Monopoles, scattering, generalized symmetries.

Diego Delmastro

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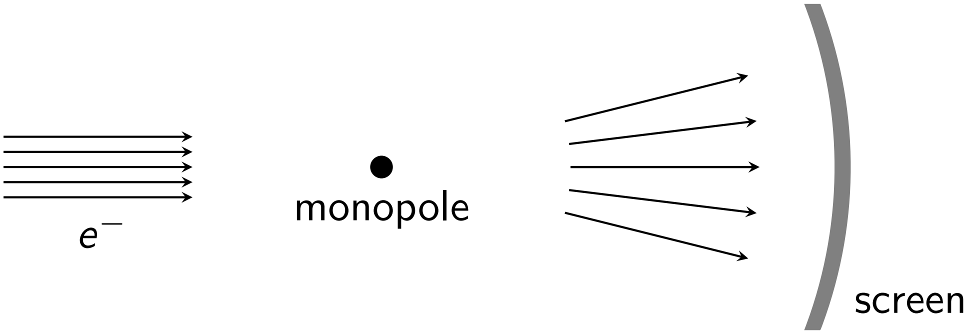
arXiv:2306.07318 with M. van Beest, P. Boyle Smith, Z. Komargodski, D. Tong.
arXiv:2312.17746 with M. van Beest, P. Boyle Smith, R. Mouland, D. Tong.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Background.

Background.

- Main motivation: recently understood symmetries of gauge theories.

- Massless QED in $4d$: gauge group $U(1)$ plus N_f Dirac fermions.
  - 40's: $G = SU(N_f)_L \times SU(N_f)_R \times U(1)_A$.
  - 60's, triangle diagrams: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$ and $U(1)_A \to \varnothing$. [Adler–Bell–Jackiw]
  - '14, higher-form: $U(1)^{(1)}$ acting on 't Hooft lines. [Gaiotto–Kapustin–Seiberg–Willett]
  - '18, higher groups: $U(1)^{(1)} \hookrightarrow G \twoheadrightarrow SU(N_f)_L \times SU(N_f)_R$ is not broken. [Córdova–Dumitrescu]
  - '22, higher-categorical: $U(1)_A$ is not broken, but acts (non-invertibly) on 't Hooft lines. [Choi–Lam–Shao, Córdova–Ohmori]

- In regular scattering theory, with only electrically charged particles, the generalized symmetries behave like regular symmetries.

- In monopole scattering, these new features play an essential role.

Background.

- Classical scattering of electric particles off magnetic targets:

$$V(r) = \frac{\ell(\ell+1) - (mq/2)^2 - mq\,\hat{\boldsymbol{r}}\cdot\boldsymbol{S}}{r^2}$$

[Aharony–Cuomo–Komargodski–Mezei–Raviv-Moshe]

- Same as electric-electric scattering, except:
  - Angular momentum is bounded $\ell \geq \frac{1}{2}|mq|$, since $\boldsymbol{L} = \boldsymbol{L}_{\text{orb}} + \boldsymbol{L}_{\text{EM}}$ with $|\boldsymbol{L}_{\text{EM}}| \sim \frac{1}{2}|\boldsymbol{B}\times\boldsymbol{E}| \sim \frac{1}{2}|mq|$.
  - Electric-magnetic interaction decays like $1/r^2$ instead of $1/r$.

Background.

- Scalars: $\hat{\boldsymbol{r}} \cdot \boldsymbol{S} = 0$. Then, $V(r) \sim +1/r^2$, repulsive.
- Fermions: $\hat{\boldsymbol{r}} \cdot \boldsymbol{S} = \pm 1/2$. Same, $V(r) \sim +1/r^2$, except for helicity-polarized s-wave:

$$\ell = \frac{1}{2}|qm| \text{ and } \operatorname{sign}(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}) = \operatorname{sign}(mq) \qquad \Rightarrow \qquad V(r) \equiv 0$$

- Conclusion: electric-magnetic scattering is boring, except for helicity-polarized, spherically symmetric fermions. These experience no classical force. Quite sensitive to quantum effects and UV physics.

Background.

- Callan and Rubakov studied the quantum dynamics of such fermions.
- Main conclusion: protons decay at $\Gamma \sim 1/\Lambda_{\text{QCD}}$.
- What do they decay into? Private letter from Witten to Callan: if you follow their math, it seems that $p + M \longrightarrow M + \frac{1}{2}\pi^+ + \frac{1}{2}e^+$.

$$e^+_L + M \to M + \tfrac{1}{2}e^+_R + \tfrac{1}{2}u_{1_R} + \tfrac{1}{2}u_{2_R} + \tfrac{1}{2}d_{3_L}$$ [Callan]

$$u_{1R} + M \to \tfrac{1}{2}(u_{1L}\bar{u}_{2R}\bar{d}_{3L}e^+_L) + M,$$ [Peskin]

- Generalized symmetries resolve the issue.

A simple toy model.

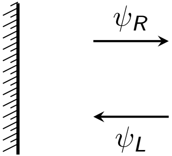
A simple toy model.

- Consider a bunch of free fermions in 1+1 dimensions. The Dirac equation reads

$$(\partial_t + \gamma^\star \partial_x)\psi = 0 \qquad \Rightarrow \qquad \begin{array}{c} \psi_L = \psi_L(t-x) \\ \psi_R = \psi_R(t+x) \end{array}$$

  Left-handed particles move to the left, and right-handed particles to the right.

- Symmetry: $O(N)_L \times O(N)_R$.
- Let us put the system on the half line, with some boundary condition at $x = 0$



A simple toy model.

- If we send ψ_L towards the boundary, it will bounce off and become some excitation of ψ_R

$$\psi_L \longrightarrow \mathcal{O}(\psi_R)$$

whose details depend on the choice of boundary condition.

- Naive puzzle: such scattering processes seem incompatible with $O(N)_L \times O(N)_R$ conservation. The in-state is charged under $O(N)_L$ but not $O(N)_R$, and the other way around for the out-state. It is impossible to write an operator $\mathcal{O}(\psi_R)$ that has the same quantum numbers as ψ_L.

- The resolution is straightforward: the symmetry $O(N)_L \times O(N)_R$ has an 't Hooft anomaly, hence there are no symmetric boundary conditions. The boundary explicitly breaks this symmetry.

A simple toy model.

- Sketch: the currents that generate $O(N)_L \times O(R)_R$ are

$$j_L = \psi_L {\psi_L}^\dagger, \qquad j_R = \psi_R {\psi_R}^\dagger$$

- These are composites. Because of short-distance divergences, $j_L' = \delta'(x-t)$ and $j_R' = \delta'(x+t)$.

- Roughly speaking, the contact term makes the boundary charged under the symmetry:

$$Q_L([0,\epsilon]) = \int_0^\epsilon j_L \mathrm{d}x = \int_0^\epsilon x j_L' \mathrm{d}x \equiv \frac{1}{2}$$

- See [Thorngren-Wang] for the proper proof.

A simple toy model.

- A more subtle puzzle: consider the subgroup $U(1) \subset O(N)_L \times O(N)_R$ with charges

$$L: \ \psi_3, \ \psi_4, \qquad R: \ \psi_5$$

- This subgroup is anomaly-free:

$$j' = (3^2 + 4^2 - 5^2)\delta' \equiv 0$$

- As such, it does admit symmetric boundary conditions. It is possible to conserve this symmetry in scattering processes.

A simple toy model.

- This choice of boundary condition leads to an apparent paradox. Let us throw ψ_3 towards the boundary. Then,
  - Energy is conserved, so something has to be reflected back.
  - $U(1)$ is conserved, so the out-going state must have charge 3.
  - But the only right-moving particle ψ_5 has charge 5!
- Naively, the only consistent scattering process is

$$\psi_3 \longrightarrow \frac{3}{5}\psi_5$$

  Conservation laws seem to ask for a fractional out-state!

- There is no operator in the right-moving Fock space with charges 3 or 4,

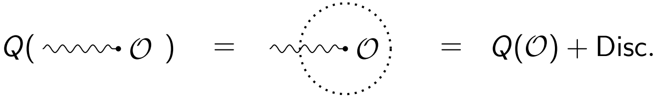
$$Q(\psi_5 \cdots \psi_5) \propto 5$$

A simple toy model.

- Resolution: the spectrum of local excitations is much larger than the Fock space of ψ_L, ψ_R.
- In string theory, these additional states are known as the twisted sector, and their defining property is that they are multi-valued (their correlation functions have branch cuts):

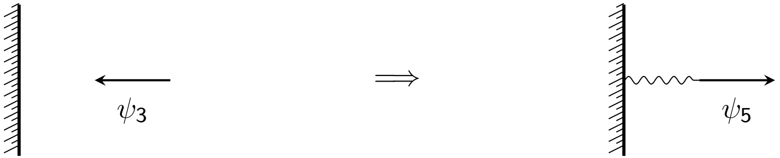
$$\mathcal{O}(e^{2\pi i} z) = e^{2\pi i \eta} \mathcal{O}(z)$$

- The branch cut adds charge to the endpoint:



A simple toy model.

- One can show that there is a unique twist operator with the same charge as ψ_3, namely ψ_5 with twist $\eta = 1/5$.
- The scattering process then looks like this:



- The S-matrix is somewhat non-standard: it turns regular (local) operators into twist fields:

$$S : \mathcal{H} \to \mathcal{H}_{1/5}$$

- This is fine: twist fields behave, for the most part, like regular fields, the only difference are extra phases as we move them around each other.

Monopole scattering.

Monopole scattering.

- Consider N_f Dirac fermions in $3+1d$. The symmetries are

$$(SU(N_f)_L \times SU(N_f)_R \times U(1)_A) \ltimes U(1)^{(1)}_m$$

- A Dirac fermion has two chiral components, e_L, e_R, whose charges under the gauge group and symmetry group are

	e_L	e_R
$U(1)_{\text{EM}}$	1	-1
$SU(N_f)_L$	$\square$	$\bullet$
$SU(N_f)_R$	$\bullet$	$\square$
$U(1)_A$	1	1

Monopole scattering.

- Let us take a heavy monopole and place it at the origin. We send a lepton, either e_L or e_R, and measure the outcome. The scattering process is

$$\psi + M \longrightarrow M + \mathcal{O}$$

where $\mathcal{O}$ is some operator with the same charges as ψ. Our task is to identify this operator.

- The Dirac equation reads

$$(i\not{\partial} + \not{A})\psi = 0, \qquad A_\phi = \frac{m}{r}(1 - \cos\theta)$$

where $m \in \mathbb{Z}$ is the magnetic charge of the monopole.

Monopole scattering.

- As reviewed in the introduction, the helicity-polarized *s*-wave is special.
- In QFT language

$$(\partial_t + \gamma^\star \partial_r) \int_{S^2} \psi \equiv 0$$

- Hence, e_L describes incoming radiation and e_R describes outgoing radiation.

Monopole scattering.

- The s-wave carries the following quantum numbers:

		$U(1)_{\text{EM}}$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_A$
Incoming:	$\int_{S^2} e_L$	1	$\square$	$\bullet$	1
Outgoing:	$\int_{S^2} e_R$	-1	$\bullet$	$\square$	1

- Formally identical to our toy model: we have perturbations that move in a single direction, but they carry different quantum numbers. The monopole plays the role of the boundary.
- Here we face out first puzzle. The incoming wave is charged under $SU(N_f)_L$, but the outgoing one is not, so the out-state will never conserve $SU(N_f)_L \times SU(N_f)_R$!

Monopole scattering.

- Resolution: the symmetry $SU(N_f)_L \times SU(N_f)_R$ is in a 2-group with $U(1)^{(1)}$.
- This implies that the symmetry is not conserved in scattering processes involving magnetically charged matter. The monopole explicitly breaks this symmetry.
- Only the anomaly-free subgroup $SU(N_F)_V$ is conserved. So we should look at

		$U(1)_{\text{EM}}$	$SU(N_f)_V$	$U(1)_A$
Incoming:	$\int_{S^2} e_L$	1	$\square$	1
Outgoing:	$\int_{S^2} e_R$	-1	$\square$	1

Monopole scattering.

- It is still unclear what the correct out-state is:

$$M + e_L \longrightarrow M + e_R \qquad U(1)_{\mathrm{EM}} \quad SU(N_f)_V \quad U(1)_A$$
$$M + e_L \longrightarrow M + {e_R}^\dagger \qquad U(1)_{\mathrm{EM}} \quad SU(N_f)_V \quad U(1)_A$$

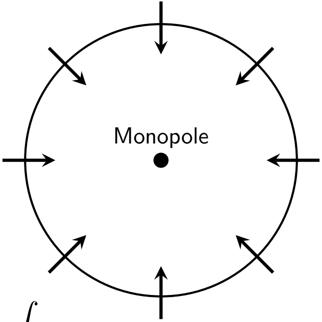
- No operator

$$\mathcal{O} = e_R \cdots e_R {e_R}^\dagger \cdots {e_R}^\dagger$$

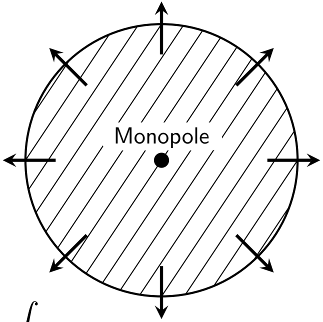
has the required quantum numbers under the three symmetries at the same time.

Monopole scattering.

- Exact same puzzle as in the toy model. People in the 80s proposed fractional out-states.
- Our claim: the out-state is a twist field. We propose an out-state of the form



$$\int_{S^2} e_L$$



$$\int_{S^2} e_R$$

Monopole scattering.

- In the interior of S^2 we place a $3d$ branch cut that implements a rotation

$$e_L \mapsto e^{2\pi i/mN_f} e_L, \qquad e_R \mapsto e^{2\pi i/mN_f} e_R$$

- The defect is just an axial rotation by an angle $1/mN_f$. This defect is generically non-invertible.

- In the paper we give two arguments for this: 1) we compute the charge carried by Wilson lines in the $3d$ Hall state TQFT and 2) we reduce on S^2 to yield a $2d$ problem very similar to the toy model from before.

Monopole scattering.

- The take-home-message is: monopole scattering requires the full machinery of generalized symmetries.
- Without these new symmetries there is an apparent paradox in which there is no possible out-state consistent with the conservation laws.
- If we take into account the full set of symmetries, a consistent answer does exist, albeit a rather non-trivial one: the S-matrix maps the regular Fock space into a twisted Fock space.
- In other words, incoming radiation formed by regular leptons becomes outgoing radiation formed by a field in a twisted sector, and there is a topological defect trailing it.
- This defect is non-invertible and hosts a $3d$ TQFT inside (the Hall state).

Standard Model.

Standard Model.

- The gauge group is $SU(3) \times SU(2) \times U(1)$. Maximal torus $U(1)^4$ so $\mathbb{Z}^4$ classification of monopoles.

- Because of $W^{\pm}$-condensation, most of these are unstable:

$$V(r) = \frac{\ell(\ell+1) - (mq/2)^2 - mq\,\hat{\boldsymbol{r}} \cdot \boldsymbol{S}}{r^2} \leq 0$$

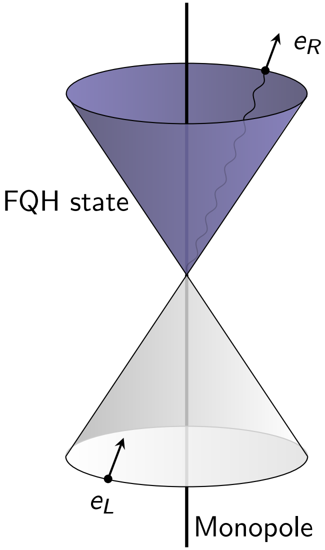
- Leptons are massive, and quarks are confined. Neglect this by working at $E \gg 1$ TeV.

- Modulo this, same setting.

Standard Model.

- Minimal stable monopole $m = 1$ under $U(1)_{\text{EM}}$. Out-state is twisted by a gauge symmetry instead of a flavor one. Also, poorly understood story involving zero-modes.
- Higher-charge monopoles $m > 1$: the outcome is not fixed by symmetry alone, it depends on the UV completion.
- A preliminary argument indicates that no twist operator has the correct quantum numbers so the answer must involve something entirely new. Back to the drawing board...

Monopole scattering.



Thanks!