Ph y s i c s 3 3 0 4  
F a l l  1 9 9 7  
S e c o n d E x a m  
N o v e m b e r  1 3 ,  1 9 9 7 

N a m e:/ ____________________________________________

S o c i a l S e c u r i t y #: ________________________________

1. W r i t e y o u r n a m e a n d s o c i a l s e c u r i t y n u m b e r o n t h i s p a g e .

2. T h e p o i n t v a l u e s f o r e a c h p r o b l e m a r e i n d i c a t e d o n t h e e x a m . T h e t o t a l i s 1 0 0 p o i n t s .

3. T h e e x a m w i l l b e h a n d - g r a d e d f o r p a r t i a l c r e d i t , s o s h o w a l l y o u r w o r k .

4. D u r i n g t h e e x a m y o u c a n u s e a c a l c u l a t o r a n d a s h e e t ( 8 . 5 x 1 1 i n c h e s ) w i t h a n y i n f o r m a t i o n y o u w a n t w r i t t e n o n b o t h s i d e s .

5. Y o u w i l l h a v e 7 5 m i n u t e s t o w o r k o n t h e e x a m .

6. B y w r i t i n g y o u r n a m e o n t h i s e x a m , y o u p l e d g e t h a t y o u h a v e u p h e l d t h e H o n o r C o d e a n d h a v e a r r i v e d a t e a c h a n s w e r s o l e l y b y y o u r o w n w o r k .

S o m e p o t e n t i a l l y u s e f u l i n f o r m a t i o n :

\[
\begin{align*}
\hbar c &= 1 2 4 0 \text{ eV nm} = 1 2 4 0 \text{ MeV fm} \\
\hbar c &= 1 9 7 \text{ eV nm} = 1 9 7 \text{ MeV fm} \\
c &= 2 . 9 9 8 \times 1 0 ^ { 8 } \text{ m/s} \\
k &= 8 . 6 1 7 \times 1 0 ^ { - 5 } \text{ eV/K} \\
h &= 4 . 1 3 6 \times 1 0 ^ { - 1 5 } \text{ eV s} \\
e l e c t r o n m a s s &= m _ { e } = 5 . 4 9 \times 1 0 ^ { - 4 } \text{ u} = 0 . 5 1 1 \text{ MeV/c}^2 \\
proton m a s s &= M _ { p } = 1 . 0 0 7 2 7 6 \text{ u} = 9 3 8 . 3 \text{ MeV/c}^2 \\
neutron m a s s &= M _ { n } = 1 . 0 0 8 6 6 5 \text{ u} = 9 3 9 . 6 \text{ MeV/c}^2 \\
\text{mass of } ^ { 3 } \text{He atom} &= 3 . 0 1 6 \text{ u} \\
\text{mass of } ^ { 2 3 8 } \text{U atom} &= 2 3 8 . 1 \text{ u} \\
1 \text{ atomic unit} &= 1 \text{ u} = 9 3 1 . 5 \text{ MeV/c}^2 = 1 . 6 6 \times 1 0 ^ { - 2 7 } \text{ kg} \\
\frac { e ^ { 2 } } { 4 \pi e _ { 0 } } &= 1 . 4 4 0 \text{ eV nm} \\
G &= 6 . 6 7 \times 1 0 ^ { - 1 1 } \text{ Nm}^2/\text{kg}^2 \\
1 \text{ eV} &= 1 . 6 0 2 \times 1 0 ^ { - 1 9 } \text{ J} \\
G M _ { p } m _ { e } &= 6 . 3 4 \times 1 0 ^ { - 4 0 } \text{ eV nm} \\
1 \text{ femto} &= 1 \text{ f} = 1 0 ^ { - 1 5 } \\
1 \text{ pico} &= 1 \text{ p} = 1 0 ^ { - 1 2 } \\
1 \text{ nano} &= 1 \text{ n} = 1 0 ^ { - 9 } \\
1 \text{ micro} &= 1 \mu = 1 0 ^ { - 6 }
\end{align*}
\]
Some hydrogen atom wavefunctions:

\[
\begin{array}{cccccc}
 n & l & m_l & R(r) & \Theta(\theta) & \Phi(\phi) \\
1 & 0 & 0 & \frac{2}{a_0^{3/2}} e^{-r/a_0} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2\pi}} \\
2 & 0 & 0 & \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2\pi}} \\
2 & 1 & 0 & \frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0} & \sqrt{\frac{3}{2}} \cos \theta & \frac{1}{\sqrt{2\pi}} \\
2 & 1 & \pm 1 & \frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0} & \sqrt{\frac{3}{2}} \sin \theta & \frac{1}{\sqrt{2\pi}} e^{\pm i\phi}
\end{array}
\]

Some potentially useful indefinite integrals:

\[
\int x^4 e^{-x} \, dx = (-x^4 - 4x^3 - 12x^2 - 24x - 24) e^{-x}
\]
\[
\int x^3 e^{-x} \, dx = (-x^3 - 3x^2 - 6x - 6) e^{-x}
\]
\[
\int x^2 e^{-x} \, dx = (-x^2 - 2x - 2) e^{-x}
\]
\[
\int x e^{-x} \, dx = (-x - 1) e^{-x}
\]
1. (8 points) A cobalt ($Z = 27$) target is bombarded with electrons and the wavelengths of its characteristic X-ray spectrum are measured. A second, fainter, characteristic spectrum is also found, due to an impurity in the target. The wavelengths of the $K_\alpha$ lines are .1793 nm (for the cobalt) and .1441 nm (for the impurity). What is the atomic number ($Z$) of the impurity?

2. Determine the ground state quantum numbers ($L$ and $S$) for the following elements:

   (a) (6 points) Tb: [Xe]$4f^96s^2$
(b) (6 points) Np: [Rn]5f⁴6d¹7s²

3. (a) (9 points) What is the probability for the electron to be further than 6a₀ from the nucleus in the ground state of the hydrogen atom?
(b) (3 points) How would your answer to part a) change if we were instead talking about the electron in a singly-ionized helium atom (He⁺)?

4. (9 points) Assume that we live in a world where there is no electric charge. In that world, the electron and proton in the hydrogen atom would be held together by the gravitational attraction between the two masses. What would the ground state energy of the hydrogen atom be in such a world? Recall that the gravitational potential energy can be written as

$$ U = -\frac{Gm_e M_p}{r} $$

where $G$ is Newton's gravitational constant, and $m_e, M_p$ are the masses of the electron and proton, respectively.
5. (8 points) In scattering of light particles from a heavy nucleus at rest, we can get the closest to the nucleus when the impact parameter $b$ is zero. Suppose the radius of a gold nucleus ($Z = 79$) is 7.0 fm; what is the necessary energy for an incident proton to just reach the surface of the nucleus?

6. Assume that a hydrogen atom in its ground state absorbs a 13.06 eV photon.

(a) (7 points) After absorption, in which excited state will it be in (specify both the $n$ and $l$ value)? Note that the selection rules for absorption of photons are the same as for emission.
(b) (7 points) From the excited state in part a) the hydrogen atom makes a series of electromagnetic transitions, ending in the ground state. Make an energy level diagram and indicate all the allowed transition paths leading from the excited state to the ground state.

7. (a) (8 points) An electron is in the \( n = 7 \) excited level of the 25-times ionized iron \((Z = 26)\) ion, \( \text{Fe}^{25+} \). Among all the decay possibilities of this state, what are the two shortest wavelength photons that will be emitted?
(b) (7 points) For the longest wavelength photon of the two photons in a) (if you didn’t get a number just leave it as a variable for this part), what is the maximum $Z$ “hydrogen-like” ion (nucleus of charge $Ze + a$ single electron) which can be ionized by that photon?

8. Assume we have an ideal gas of “hydrogen-like” uranium ($U^{91+}$), i.e. a uranium nucleus ($Z = 92$) with all electrons but 1 stripped off.

(a) (8 points) Assume that all the atoms are either in the ground state or the first excited state. At what temperature would we expect to find 5% of the atoms in the first excited state? You can assume that classical statistics is valid here.
(b) (6 points) Justify the assumption in part a) that classical statistics are valid for this problem. Assume that the average spacing between the atoms in the gas is 3 nm, and assume that the average kinetic energy of the atoms in the gas is given by \( \frac{3}{2} kT \), where \( k \) is the Boltzmann constant and \( T \) is the temperature determined in part a).

9. (8 points) Find the Fermi energy and the average energy of the protons in an \(^{37}\text{Ar}\) nucleus (\( Z = 18 \)). The radius of the \(^{37}\text{Ar}\) nucleus is 4.3 fm.