference increases, as indicated in (d) and (e) of the figure.

If the mirrors $M_1$ and $M_2$ are not exactly parallel, fringes will still be seen with monochromatic light for path differences not exceeding a few millimeters. The fringes are in general curved and are convex toward a point off the screen. Then with a certain value of $d$, you might observe fringes shaped like those of Fig 3g. If the separation of the mirrors is decreased, the fringes will move to the left across the field, a new fringe crossing the center each time $d$ changes by $1/2$. As we approach zero path difference, the fringes become straighter, until the point is reached where $M_1'$ actually intersects $M_2$, when they are perfectly straight, as in (h). Beyond this point, they begin to curve in the opposite direction, as shown in (i). The blank fields (f) and (j) indicate that this type of fringe cannot be observed for large path differences.

**Experiment**

**Caution:** The interferometer is a precision optical instrument. It can be badly damaged by improper use. Please use care in handling all components.

1. **DO NOT TOUCH OR BREATHE ON ANY OF THE GLASS SURFACES.** The mirrors have a reflecting coating on their front surfaces, which can be damaged by a finger print or damp air.

2. Do not force any of the adjustments that seem difficult to turn. Only gentle pressure with one finger and thumb should be sufficient. If you have doubt about an adjustment, ask your instructor.

Look at the interferometer and compare it with Fig. 1. Identify the various elements of Fig. 1 with their physical counterpart on your equipment.

The mirror $M_2$ is moved by a lever which is in turn operated by a micrometer. The mirror moves in a straight line toward or away from the beam splitter. The geometry of the lever is such that the mirror moves $1/5$ the distance that the micrometer travels. Since the micrometer is graduated to 0.01 mm, each division corresponds to 0.002 mm of mirror movement.

Note the two screws on the back of the mount for mirror $M_1$. These are tilt adjustment for the mirror, and they can be used to align it perpendicular to the moving mirror. Only small adjustments will be necessary.

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A diffusing plate is placed to the left of the beam splitter at the point where light enters the interferometer. This gives a diffuse source of light for observing fringes. A screen is used as a detector to observe fringes.

Alignment

Remove the diffusing plate. Project the laser beam into the Michelson interferometer and onto the detector screen. Make certain that the beam strikes the center of both mirrors.

You will see two intense spots on the screen in addition to a number of subsidiary spots. Adjust the orientation of the adjustable mirror (M₁) until the two intense spots exactly overlap.

Insert the diffusing plate and view the interference pattern on the detecting screen. Rotate the micrometer drive that is linked mechanically to mirror M₂ until the micrometer reads about 21 mm.

Adjust the alignment of mirror M₁ until the center of the fringes appears on the screen. You should see a set of circular fringes centered on the screen. If the center of the fringe pattern is not visible, carefully adjust the mirror until the center of the interference fringes appears on the screen. See Figure 3 for help in the adjustment.

Rotate the micrometer drive in such a direction that the circular fringes appear to move inward. Continue to rotate the drive in this direction until only three or four dark fringes appear. Occasional adjustments of mirror M₁ may be necessary to keep the pattern centered.

Measurement of the wavelength of laser light

Adjust the micrometer drive to produce a null in the center of a pattern with two dark fringes. Record the micrometer setting.

While counting the nulls in the fringes, move the micrometer drive in the direction which causes the fringes to expand in size. Move through 100 successive nulls. It is best to do these in sets of about 20 in order to maintain a good count. Record the micrometer setting.

Assuming the mirror, because of mechanical linkages, moves 1/5 of the distance recorded by the micrometer, compute the wavelength of the laser light. Note that the pathlength changes by twice the mirror displacement. The accepted value for the wavelength of the He-Ne laser is 632.8 nm.
Measurement of the wavelength difference between the sodium D lines

Turn on the sodium lamp and let it warm up. Install the diffusing plate and replace the laser with the sodium light. You will now have to view the fringes by looking into the beam splitter.

As the path difference changes, the fringes with sodium light proceed through cycles of fuzziness and sharpness as the micrometer screw is traversed. The cycles repeat every several hundred fringes or so. This effect is due to the yellow sodium light having two spectral lines very near to one another in wavelength. The alternate fuzzing and sharpening of fringes is an effect similar to the beats you hear between two sound sources of similar frequency. For some path differences, the bright fringes for one line are on top of the dark fringes from the other, thus washing out the pattern to create a fuzzy image.

To determine the relation between the difference in wavelength and interferometer travel, consider two wavelengths $\lambda_1$ and $\lambda_2$. The fringes from $\lambda_1$ are spaced $\lambda_1/2$ apart in mirror travel while those from line $\lambda_2$ are spaced $\lambda_2/2$ apart. In order to produce a shift from one fuzzy image to a clear image and back to a fuzzy image, one fringe train must fall one whole fringe behind the other. If this takes a pathlength $d$, then the number of fringes that have passed due to each line is

$$N_1 = \frac{d}{\lambda_1} \quad \text{and} \quad N_2 = \frac{d}{\lambda_2}$$

Now if this fuzzy image is one cycle from another one, we must have $N_1 = N_2 + 1$, or

$$\frac{d}{\lambda_1} = \frac{d}{\lambda_2} + 1$$

Rearranging, we get

$$\frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\lambda_2}{\lambda_1} = \frac{d}{\lambda_1}$$

If $\Delta \lambda = \lambda_2 - \lambda_1$ is much less than $\lambda_1$ or $\lambda_2$, then to a good approximation

$$\frac{\Delta \lambda}{\lambda} = \frac{\lambda}{d}$$

The interferometer is the most precise means of measuring $\Delta \lambda$, the separation between these two lines. Measure the micrometer travel between points where the fringe pattern has a maximum fuzziness in its image. Better yet, measure the travel over several cycles from fuzzy to sharp and

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back to fuzz. The accepted values for the wavelengths of the sodium "D" lines are 589.0 and 589.6 nm. Remember that the actual mirror travel is 1/5 of the micrometer travel due to the lever arrangement.

**White light fringes**

Since white light has a broad band of frequencies, fringes are obtained only when the order of interference is close to zero. For a perfectly monochromatic wave, such as with laser light, fringes can be seen for any path difference. But when a finite band of wavelengths is present, as with white light, a sufficiently large path difference will result in a large range of phase differences for the various wavelengths. The maxima of one wavelength will fall on the minimum of another and the fringes disappear. However the fringes can be observed in the interferometer when the two arms are of equal optical length.

The condition of equal optical length is the one in which the optical path length in the mirror M₁ branch is exactly equal to that in the mirror M₂ branch. Eq. (1) is satisfied for all λ for this condition, consequently white light constructive interference occurs. This condition also requires that n = 0. It produces a fringe that appears yellow.

In order to find these conditions, start with the sodium lamp.

Move mirror M₂ in a direction that makes the circles collapse toward the center. (Why is this the direction to move? See question 1 at the end of the report.) As you approach zero path difference, the fringes will get coarser, and fewer will be seen in the field, until finally one fringe will fill the field. At this point you are very near zero path difference. Now go to the incandescent light and move the micrometer very slowly until a group of colored fringes is seen. Look sharp, because they only exist for path differences of the order of λ. Since white light consists of a band of wavelengths, each of the wavelengths will only be in phase near zero-path difference. Look for the central white fringe in the colored fringe pattern. (This fringe is actually yellow in appearance.)

This is the only fringe where all the wavelengths are in the same phase, so it corresponds to zero path difference. With this fringe set in the center of the field, the two path lengths are equal within a fraction of the wavelength of light. This is an example of the precision obtainable by these optical techniques. To get an idea of how precise it is, express the accuracy as a percentage of the total path length, i.e. the distance from the beam splitter to either mirror and back.

This procedure can be very time consuming because of the small number of colored fringes observable. Therefore you may not be able to locate the zero-path length condition before the end of the period.
Lab report guidelines for Michelson Interferometer Experiment

Please include the following items in your lab report. The report will be worth 10 points total, with 1 point for the prelab.

1. **Introduction**: Include a brief description of the experiment you have done and the measurements you report.

2. **Measurement of wavelength of laser light**: Explain (using the appropriate equation) why this measurement determines the wavelength of laser light. Summarize the numerical values from your measurements and show your calculation of the wavelength of the laser light. Make an estimate of the error in the measurement. Comment on the agreement with the accepted value.

3. **Measurement of the wavelength difference between the sodium D lines**: Explain (using the appropriate equation) why this measurement determines the wavelength difference. Summarize the numerical values from your measurements and show your calculation of the wavelength difference. Make an estimate of the error in the measurement. Comment on the agreement with the accepted wavelength difference.

4. **White light fringes** When looking for these white light fringes, why do you move the mirror M2 in the direction which causes the fringes to collapse toward the center? To get an idea of how precisely you positioned the mirror for this part, express the positioning precision as a percentage of the total path length, i.e. the distance from the beam splitter to either mirror and back.

5. **Conclusion**: Write a brief conclusion summarizing the major findings of your experiment.