Physics 3304
Assignment 2 solutions

Grading: The problems that will be graded in detail for this assignment are Ch. 2, number 35 and Ch. 3, number 9 (each worth 5 points total). You will receive 1 point for each of the other problems if you have made a reasonable attempt at a solution. (Total points for this assignment=18)

Ch. 2, Problem 25

a) Use the relativistic velocity transformation to determine the velocity \( v' \) as measured in the frame \( O' \):

\[
v' = \frac{v - u}{1 - \frac{uv}{c^2}}
\]

Then the energy becomes:

\[
E' = \gamma' mc^2 = \frac{mc^2}{\sqrt{1 - (v')^2/c^2}} = \frac{mc^2}{\sqrt{1 - \left[\frac{(v-u)/c}{1-w/c} \right]^2}}
\]

and the momentum is:

\[
p' = \gamma' mv' = \frac{mv'}{\sqrt{1 - (v')^2/c^2}} = \frac{m\left[\frac{(v-u)/c}{1-w/c} \right]}{\sqrt{1 - \left[\frac{(v-u)/c}{1-w/c} \right]^2}}
\]

b)

\[
E'^2 - p'^2c^2 = \frac{mc^4 - m^2c^2 \left[\frac{(v-u)/c}{1-w/c} \right]^{2}}{1 - \left[\frac{(v-u)/c}{1-w/c} \right]^2} = m^2c^4
\]

so the quantity \( E'^2 - p'^2c^2 \) is an invariant, meaning it has the same constant value (square of the particle’s rest energy) in any inertial reference frame. The rest mass \( m \) is sometimes referred to as the invariant mass.

Ch. 2, Problem 31

a) The first stage of acceleration takes the electron from rest \( (E_0 = mc^2 = 0.511 \text{ MeV}) \) up to the energy:

\[
E_1 = \gamma mc^2 = \frac{0.511 \text{ MeV}}{\sqrt{1 - (0.99)^2}} = 3.6 \text{ MeV}
\]

so the first stage adds \( 3.6 - 0.511 = 3.1 \text{ MeV} \) to the energy of the electron.

b) The second stage takes the electron up to an energy:

\[
E_2 = \gamma mc^2 = \frac{0.511 \text{ MeV}}{\sqrt{1 - (0.999)^2}} = 11.4 \text{ MeV}
\]

so the second stage adds \( 11.4 - 3.6 = 7.8 \text{ MeV} \) to the energy of the electron.
Ch. 2, Problem 32

After acceleration through 10 MV both the electron and proton will each have a kinetic energy of 10 MeV. For the electron,

\[ E = K + mc^2 = 10.5 \text{ MeV} \]
\[ p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2} = 10.5 \text{ MeV}/c \]

while the classical relation between kinetic energy and momentum gives:

\[ p = \sqrt{2mE} = \frac{1}{c}\sqrt{2mc^2K} = 3.20 \text{ MeV}/c \]

Since the electron is very relativistic \((pc >> mc^2)\) the classical expression fails badly. For the proton,

\[ E = K + mc^2 = 10.0 \text{ MeV} + 938.3 \text{ MeV} = 948.3 \text{ MeV} \]
\[ p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2} = 137 \text{ MeV}/c \]

while the classical relation between kinetic energy and momentum gives:

\[ p = \sqrt{2mE} = \frac{1}{c}\sqrt{2mc^2K} = 137 \text{ MeV}/c \]

Since the proton is non-relativistic \((pc << mc^2)\) the classical expression works well in this case.

Ch. 2, Problem 35

\[ E_p + E_\vec{\pi} = 9700 \text{ MeV} \]

Since the proton and antiproton have the same speed (and rest mass), their energies are equal:

\[ E_p = E_\vec{\pi} = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \]
\[ 1 - \frac{v^2}{c^2} = \left[ \frac{938 \text{ MeV}}{(9700 \text{ MeV}/2)} \right]^2 \text{ so} \]
\[ v = 0.981c \]

Ch. 2, Problem 36

Since the pion is moving at the speed 0.98c, its energy and momentum measured in the laboratory are:

\[ E_\pi = \gamma mc^2 = \frac{135 \text{ MeV}}{\sqrt{1 - (0.98)^2}} = 678 \text{ MeV} \]
\[ p_\pi = \frac{1}{c}\sqrt{E_\pi^2 - (mc^2)^2} = 664 \text{ MeV}/c \]
The problem says that the decay gamma rays have equal energies so they share the 678 MeV equally: \( E^\gamma = 339 \text{ MeV} \). The angle of the two \( \gamma \)-rays can be obtained from the conservation of momentum:

\[
p_x^+ = 664 \text{ MeV}/c = 2p_y^+
= 2p_\gamma \cos \theta = 2(E_\gamma/c) \cos \theta
\theta = 11.7^\circ
\]

Ch. 2, Problem 37

As in Example 2.16, the total energy of the kaon is 823 MeV. The speed of the kaon relative to the lab frame can be found from the expression for the energy:

\[
E = \gamma mc^2
823 \text{ MeV} = \gamma (498 \text{ MeV})
\gamma = 1.6526
v/c = 0.796
\]

This is also the transformation speed \( u \) that is required to transform from the lab frame to a frame where the initial kaon is at rest. When the decay of the kaon is observed in its rest frame, the two pions share the decay energy equally and they move in opposite directions with equal and opposite velocities. So we have:

\[
E_{\pi_1} = E_{\pi_2} = \frac{1}{2}m_Kc^2 = 249 \text{ MeV} = \gamma m_{\pi}c^2 = \frac{m_{\pi}c^2}{\sqrt{1 - v^2/c^2}}
\]

Solving gives that the speed of each of the pions is \( v = 0.827c \). Now we transform back to the lab frame by making a Lorentz velocity transformation using the speed \( u = 0.796c \) which is the relative velocity between the lab \( (O) \) and kaon rest \( (O') \) frames. We assume in the kaon rest frame one pion moves along the \(+x'\) axis with speed \( v_1' = +0.827c \) and the other moves along the \(-x'\) axis with speed \( v_2' = -0.827c \). So we then have for the speeds of the pions in the lab frame:

\[
v_1 = \frac{v_1' + u}{1 + uv_1'/c^2} = \frac{0.827c + 0.796c}{1 + (0.796)(0.827)} = 0.9787c
\gamma_1 = 4.873
v_2 = \frac{v_2' + u}{1 + uv_2'/c^2} = \frac{-0.827c + 0.796c}{1 + (0.796)(-0.827)} = -0.0907c
\gamma_2 = 1.004
\]

Then the kinetic energies of the pions in the lab frame are given by:

\[
K_1 = (\gamma_1 - 1)m_{\pi}c^2 = (4.873 - 1)(140 \text{ MeV}) = 542 \text{ MeV}
K_2 = (\gamma_2 - 1)m_{\pi}c^2 = (1.004 - 1)(140 \text{ MeV}) = 0.6 \text{ MeV}
\]

Ch. 3, Problem 3
\[ \lambda = \frac{2d \sin \theta}{n} = \frac{2(0.347 \text{ nm}) \sin 34.0^\circ}{1} = 0.388 \text{ nm} \]

b) The spacing between the planes oriented at 45° is:

\[ d \sin 45^\circ = (0.347 \text{ nm}) \frac{\sqrt{2}}{2} = 0.245 \text{ nm} \]

\[ \sin \theta = \frac{\lambda}{2d} = \frac{0.388 \text{ nm}}{2(0.245 \text{ nm})} = 0.791 \text{ or } \theta = 52.2^\circ \]

This is the angle measured with respect to the crystal planes. Referring to Figure 3.6 in the book, the angle of incidence measure with respect to the crystal surface is \( \theta - 45^\circ = 7.2^\circ \), while the emerging beam makes an angle of \( \theta + 45^\circ = 97.2^\circ \) with respect to the surface (measured from the opposite side of the surface).

**Ch. 3, Problem 6**

a) \[ \lambda = \frac{hc}{E} = \frac{1240 \text{ eV-nm}}{1.00 \times 10^4 \text{ eV}} = 0.124 \text{ nm} \]

b) \[ \lambda = \frac{1240 \text{ eV-nm}}{1.00 \times 10^6 \text{ eV}} = 1.24 \times 10^{-3} \text{ nm} \]

c) \[ 350 \text{ nm : } E = \frac{hc}{\lambda} = \frac{1240 \text{ eV-nm}}{350 \text{ nm}} = 3.5 \text{ eV} \]

\[ 700 \text{ nm : } E = \frac{hc}{\lambda} = \frac{1240 \text{ eV-nm}}{700 \text{ nm}} = 1.8 \text{ eV} \]

so the range is 1.8 eV to 3.5 eV.

**Ch. 3, Problem 9**

The stopping potential (\( V_s \)) is related to the work function and wavelength through the relation:

\[ eV_s = K_{max} = \frac{hc}{\lambda} - \phi \text{ so} \]

\[ 0.65 \text{ eV} = \frac{hc}{420 \text{ nm}} - \phi \]

\[ 1.69 \text{ eV} = \frac{hc}{310 \text{ nm}} - \phi \]
Subtracting the two equations allows us to solve for Planck’s constant, \( h \), using only the data given:

\[
1.04 \text{ eV} = hc \left( \frac{1}{310 \text{ nm}} - \frac{1}{420 \text{ nm}} \right)
\]

yielding \( h = 4.10 \times 10^{-15} \text{ eV-s} \)

Now either of the above two equations can be used to obtain the work function \( \phi \):

\[
\phi = \frac{hc}{310 \text{ nm}} - 1.69 \text{ eV} = 2.28 \text{ eV}
\]

Ch. 3, Problem 13

a)

\[
\phi = 4.31 \text{ eV for zinc}
\]

The largest wavelength corresponds to the minimum energy photon required to extract a photoelectron. That minimum energy is simply the work function \( \phi \).

\[
E_{\text{min}}^\gamma = \phi
\]

\[
\lambda_{\text{max}} = \frac{hc}{E_{\text{min}}^\gamma} = \frac{1240 \text{ eV-nm}}{4.31 \text{ eV}} = 288 \text{ nm}
\]

b)

\[
eV_s = K_{\text{max}} = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV-nm}}{220.0 \text{ nm}} - 4.31 \text{ eV} = 1.33 \text{ eV}
\]

\[
V_s = 1.33 \text{ volts}
\]